Developing Mathematical Knowledge and Skills through the Awareness Approach of Teaching and Learning

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Abstract
Every object we think of or encounter, whether a natural or human-made, has a regular or irregular shape. In its own intrinsic conceptual design, it has elements of mathematics, science, engineering, and arts, etc., which are part of the object's geometric shape, form and structure. Geometry is not only an important part of mathematics, but it is also an important part of daily life. However, geometry is challenging for some students, even high-achieving students. One way to help students understand geometry and its relevance in life is to engage students to discover them cognitively, then to research and identify them in real world examples and then to relate them to past, present, and future innovations that improved our way of thinking about ourselves and the world around us. This interdisciplinary activity uses the Developmental Awareness Approach of Teaching and Learning (DAATL) to help students discover principles, acquire knowledge, and learn mathematical concepts including surface area, volume, dimensions, regular and irregular plane figures, solid polygons (regular polygons and polyhedra), thinking design, and graph making, etc. It is designed to help students become acquainted with the most useful and familiar parts of mathematical geometry and its application in daily life through connections with disciplines such as science, engineering, art, design, and social studies. The Development Awareness Approach of Teaching and Learning (DAATL) capitalizes on student's natural curiosity, inclination to comprehend as well as students love of drawing, doodling, painting, thinking and talking. Throughout the learning process, students are engaged in authentic learning activities by real and concrete doing with clear purposes, thinking analytically, and evaluating their understanding of texts and ideas orally, in drawing, and in writing. This approach of teaching and learning has been tried and modified to ensure maximum effectiveness of acquiring understanding of the intended learning concepts. The activities can be used with students in elementary school up to 2-year college levels.
Keywords: Geometry, Learning Math, Developmental Discovery Approach, Active Learning, Student's Active Engagement.

1. Introduction

This interdisciplinary activity is designed to help students discover and understand a range of mathematical concepts and skills capitalizing on their own personal experiences. It uses the Developmental Awareness Approach of Teaching and Learning (DAATL) to help students discover principles, acquire knowledge, and learn mathematical concepts including surface area, volume, dimensions, regular and irregular plane figures, solid polygons (regular polygons and polyhedra), thinking design, and graph making, etc. It is designed to help students become acquainted with the most useful and familiar parts of mathematical geometry and its application in daily life through connections with disciplines such as science, engineering, art, design, and social studies.

The Development Awareness Approach of Teaching and Learning (DAATL) capitalizes on student's natural curiosity, inclination to comprehend as well as students love of drawing, doodling, painting, thinking and talking. This is simply because "learners who can make connections between the subject matter and their own experiences struggle less with paying attention, making connections, completing tasks, taking tests, [and applying what they learn in different situations]" (Boyles and Contadino, 1998, p. 2). The DAATL allows students to make the connections and also lends itself naturally to combine with other subject areas such as science, engineering, language arts, the fine arts, technology, and social issues. This approach of teaching and

learning has been tried and modified to ensure maximum effectiveness of acquiring understanding of the intended learning concepts. The activities can be used with students in elementary school up to 2-year college levels. The approach also emphasizes the importance of shared vocabulary in the development of effective communication and the importance of writing in the learning process. In short, throughout the learning process, students are engaged in authentic learning activities by real and concrete doing with clear purposes, thinking analytically, and evaluating their understanding of texts and ideas orally, in drawing, and in writing.

Every object we think of or encounter, whether a natural or human-made, has a regular or irregular shape. In its own intrinsic conceptual design, it has elements of mathematics, science, engineering, and arts, etc., which are part of the object's geometric shape, form and structure.

Geometry is not only an important part of mathematics, but it is also an important part of daily life. Furthermore, as James S. Tanton (2016b) indicated, its core skills of logic and reasoning are critical to success in school, work, and many other aspect of life. However, geometry is challenging for some students, even high-achieving students. One way to help students understand geometry and its relevance in life is to engage students to discover them cognitively, then to research and identify them in real world examples and then to relate them to past, present, and future innovations that improved our way of thinking about ourselves and the world around us. For example, geometric shapes are part of nature and can be seen in snowflakes, spider webs, diffraction ranges in water, diatoms (aquatic organisms), sunflowers, rainbows, many forms of minerals (salt and diamonds), as well as in buildings, parks, and gardens, to name a few. Furthermore, most of what humans design from pharmaceutical tablets and capsules to bicycles, cars and airplanes, to name a few, are determined by a mathematical formula involving geometry (Slater and Tobey, 1991).

By the end of successfully completing the planned activities, students will be able to define and describe regular and irregular plane figures as well as solid shapes and objects. They will also be able to verbalize what distinguishes one plane figure from another, and one solid object from another, and also be able to describe their characteristics using sides, corners, and angles for those figures which have them. They will also be able to distinguish between circles and Triangles, rectangles and squares, etc. and what similarities and differences they have. But most of all, they students will build confidence in their cognitive abilities by applying what they have learned from identifying the plane figures and solid shapes around them to the world in which they live.

They will understand how to use the sides, corners, angles, size, and surface areas as attributes for understanding and identifying plane figures and solid objects. For example, a side is a straight line that makes part of the shape and a corner is where two sides meet. An angle is the space or the amount of turn (usually measured in degrees) between two intersecting lines or surfaces at or close to the end point where they meet (the vertex). An area is defined as the number of square units that covers a closed figure, and the surface area of an object is calculated as the total area of the surface of a three-dimensional (3-D) object. The volume of an object, which is also known as capacity, is the amount of 3-dimensional space that a given object occupies. Shared vocabulary and meaning of things among students are essential for effective communication and meaningful learning.

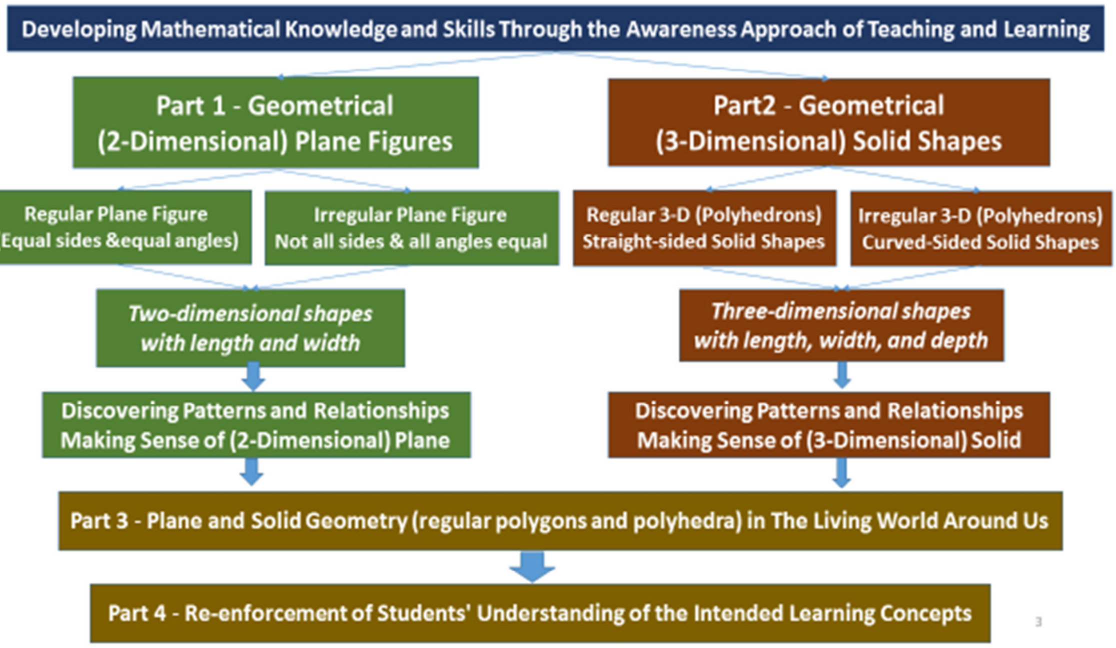
2. Discovery-Guided Learning Activities: Using the Development Awareness Approach of Teaching and Learning

As seen in Figure 1 and Table 1, this set of related learning activities is divided to four main parts. Each part is divided into a number of stages.

- Part one is called Developing an Awareness of 2-Dimentioanl Plane Figures.
- Part two is called Developing an Awareness of 3-Dimensional Solid Objects.
- Part three deals with discovering regular and solid geometry (regular polygons and polyhedra) in the living world around us.
- Part four deals with the assessment and the re-enforcement of students' understanding of the intended learning concepts both in the classroom and in daily life.

Journal of Education and Practice
ISSN 2222-1735 (Paper) ISSN 2222-288X (Online)
Vol.8, No.13, 2017

Figure 1
Concept Map for Developing Geometrical Knowledge and Skills
Through the Awareness Approach of Teaching



The activities and the exercises are purposely designed to start from simple to more challenging ones building on each other's gained information and learned knowledge, skills, and understanding. Figure 1 describes the four parts depicted in the concept map and the pedagogical relationship between them.

Table 1
Parts and Their Stages of the Learning Activities For Developing Geometrical Knowledge and Skills Through the Awareness Approach of Teaching

Part of the Learning Activities		**Stages of the Learning Activities**	
One	Developing an Awareness of 2-Dimensional Plane Figures	**I**	Developing an Awareness of Plane Figures (**regular polygons)** Around You.
		II	Knowing Your Plane Figures (**regular polygons)** Better.
		II	Discovering Patterns and Relationships Among Plane Figures (**regular polygons).**
Two	Developing an Awareness of 3-Dimensional Solid Objects	**I**	Developing an Awareness of 3-dimensional shapes **(solid polygon known as polyhedra)** Around You.
		II	Knowing Your 3-dimensional shapes Figures **(solid polygon known as polyhedra)** Better.
		III	Discovering Patterns and Relationships Among 3-dimensional shapes **(solid polygon known as polyhedra).**
Three	Discovering regular and solid geometry (regular **polygons and polyhedra**) in the living world around us.	**I**	Developing Learning Skills of Constructing and Calculating Plane and Solid Diagrams.
		II	Regular and solid geometry (regular **polygons and polyhedra**) in The Living World Around Us.
Four	Assessment and the re-enforcement of students' understanding of the intended learning concepts both in the classroom and in daily life.	**I**	It's Time For Geometry, Art & Fun
		II	General Questions for the Re-enforcement of Students' Understanding of the Intended Learning Concepts.

3. Part One: Developing an Awareness of 2-Dimensional Plane Figures Around You

Part one is divided into three stages of related learning activities.

3.1. Part One – Stage I: Developing an Awareness of Plane Figures Around You

In this stage of the activity, students will work mainly with regular plane figures. A plane figure is defined and described as a flat, closed figure or shape. It is a 2-dimensional shape with width and breadth, but no thickness.

It can be made of straight lines, curved lines, or both straight and curved lines of a closed figure. On the other hand, a given plane figure with no sides or curves that are equal is called an irregular polygon or an irregular plane figure. Students will first construct a given closed plane figure using bending drinking straws or popsicle sticks, then draw and name the given figure as instructed by the procedures and/or their teacher. In the process, students develop shared vocabulary, skills, and understanding that is needed for meaningful learning.

Procedures: In this stage of the learning activity, students will first construct a given closed plane figure using bending drinking straws or popsicle sticks, then draw and name the given figure as instructed below. (Note that the examples that are provided in bold and between two parentheses are for teachers and not for students. Also the students need to be divided in a number of groups equal to the number of closed planes figures that will be explored. In the following procedure ten closed plane figures are explored, therefore at least, ten groups of students are required.)

Using bending drinking straws (or popsicle sticks), then one of the 8 1/2 X 11 inch papers (e.g. white paper) ask each member of:

1. Group one to first construct (using bending straws), and then draw and name a plane figure formed from ***three*** equal line segments (sides) so that each pair of two line segments shares an endpoint. (e.g., **triangle**).
2. Group two to first construct (using bending straws), and then draw and name a plane figure formed from ***four*** equal line segments (sides) so that each pair of two line segments share an endpoint. (e.g., **quadrilateral**).
3. Group three to first construct (using bending straws), and then draw and name a plane figure formed from ***five*** equal line segments (sides) so that each pair of two line segments share an endpoint. (e.g., **pentagon**).
4. Group four to first construct (using bending straws), and then draw and name a plane figure formed from ***six*** equal line segments (sides) so that each pair of two line segments share an endpoint. (e.g., **hexagon**).
5. Group five to first construct (using bending straws), and then draw and name a plane figure formed from ***seven*** equal line segments (sides) so that each pair of two line segments share an endpoint. (e.g., **heptagon**).
6. Group six to first construct (using bending straws), and then draw and name a plane figure formed from ***eight*** equal line segments (sides) so that each pair of two line segments share an endpoint. (e.g., **octagon**).
7. Group seven to first construct (using bending straws), and then draw and name a plane figure formed from ***nine*** equal line segments (sides) so that each pair of two line segments share an endpoint. (e.g., **nonagon**).
8. Group eight to first construct (using bending straws), and then draw and name a plane figure formed from *ten* equal line segments (sides) so that each pair of two line segments share an endpoint. (e.g., **decagon**).
9. Group nine to first construct (using bending straws), and then draw and name a plane figure formed from *more equal line segments* (sides) *than the one mentioned above*, so that each pair of two line segments share an endpoint (e.g., **polygon**).
10. Group ten to first construct (using bending straws), and then draw and name a plane figure that is the *set of points equidistant from a given point*. (e.g., **circle**).

Give all the students extra 10-15 minutes to finish any kind of final touch they would like to add to their construction and/or drawing of a given plane figure and to look at each other's constructions, drawings and the choice of geometrical shapes.

By now, each group of students has constructed, named and drawn a plane figure. Note however, if you don't have enough students in your class to divide into ten groups, you can divide stage one into: Stage 1-A (steps 1-5) and stage 1-B (steps 6-10). In this way, each group of students will have the opportunity to work with two different plane figures instead of only one. Thus they will develop a better sense of the nature and the concept of plane figures and their characteristics, properties, and their application and implication in the environment around us.

3.2. Part One – Stage II: Knowing Your Own Chosen Plane Figure Better

Procedures: Stage two of this learning activity is divided into three integrated phases. In this stage of the activity, all the questions are directed to and must be answered by all the groups. Thus members of each group are going to work together collaboratively to answer the questions in both writing and in drawing form. Therefore, members of each group must take turns recording questions, writing answers, and making drawings.

3.2.1. Phase 1: Naming Your Chosen Plane Figure (Regular polygons):

Give each group of students a copy of Table 2, but without the information in the second column that is each figure's name and ask the member of each group to:

1. Select a name for their chosen plane figure and justify their selection.
2. Provide a reasonable explanation for naming a given plane figure is important because students might chose to give a name to their figure that has no relationship to the shape and or the number of the sides, corners or points.

Furthermore, challenging students' choice of names and their reasons behind them forces students to think of

names that have a relationship to some aspect of the figure itself (see Table 1). To do so, they are forced not only to think but also to bring and integrate their own prior experiences as well as actual observations from their surrounding environment into the learning process.

Table 2: The Relationship Between the Number of Sides in Regular Plane Figures and the Figures' Names

	Number of Equal Sides in a Given Plane Figure (Regular Polygon)	**Figure's Name**
1	***Three*** equal line segments so that each pair of two line segments share an endpoint	Triangle
2	***Four*** equal line segments so that each pair of two line segments share an endpoint	Quadrilateral
3	***Five*** **equal** line segments so that each pair of two line segments share an endpoint	Pentagon
4	***Six*** **equal** line segments so that each pair of two line segments share an endpoint	Hexagon
5	***Seven*** equal line segments so that each pair of two line segments share an endpoint	Heptagon/ Heptagon
6	***Eight*** **equal** line segments so that each pair of two line segments share an endpoint	Octagon
7	***Nine*** **equal** line segments so that each pair of two line segments share an endpoint	Nonagon
8	***Ten equal*** line segments so that each pair of two line segments share an endpoint	Decagon
9	***Twelve equal*** line segments so that each pair of two line segments share an endpoint	Dodecagon
10	***More equal line segments than mentioned above***, so that each pair of two line segments share an endpoint	Polygon
11	Locus of points at a given distance from a given point in a 2-dimensional plane and meet a given condition	Circle

3.2.2. Phase 2: Defining and Describing Your Chosen Plane Figure (Regular polygons)

To define is to specify the essential nature, purpose, or basic qualities of something by placing it in a category and then distinguishing it from other members of that category. In other words, it is to exactly state the nature, scope, or meaning of something by giving the general meaning of that something. To describe is to give essential characteristics and features of something, in other words, to give a detailed account of something (Hasa, 2016). Distinguishing between the concepts of definition (to define) and description (to describe) always presents challenges to students of all educational levels. When students are asked to define a given object, quite often they come up with statements that reflect more of a description than a definition of that object (Cherif and Adams 2016). Because of this, students are asked to define and to describe their chosen figures in writing. In addition, writing is an effective way of learning, providing clarity of mind, and targeted focus on aims and purposes. With this in mind, give a copy of the following Table 3 to the members of each group, and ask the students:

1. Using Table 3, and in written words, define and describe your chosen plane figure using the least amount of words possible, but as accurate a description and definition as possible.
2. Discuss your definition and description of your figure with the whole class and modify them accordingly (if needed). If you made any changes, please enter them in the second row of table 2.
3. How do the definition and the description of your chosen figure differ?
4. Write one paragraph explaining what you have learned from actively engaging in this learning exercise.

Table 3 Defining and Describing a Given Closed Plane Figure

Given Plane Figure		**Definition**	**Description**
Name	**# of Sides**		

3.2.3. Phase 3: Knowing Your Plane Figure Better

The aim of this phase of stage one is for the students to gather and generate as much data and information as they can about their chosen plane figures. It is also useful for students to develop a shared vocabulary which is needed for students to develop successful communication and meaningful understanding. The phase is divided into four related learning parts.

<u>Phase 3-A: How Well Do You Know Your Own Selected Plane Figure?</u>

The questions in this phase are directed to and should be answered by the members of all the groups. Ask the members of each group to answer the following questions;

1. What name have you given to your own chosen plane figure?
2. What is the scientific or mathematical name of your chosen plane figure?
3. Is this name similar of different from the name that you have already selected for your chosen plane

figure?
4. How many line segments (sides) does your plane figure consist of?
5. How many angles does your plane figure have?
6. What is the size of each angle and what is each angle called?
7. What is the total measurement of all the angles in your plane figure?
8. What is the length of each line segment (side) in your plane figure?
9. What is the total length of all the line segments (sides) of your plane figure?
10. What is the area of your plane figure?
11. Why do you think a plane figure like the one you drew is called a 'Closed-plane Figure'?
12. Why do you think a plane figure like the one you drew is called a 'Regular closed-plane Figure'?
13. What type of conclusion or relationship can you draw between the number of angles and the number of straight line-segments (sides) of a given plane figure?
14. Write one paragraph describing what you have learned from actively engaging in this hands-on learning exercise.

Challenge Yourself:

A. Why a drawing that is made of only two lines (sides) cannot be considered as "a closed plane figure"?
B. Can you draw your plane figure in such a way that the total sum of the angles will not be equal to the total sum of angles in the original regular plane figure?

For example, Carl Friedrich Gauss (1777-1855) introduced the geometry of curved surface (non-Euclidean geometry) in which the sum of the angles of a triangle is less than 180 degrees.

Any closed figure with straight sides is called a polygon, as mentioned before. The number of angles in a polygon is the same as the number of sides. A polygon may be divided into triangles, each of which contains 180°. To get the number of degrees in the sum of the angles of any polygon, take two less than the number of sides, and multiply by 180° i.e. $(n-2)180^\circ$, where n stands for the number of sides of the polygon (Alder, 1960, p. 25).

The following Table 4 shows examples of the relationship between the figures' names, the number of sides, and the sum of angles in regular plane figures and circles by using the previously mentioned formula.

Table 4

The Relationship Between the Regular Plane Figures' Names, the Number of Sides, and the Sum of Angles in Regular Plane Figures and Circles

	Figure's Name	**Number of Sides $n, n \geq 3$**	**Number of Angles $n, n \geq 3$**	**Sum of Angles in Degrees $(n-2)180^\circ, n \geq 3$**
1	Triangle	3	3	1 X 180° = 180°
2	Quadrilateral	4	4	2 X 180° = 360°
3	Pentagon	5	5	3 X 180° = 540°
4	Hexagon	6	6	4 X 180° = 720°
5	Heptagon/Septagon	7	7	5 X 180° = 900°
6	Octagon	8	8	6 X 180° = 1080°
7	Nonagon	9	9	7 X 180° = 1260°
8	Decagon	10	10	8 X 180° = 1440°
9	Polygon	N	N	(N-2) X 180°
10	Circle	1	0	360°

Phase 3-B: Generating More Plane Figures from the Original Selected Plane Figure

For the following learning exercise, you need to give the members of each group three large drawings (copies) of their original plane figure, colored pencils, or colored felt pens, and two 12 inch rulers.

Ask the members of each group to answer the following questions:

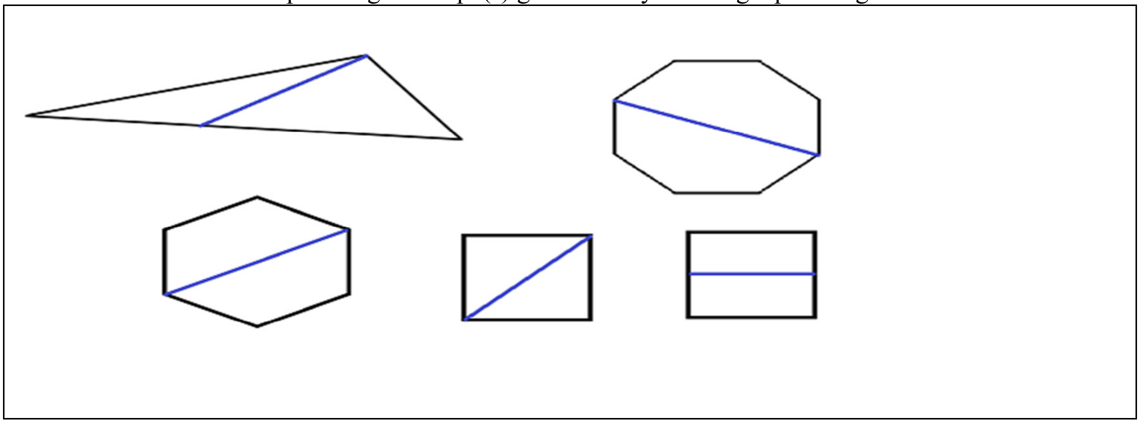
1. What kind of closed plane figure shape(s) can you get by dividing your original plane figure into two areas
   (a) By drawing a straight line from one angle to the faced side (diagonal)?
   (b) By drawing a straight line between two opposite sides (perpendicular bisector)?
   For example, some students will get 2-triangles from one triangle, 1-triangle and 1- trapezoid from one triangle, 2-triangles or 2-rectangles from a square, 2-trapezoids from one hexagon, 2-pentagons from an octagon, to name a few. See Figure 2.
2. Draw straight lines directly from a one single angle or corner into each of the other corners or angles in your chosen closed plane figure. Using Table 5 below, answer the following questions:
   a. How many straight sides and angles does your chosen closed figure have?
   b. How many straight lines can be drawn from a single angle or corner into each of the other corners or angles in the same closed plane figure?
   c. How many closed plane figures can be produced by drawing straight lines from one single angle or

corner into each of the other corners or angles in the same closed plane figure (Poonen and Rubinstein 1998)

d. What types of closed plane figures are produced as a result of drawing straight lines from one single angle or corner into each of the other corners or angles in the same closed plane figure?

Figure 2

The kind of closed plane figure shape(s) generated by dividing a plane figure into two areas



3. What conclusion can you come up with as a result of analyzing your answers to the questions above and recorded data in Table 5 below?
4. What have you learned from actively engaging in this learning exercise?

For example, the students will be able to figure out and conclude that:

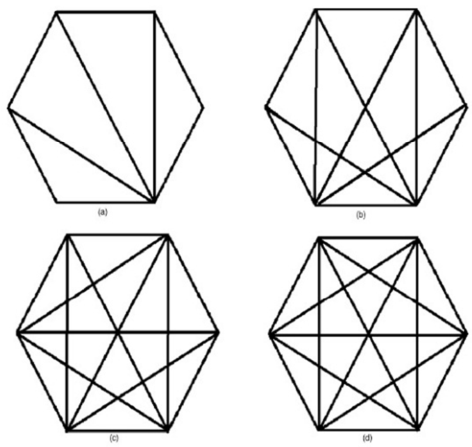
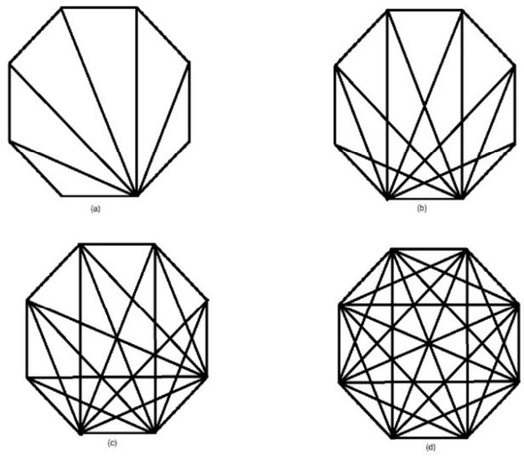
1. With the exception to the triangle, the number of straight lines that can be drawn from a single angle or corner into each of the other corners or angles is always _three less than_ the actual number of sides of the given plane figure. *(Actual # of sides – 3); (See Figure 3 for an example).*
2. The number of closed plane figures that resulted from drawing from a single angle or corner into each of the other corners or angles is always _two less than_ the actual number of sides of the given plane figure. *(Actual # of sides – 2); (See Figure 3 for an example).*
3. The types of the plane figures that result from drawing from a single angle or corner into each of the other corners or angles is always _one type – a triangle_. *(See Figure 3 for an example).*

Table 5:

Students' Predications and their Actual Findings in drawing straight lines from one single angle or corner into each of the other corners or angles

	Name of closed plane figure	Number of sides, points or angles in the closed plane figure $n, n \geq 3$	Number of straight lines that can be drawn from a single angle or corner into each of the other corners or angles (diagonals) $n-3, n \geq 3$	Number of closed plane figures that resulted from taking the action $n-2, n \geq 3$	Types of closed plane figures that can be made in a chosen plane figure
1	Triangle	3	0	1	Triangles
2	Square	4	1	2	Triangles
3	Pentagon	5	2	3	Triangles
4	Hexagon	6	3	4	Triangles
5	Heptagon/ Septagon	7	4	5	Triangles
6	Octagon	8	5	6	Triangles
7	Nonagon	9	6	7	Triangles
8	Decagon	10	7	8	Triangles
9	Hendecagon	11	8	9	Triangles
10	Dodecagon	12	9	10	Triangles
11	Octadecagon	18	15	16	Triangles
12	Icositetragon	24	21	22	Triangles
13	Triacontagon	30	27	28	Triangles
14	Tetracontagon	40	37	38	Triangles
15	Pentacontagon	50	47	48	Triangles

Figure 3
Drawing straight lines from one single angle or corner into each of the other corners
or angles and from each angle or corner into each of the other corners or angles

Regular Hexagons	Regular Octagons
	

<u>*Challenging Activity: From Simplicity to Complexity*</u>
To ensure that the students understand and comprehend what they have learned so far, challenge them with the following question based on what the students learned, were able to do, and were able to conclude.

1. Ask the members of the group who <u>worked with the triangle</u> plane figure to imagine that they draw a straight line from each corner (angle) that is perpendicular to the opposite side in their chosen triangle (plane figure).
2. Ask the members of the <u>rest of the groups</u> to imagine drawing straight lines from each of the angles to each of the other angles in their chosen plane figure.
3. Give each group of students a copy of Table 7-(i), then ask the members of <u>*all the groups*</u>, based on your imagination and following the instructions you have been given above, predict and record your predictions in Table 7-(i):
   A. How many straight lines can you draw in your chosen plane figure?
   B. How many closed plane figures can you make in your chosen plane figure?
   C. How many different types of closed plane figures can you get into your chosen plane figure?
4. Using the large drawing (copies) of their original plane figure, colored pencils (or colored felt pens), and rulers, ask the members of all the groups to test their predications by actually:
   A. Drawing the needed straight lines in their chosen plane figures.
   B. Using the colored pencils or colored felt pens to color the newly created closed plane figures.
   C. Recording their findings in Table 7-(i).
5. Do your predications agree or disagree with your actual findings? Explain.
6. What types of inference and or conclusion can you make from your obtained data and findings?
7. What have you learned from actively engaging in this hands-on learning exercise?

Table 7-(i)
Students' Predications and Their Actual Findings in Generating
More Plane Figures from their Original Closed Plane Figures

		Predictions	**Actual Findings**
1	How many different straight lines can you make from each angle in your chosen plane figure?		
2	As a result of making straight lines from each angle, how many closed plane figures can you make in your chosen plane figure?		
3	As a result of making straight lines from each angle, how many different closed plane figures can you get in your chosen plane figure?		
4	What have you learned from actively engaging in this learning exercise?		

For teachers: Figure 3 and the following Table 7(ii) are resources for teachers to help them guide their students in the discovery process of Table 7-(i).

Table 7-(ii)

Notes and resource for teachers to help guide their students in the discovery process

	Name of closed plane figure	Number of sides, points, angles n	Number of straight lines emerging from a single angle $n-3$	A total of all the straight lines that can be made from each angle into the other angles in a chosen plane figure (diagonals) $\frac{n(n-3)}{2}$	Number of closed plane figures that can be made in a chosen plane figure	Different types of closed plane figures that can be made in a chosen plane figure
1	Triangle	3	0	0	0	Triangle
2	Square	4	1	2	4	Triangles
3	Pentagon	5	2	5	11	Triangles and pentagon
4	Hexagon	6	3	9	24	Triangles and quadrilaterals
5	Heptagon/ Septagon	7	4	14	50	Triangles, quadrilaterals, pentagons and heptagon
6	Octagon	8	5	20	80	Triangles, quadrilaterals, pentagons, hexagons and nonagon
7	Nonagon	9	6	27	154	Triangles and quadrilaterals
8	Decagon	10	7	35	220	Triangles, quadrilaterals and pentagon

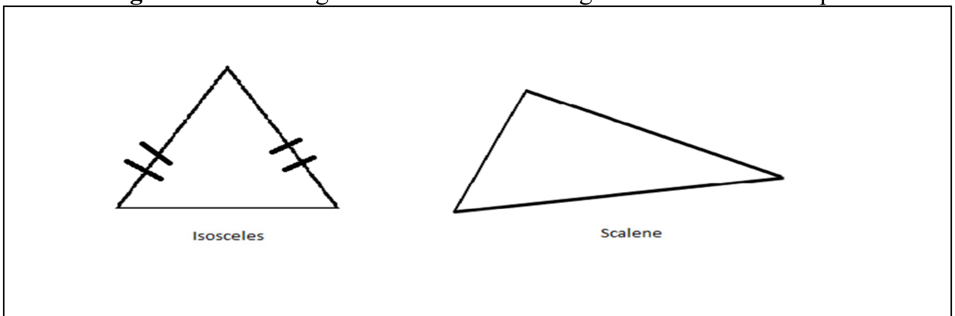
Phase 3-C: Fun With Manipulating the Sides of the Original Closed Plane Figure

Ask the members of each group to answer the following questions:

1. How many different types of closed plane figures can you get by changing the length of the equal sides of your original plane figure from equal sides to unequal sides? Draw and name these different types of figures that all still have the same number of line segments but they are all now in unequal lengths.

   Example 1. An equilateral triangle has all its sides the same length, an isosceles triangle has two of the three of its sides the same length, and the scalene triangle has all of its three sides different lengths. See the following Figure 4.

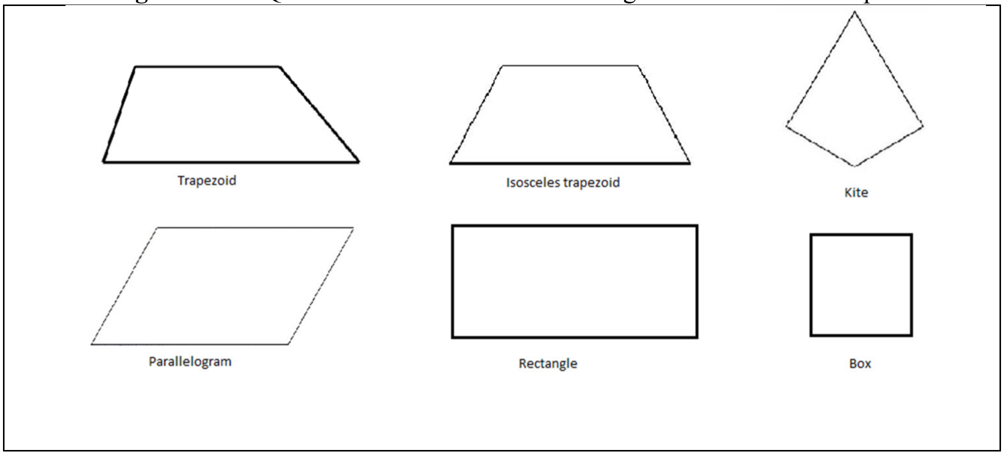
Figure 4: All Triangles Have Three Line Segments that Share Endpoints



Example 2. All of the following quadrilaterals have four line segments that share endpoints: A trapezoid (has two parallel sides), an isosceles trapezoid (has congruent nonparallel sides), a kite (has two pairs of adjacent congruent sides), a parallelogram (has pairs of opposite sides parallel), a rectangle (has a right angle), a square (has a pair of adjacent congruent sides). See Figure 5.

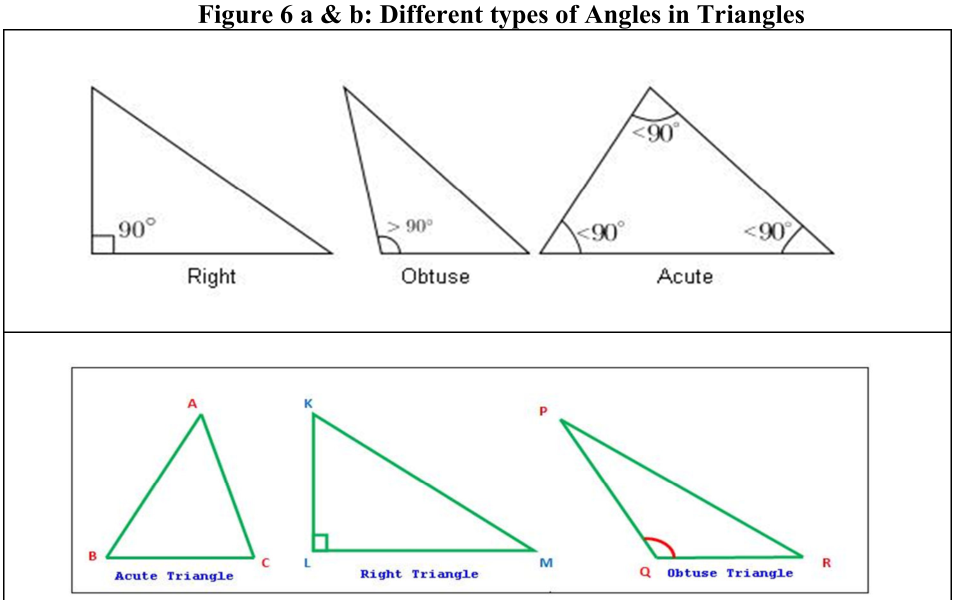
Journal of Education and Practice
ISSN 2222-1735 (Paper) ISSN 2222-288X (Online)
Vol.8, No.13, 2017

Figure 5: All Quadrilaterals Have Four Line Segments that Share Endpoints



2. How many plane figures can you get by changing the sizes of the angles of your figure? Draw and name these different figures that all still have the same number of angles and the same total sum of the angles as your original plane figure.
   Example 3. The right triangle has one angle measuring 90˚, an obtuse triangle has one angle measuring more than 90˚ and the acute triangle has all its angles measuring less then 90˚. Yet, the total sum of the angles of each triangle is still the same. See Figure 6.
3. Knowing two angles in a given closed plane figure, how can you calculate the rest of the angles in the same figure? Illustrate your answer in writing and drawing.
4. Write one paragraph illustrating what you have learned from actively engaging in this learning hands-on exercise. Include in your writing a concept map on how many way you can classify Triangles, for example, students can classify triangles based on sides or angles (see figure 7 below).

Figure 6 a & b: Different types of Angles in Triangles

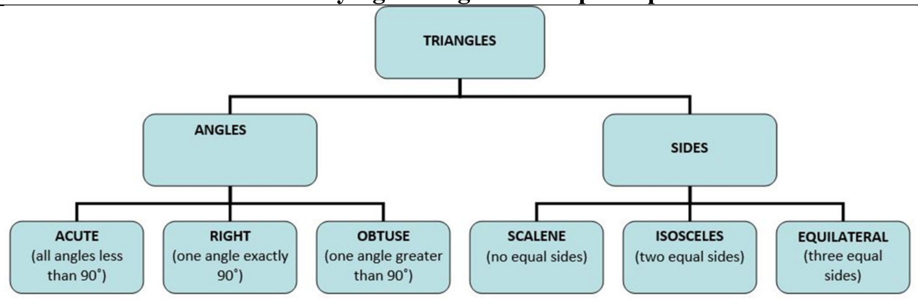


Source: http://www.mathcaptain.com/geometry/triangle.html

Journal of Education and Practice
ISSN 2222-1735 (Paper) ISSN 2222-288X (Online)
Vol.8, No.13, 2017

Figure 7: Classifying Triangles Concept Map

Classifying Triangles Concept Map



Source: http://nextgen.wvnet.edu/Courses/lesson.php?c=7&u=23&l=238&t=Lesson

Phase 3-D: The Power of Informative Prediction In the Development of Cognitive Thinking Skills:

So far, in all parts of Phase III we worked with various plane figures that had an equal number of segments or sides. The questions are:

1. What do you think will happen if you change the length of one or two sides in your original selected plane figure? Write your predictions down. Explain.
2. What do you think will happen if you change the size of one or two angles in your original selected plane figure? Write your predictions down. Explain.

Phase 3-E: Testing Your Understanding by Playing with the Triangle

The following hands-on learning exercise is for all the students to do. A triangle is a polygon made up of three segments, or sides. However, based on the length of the sides and the size of the angles, triangles can be classified into six types as shown in Table 7. A triangle with sides A, B, C can be read and or consider as *ΔABC.* Using Table 7, the members of each group will follow the directions in the first column (in Table 7) to draw the corresponding closed plane figure in column three of the same table.

Table 7: Triangles, Types and Description

	Description of Sides and or Angles	**Type of Triangles**	**Draw the shape of this triangle**
1	All sides are the same length	Equilateral triangle	
2	Two or more sides are the same	Isosceles triangle	
3	All sides are different lengths	Scalene triangle	
4	One angle is a right angle	Right triangle	
5	One angle is an obtuse angle	Obtuse triangle	
6	All three angles are acute	Acute triangle	

Based on what you learned and the information in Table 7:

1. Identify which of the following statements are accurate or not and why:
   a. All equilateral triangles are also isosceles?
   b. All isosceles triangles are equilateral?
2. A triangle is a polygon made of three straight segments, or sides and angles. Where do you see a real-world application of angles of various types?
3. An angle is a set of points consisting of two rays, or half-lines, with a common endpoint (known as the vertex). Where do you see a real-world application of angles of various types?
4. Since the sum of the angles of any triangle is 180˚, and if you know the measurement of two angles, how can you find out or calculate the third angle?
5. Knowing that the sum of the angles of any triangle is 180˚, and a triangle is a polygon made of three segments, or sides, how can you use this information to find out the sum of the angles of other types of regular plane figures (Regular Polygon). Demonstrate your explanation with actual illustrations.
   (Hint: If n = the number of sides of a given polygon, then it can be divided into n-2 triangles, each having 180˚ as the sum of its angles. Thus the sum of the angle is (n-2) X 180˚. For example, a five-sided figure (pentagon) can be divided into 5-2= 3 triangles. The sum of the angles of each triangle is 180˚. Thus the sum of the angle of the five-sided figure (polygon) is 3 X 180˚ = 3,540˚).
6. Your teacher will give you a copy of the map of the United States with its 50 unnamed states. Using this map of the United States:
   a. Label as many U.S. states as you can. Talk with you classmates and or use an atlas if needed to finish labeling the states.
   b. Select three states that you like and/ or would love to visit.
   c. Match each of your three selected states to the closest shape of a given regular plane figure (polygon).

d. Which of your three selected state shapes can be easily matched to a shape of a given polygon? Explain.
e. Which of your three-selected state shape cannot easily be matched to a shape of a given polygon? Explain.

7. Re-draw the map of the U.S.A. using regular and irregular geometrical plane figures to illustrate the states.
8. Write one paragraph explaining what you have learned by engaging in this hands-on learning exercise.

Activity: One Plane Figure - Multiple Names

Regular plane figures can be identified with more than one term. The table below lists plane figures in the first column.

1. Study the plane figures listed in the first column in Table 8. Then draw the figure in column two.
2. In column three, select and list all the term(s) that can also be used to identify the plane figure listed in column one.

Table 8

Regular plane figures can be identified with more than one term

	Regular plane figure	Draw the shape of the regular plane figure	Which of the following terms can also be used to identify this regular plane figure? (*Hexagon, kite, parallelogram, quadrilateral, rectangle, regular polygon, rhombus, square, trapezoid and triangle*)
1	Square		
2	Triangle		
3	Kite		
4	Rectangle		
5	Parallelogram		
6	Quadrilateral		
7	Hexagon		
8	Right triangles		
9	Trapezoid		
10	Rhombus		
11	Circle		

3.3. Part One– Stage III: Discovering Patterns and Relationships in Plane Figures (Regular Geometrical Shapes)

Procedures for Stage Three: As in all the previous stages, in this activity, <u>all the questions are directed to and must be answered by all the groups in writing and in drawing</u>.

1. Give each student a copy of Table 9. Then divide the blackboard or the whiteboard into two large sections and divide each section into three subsections as shown in Table 8.
2. Ask all the students to fill Table 8, using the information they have already generated. Then ask representatives from each group to come and transfer the information from their own Table 8 onto the similar table on the blackboard.

Table 9 - Discovering Patterns and Relationships in Plane Figures

Original Plane Figure			The New Modified Plane Figures that have been derived from the Original Plane Figure by drawing straight lines from one single angle or corner into each of the other corners or angles		
Name of the figure	Number of sides	Number of angles	Name of the figure	Number of sides	Number of angles

3. Ask all the groups to study Table 9 by analyzing and comparing the number of sides and the number of

angles in the plane figures. Then ask the following questions:

a. What kind of inference can you make about the relationship between the number of sides and the number of angles in a given plane figure (by using what you observed to explain what you discovered?)
b. Can you predict how many angles a plane figure with 24 sides will have?
c. Can you predict how many sides a plane figure with 18 angles will have?
d. Can you find a relationship between the kind of angle and the number of angles, or the number of sides in a plane figure?
e. Can you predict which plane figure can or cannot be divided into two, three, or four equal parts when all the sides are equal? Use Table 10 to answer this question.

Table 10
Identifying Patterns and Relationships in Plane Figures

Original Plane Figure	A Figure Divided into 2 Equal Parts When All The Sides are Equal	A Figure Divided into more than 2 Equal Parts When All The Sides are Equal	A Figure that cannot be Divided into more than 2 Equal Parts When All The Sides are Equal

4. Part Two: Developing an Awareness of Three - Dimensional (3-D) Solid Shapes
4.1. Description For Part Two:
In part 2 of the learning activities students will have the opportunity to transfer 2-dimensional plane figures into 3-dimensional shapes. In doing so, the students will learn to differentiate between 2-D and 3-D figures, identify the characteristics, properties, use, and implementation of the 3-D shapes in real life, and how to construct 3-dimensional shapes using straws and or popsicle sticks. As in part one, part two is also divided into three stages of related learning activities.

A three dimensional (3D) shape or object is an object that has width, depth and height, and thus identified as solid geometry (the geometry of three-dimensional space). But unlike the 2-dimensional shape, the 3-dimensional shapes, by their nature, have an inside and an outside, separated by a surface.

There are two types of 3-dimensional figures: (a) those that have straight sides and are identified as polyhedrons, (based on polygons), and (b) those solids objects which have curved sides. Spheres, cubes, cones, pyramids, rectangular prisms, and cylinders, are only a few of many examples of 3D-objects that we encounter on a daily basis. Solid objects (3-D) have special properties that distinguish them from regular plane figures such as volume and surface area, as well as the number of vertices (points), faces, and edges. Polyhedrons are defined as having straight edges, flat sides called faces, and corners, called vertices. They are differentiated by whether their faces are the same shape and/or size.

Like in Part 1, which dealt strictly with 2-dimensional plane figures, Part 2 consists of a number of stages that are designed so that what students discover and learn in one stage can be used for engaging students in the discovery and learning of the following stages.

4.2. Part Two – Stage I: Developing an Awareness of Three - Dimensional (3-D) Solid Shapes (Objects) Around You
In this stage of the activity all the questions will be directed to all the groups. Thus, ask members of each group to:

1. Draw and name your original plane figure as a 3- dimensional shape (solid object).
   Example: cylinder, sphere, cube, triangular pyramid, rectangular pyramid, rectangular prism, hexagonal prism, triangular prism, square pyramid, or cone. See Figure 8.
   There are a number of hands-on techniques that teachers can use to engage students to construct 3-dimensional shapes from 2-dimensional figures. The following is one way that we found to be helpful.
   a. Draw a plane figure on graph paper using a dark marker. Then place a sheet of transparent or wax paper on top of the first paper and outline the plane figure you have already drawn, using a different

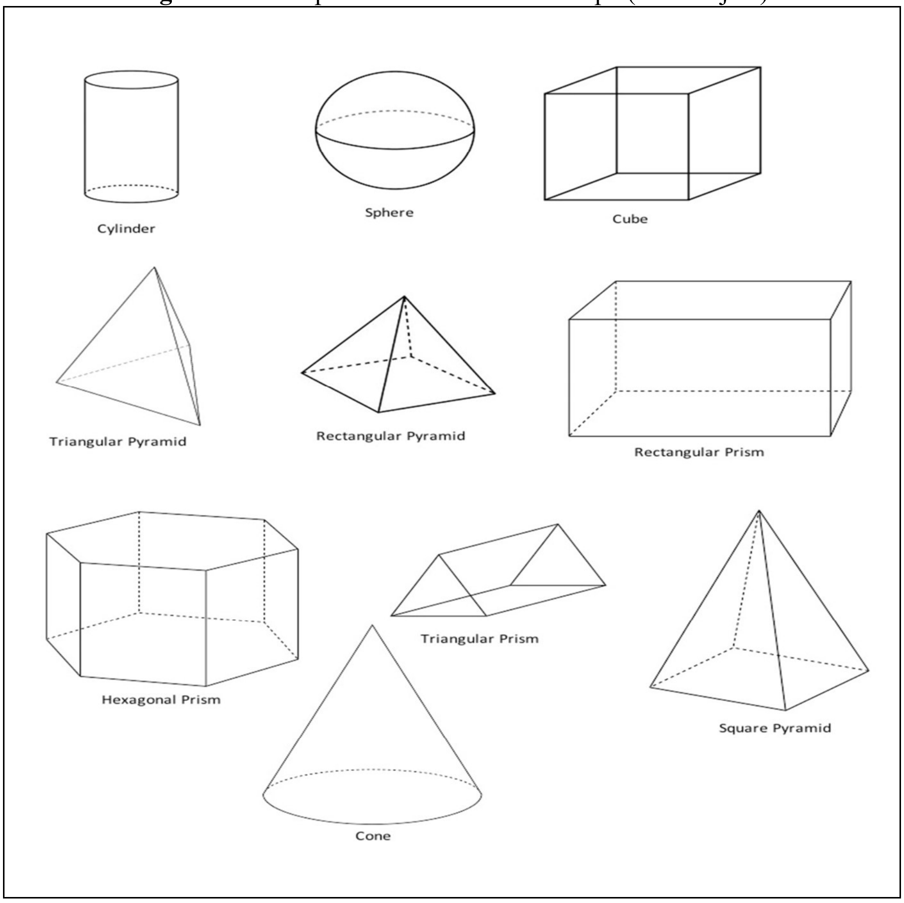
color felt pen.

b. Move the transparency or wax paper in two directions at the same distance and perpendicular to each other.

c. Connect the symmetrical points of the two plane figures; each point in the first figure (black figure on the graphic paper) with its corresponding point in the second figure (the colored figure on the transparent sheet).

d. The result will be a three dimensional figure of your original plane figure on the sheet of transparent or wax paper.

Figure 8 - Examples of 3- dimensional shape (solid object)



After you give your student enough time to practice using this approach to create a 3-dimensional figure from their original plane figure (2-D), ask them:

1. Can you think of other ways or techniques that you could use to create a 3-dimensional shape from your original plane figure?
2. How many different 3-dimensional shapes can you make from your original plane figure?
3. What are the differences and similarities between all the 3-dimensional shapes that you created from your answers of step 2?
4. What are the differences and similarities between 2-dimensional plane figures and 3-dimensional shapes or objects?
5. Write down a one paragraph explanation of what you have learned from being actively engaged in this learning exercise.

4.3. Part Two – Stage II: Knowing Your Chosen 3-Dimensional Shapes Better

Procedures: In this stage of the activity all the questions are directed to, and must be answered, by all the groups. Thus, members of each group are going to work collaboratively to answer the questions in both written and in drawing form. Members of each group must take turns recording questions, writing answers, and making drawings. Ask all the groups to:

4.3.1. Phase – 1: 3-Dimensional Shapes (Solid Objects):

Phase – 1-A: Defining and Describing 3-Dimensional Shapes (Solid Objects):

1. In written words, define and describe your 3-Dimensional shape using as less words as possible, but as accurate a description as possible.

2. Discuss your definition and description of your 3-Dimensional shape with the whole class and modify it accordingly (if you have to).

Phase – I-B: Naming Your 3-Dimensional shapes:

Give each group a copy of the Table 11 but without the information in the third column. Then ask the member of each group to select a name for their 3-Dimensional shapes and to justify their selection. Providing a reasonable explanation for naming a given 3-D shapes or object is important because students might choose to name their 3-D shapes names that have no relationship to the width, depth and height, number of vertices (corner points), number of the faces (sides), and planes of the 3-D shapes. Furthermore, challenging students' choices of names and their reasons behind them force students to think of names that are relative to and have a relationship to some aspect of the 3-D shapes.

Table 11

Regular 3-Dimensional Solid Shapes

	Number of Sides in a Given Three- Dimensional Figure	**Number of Flat Faces (sides)**	**Figure's Name**
1	Four triangles with three of them at each corner. Each triangle with three line segments that share endpoints and with all sides equal	Four	**Tetrahedron**
2	Six squares with three of them at each corner. Each square with four line segments that share endpoints and with all sides equal.	Six	**Hexahedron (Cube)**
3	Eight triangles with four of them at each corner. Each triangle with three line segments that share endpoints and with all sides equal.	Eight	**Octahedron**
4	Twelve pentagons with three pentagons at each corner. Each pentagon with five line segments share endpoints and with all sides equal.	Twelve	**Dodecahedron**
5	Twenty triangles with five triangles at each corner. Each triangle with three line segments share endpoints and with all sides equal.	Twenty	**Icosahedrons**

For teachers:

Using Euler's Formula

$$V - E + F = 2$$

or, in words: the number of vertices (V), minus the number of edges (E), plus the number of faces (F), is equal to two. It can be derived that

$$E = V + F + 2$$

Meaning that if you add the number of corners to the number of faces of any one of these solids, you will get the number of edges in the solid plus 2. Try it with a cube. There are eight corners, and six faces, so the sum of these numbers is 14. Now count the number of edges (Adler, 1960, p. 36-37).

How to Make a Solid Object with Flat Faces:

First make an equilateral triangle, a square, and a regular pentagon on cardboard, and cut them out. Then you can make each figure as many times as you have to, and in the right position, by tracing around the cardboard form. When a pattern is complete, cut it out, and make creases on the lines. After you fold it up, seal it by binding the edges with adhesive tape (Adler, 1960, p. 36-37).

Table 12: Irregular 3-Dimensional Figures (Irregular Solid Objects)

	Number of Sides in a Given 3- Dimensional Figure	**Number of Flat Faces (sides)**
1	Two triangles and three squares. Each square with line segments that share endpoints and with all sides equal.	
2	One square and four triangles. Each triangle with line segments that share endpoints and with all sides equal.	
3	One regular pentagon and five triangles. Each triangle with line segments that share endpoints and with all sides equal.	
4	One regular hexagon and six triangles. Each triangle with line segments that share endpoints and with all sides equal.	
5	Seven regular pentagons, each with five line segments that share endpoints and with all sides equal.	

4.3.2. Phase - 2: Knowing Your 3-Dimensional Shape (Solid Object) Better:

Phase 2-A: How Well Do You Know Your New 3-Dimensional Shape (Solid Object)?

In this part students will be given the opportunity to deepen their understanding on 3-dimensional shapes by adding the two new notions of surface area and volume. The surface area of a solid object is a measure of the total area that the surface of the object occupies. Volume is the quantity of 3-Dimensional space enclosed by a

closed surface.
All the questions in this section are directed to and must be answered by the members of each group. Students may use protractors for the measurement of angles and 12-inch rulers for the measurement of sides.

1. How many sides does your new 3-Dimensional shape have?
2. How many angles does your a new 3-Dimensional shape have?
3. What is the measurement of each angle and what is each angle called?
4. What is the total measurement of all the angles in your new 3-Dimensional shape?
5. What is the length of each line segment (side) in your new 3-Dimensional shape?
6. What is the total length of all the line segments (sides) of your new 3-Dimensional shape?
7. What do you think is needed from you to know in order to find the surface area of your 3-Dimensional shape?
8. What do you think is needed from you to know in order to find the volume of your 3-Dimensional shape?
9. Put all your answers for question 1-8 in written and paragraph form.
10. Write one or two paragraphs explaining what you have learned from actively engaging in this learning exercise about your 3-Dimensional (solid) shape.

Phase 2-B: Manipulating and Generating Various 3-Dimensional Shapes from Your Original 3-Dimensional Figure

All the questions in this section are directed to and must be answered by the members of each group.

1. How many 3-Dimensional shapes can you get by changing the length of the sides of your original 3-Dimensional figure? Draw and name these different 3-Dimensional shapes.
2. How many 3-Dimensional shapes can you get by changing the sizes (measurement) of the angles of your original 3-Dimensional shape? Draw and name these different 3-Dimensional shapes.
3. Name five objects that have approximately the same shapes as your original 3-Dimensional shape.
4. Put all your answers for questions 1-3 in written and paragraph form.
5. What can you conclude or infer from your findings?
6. Write one paragraph explaining what you have learned from actively engaging in this learning exercise about your 3-Dimensional (solid) shape.

4.4. Part Two – Stage III: Discovering Patterns And Relationships in 3-Dimension Shapes (Solid Geometrical Objects)

Give each group of students one copy of Table 12. Then divide the blackboard or whiteboard into two large sections and divide each section into three subsections as shown in Table 12.

Procedures For Stage III: Like in all the previous stages, in this stage of the activity, all the questions are directed to and must be answered by all the groups in writing and in drawings.

1. Ask all the student to fill table 12 using the information they already have generated, then ask representatives from each group to come and transfer the information from their own table 12 onto the table on the blackboard.
2. Ask all the groups to study Table 13 by analyzing and comparing the number of sides and the number of angles in a given 3-Dimensional shape. Then ask the following questions:

Table 13

Discovering Patterns and Relationships in 3-Dimensional Shapes (Solid Geometrical Objects)

Original 3- Dimensional Shapes				**The New Modified 3-Dimensional Shapes that have been derived in Phase II-B**		
Name of The Figures	Number of Sides (Faces)	Number of Angles (Vertices)		Name of The Figures	Number of Sides (Faces)	Number of Angles (Vertices)

a. What kind of inference can you make about the relationship between the number of sides and the number of angles *in a 3-Dimensional shape*, by using what you have observed to explain what you discovered?

b. Can you predict how many angles *a 3-Dimensional shape* with 24 sides will have?
c. Can you predict how many sides a *3-Dimensional shape* with 18 angles will have?
d. Can you find a relationship between the kind of angle and the number of angles or the number of sides in *a given 3-Dimensional shape*?
e. Can you predict which one can or cannot be divided into two, three, and four equal parts when all the sides are equal?
f. What can you conclude or infer from your findings?
g. Write one paragraph explaining what you have learned from actively engaging in this learning exercise about 3-Dimensional (solid) shapes.

Making The Connection: Geometry and Human Societies

Geometry is not only an important part of mathematics, but it is also an important part of life. You might be surprised to discover that any physical item that you can touch is indeed a 3-Dimensional object. In the following activities, students will engage in active research finding and making sense of the real connection between geometry, life, and the world around us.

Ask all the students to:

1. Take their original plane figure and change it into a star shape with symmetrical points by drawing straight lines joining two opposite angles (diagonals).
2. Take their original 3-Dimensional shape and change it into a star shape with symmetrical points by drawing straight lines joining two opposite vertices (diagonals).
3. Try to associate a given star with a similar symbol that is used by a given human community, culture or society in our beautiful world.
4. Conduct a library search to discover if there is a society or culture that has been using the star you have created from your original plane figure and/or the 3-Dimensional shape as a cultural symbol.
5. Conduct library research to discover how many of the plane figures drawn in the class can be found in coins in this country and throughout the world.
6. Conduct marketing research to find out how many of all the plane figures and the 3-Dimensional shapes that have been drawn in the class can be seen in or are popular in jewelry making (gemstones etc.).

5. Part Three: Discovering Regular and Solid Geometry (regular polygons and polyhedra) In the Living World Around Us

Part three is divided into two stages of related learning activities.

5.1. Part Three – Stage I: Developing Learning Skills of Constructing and Calculating Plane Figures and Solid Shapes

By now, the students have enough data and information that enable them to discover patterns and relationships between plane figures, 3-Dimensional solid shapes, and the relationships between the number of sides and angles in plane figures and 3-Dimensional shapes. Because of this, the students are ready to construct regular planes and solid shape diagrams and calculate their volume, surface areas, angles, etc. as well as to apply their understanding in real world solutions.

Procedures: In this stage of the activity, all the questions are directed to and must be answered by all the groups in writing and in drawing. Thus, ask all the groups to:

1. Describe a regular polyhedron.
2. According to your previous definition in question 1, describe an irregular 3-Dimensional shape.
3. A pyramid and a prism are both named by the shape of their what?
4. What kind of shapes can be fit together to make a regular polyhedron?

Table 14: Relationship between Faces, Edges and Vertices of Polyhedrons

	Number of faces	**Number of edges**	**Number of vertices** $V = E - F + 2$	**Base face**
Triangular Prism	5	9	6	Triangular base
Rectangular prism/Cuboid	6	12	8	Rectangular base
Pentagonal Prism	7	15	10	Pentagonal base
Hexagonal Prism	8	18	12	Hexagonal base
Square Pyramid	5	8	5	Square base
Pentagonal Pyramid	6	10	6	Pentagonal base
Hexagonal Pyramid	7	12	7	Hexagonal base
Octahedron	8	12	6	Only side face that is triangular

More challenging Topic:
Platonic solids are 3-Dimensional shapes where each face is the same regular polygon and the same number of polygons meets at each vertex. The platonic solids are only 5, tetrahedron, cube, octahedron, dodecahedron, and icosahedron. The Platonic solids were known to the ancient Greeks, and were described by Plato in his *Timaeus* ca. 350 BC. Why are there only 5 platonic solids? Students can use Table 15 to answer this question.

Table 15 - Platonic Solids

Name of Polyhedron	Number of Faces	Number of Edges	Number of Vertices
Tetrahedron/Cube	4	6	4
Cube/Hexahedron	6	12	8
Octahedron	8	12	6
Dodecahedron	12	30	12
Icosahedron	20	30	20

Students' answers may vary: some of them could be each vertex must be in contact with at least three sides; the sum of the angles at each vertex must be less than 360 degrees; the angles at all vertices must be equal; and the common face can only be a triangles, square, or pentagon, as faces with six or more sides have angles that are too great to be valid.

5.2. Part Three – Stage II: Plane and Solid Geometry in the Living World

1. Conduct a library and internet research to identify five examples of plane figures and 3-dimensional (solid) shapes in:
   - Plants
   - Animals
   - Microorganisms
   - Human-made objects
   - Natural made objects
   - Buildings and bridges
   - Parks and Gardens
   - Salt, Sugar and Minerals
   - Snowflake
   - Grain of sand
   - Hive
   2. Write one paragraph to describe each identified item and support your description with an illustration or drawing.
   3. As a result of successfully completing this activity, what have you learned about plane and solid geometry in the living world?

5. **Part Four: Assessment and Reinforcement of Students' Understanding of the Intended Learning Concepts**
   Part four is divided into two stages of related learning activities.

6.1. Part Four – Stage I: It's Time For Geometry, Art & Fun

A. We Have a Job for you! Geometry, Jobs, and Professional Careers:

Procedures:

1. Read or provide the following statement to your students:
   If you are an expert in geometry and design, we are looking for an experienced Garden Designer, Jewelry Designer, and Shopping-mall Designers to help us design public gardens and parks, jewelry, and public shopping malls. The main criteria in designing these three types of places are to use geometry and plane figures in the design as well as the associated areas.
   We are looking for those with expertise in using geometrical plane figures and shapes to design intricately crafted artisan jewelry, to design highly inviting and entertaining shopping malls that appeal to all types of people of all ages, and extraordinary garden designs that can enhance the city's urban landscapes and neighborhoods.
1. Ask each student which role he or she would like to assume and play in this activity. Then put the students into groups of 3-4 students with similar interests to work together on their selected profession. Remind the students that they are jewelry designers, park and garden designers, or shopping mall designers, and they must utilize geometrical figures, shapes, and properties in their designs.
2. Remind each group to submit their two copies of their final design as well as a written description of the design and rationale and justification behind their selection.
3. When all the groups are ready, ask the groups to present their designs in the classroom.

Journal of Education and Practice www.iiste.org
ISSN 2222-1735 (Paper) ISSN 2222-288X (Online)
Vol.8, No.13, 2017

4. Provide the opportunity for the members of the other groups to ask questions at the end of each group's presentation.
5. At the end, ask all the students to assign a grade level based on 1-to-10, with 10 for the best design. Each student must also explain the grade he or she gives to a given design.

B. Let Us Make Jewelry for Those Who We Love, Respect, and Admire

Jewelry is personal, durable ornamentation, such as necklaces, rings, brooches, earrings, and bracelets. They are typically made from or contain small decorative items such as jewels and precious metals, which are specifically designed with a goal and purpose in mind. In this sense, it is the design of the jewelry that first attracts and captures the attention of customers more than what a jewelry is made of. In this learning activity, all the students assume the role of jewelry designers who are asked to design jewelry based on geometrical shapes and forms with a highly creative visual appeal for those who like to wear jewelry.

In this learning activity students are working individually, thus ask each individual student to:

1. Select 1-3 different plane figures to work with.
2. Using the 1-3 selected plane figures, design 5 different pieces of jewelry.
3. Name each design, describe its designed structure, artistic appeal, and provide a rationale for why you created these particular designs.
4. Write two paragraphs to summarize your answers in question 1-3 in a letter to a friend that you think might be interested in your designs.
5. Write one paragraph explaining what you have learned from actively engaging in this learning activity.

C. Artistic Mathematics of Perspective

Historically speaking, "humans have been having fun and games with mathematics for thousands of years. Along the way, they've discovered the amazing unity of this field – in [arts], science, engineering, finance, games of chance, and many other aspects of life [and living]" Albrecht Durer, the great German artist was widely cited by saying that "Geometry is the right foundation of painting." As Alder (1960) explained:

> *To make a painting look real, the painter thinks of his canvas as a "window" through which he is looking at a scene that is beyond it. He reasons in this way: Each point of the scene sends a ray of light to the eye of the person looking at it. These rays of light pass through the "window" between the eye and the scene. The place where a ray crosses the window is the place where the point it comes from will appear in the picture. The collection of rays going from the scene to the eye is called a projection. The picture formed where the window crosses the projection is called a section. To figure out what the section will look like is a problem of perspective. The rules of perspective were worked out with the help of geometry.[Two rules of perspectives applied]. ... The further away something is, the smaller it looks. Parallel lines that go off into the distance, like straight railroad tracks, look as though they come to a point. Mathematics helped art through the science of perspective. But then art repaid its debt. This is because the study of perspective led to the development of a new branch of mathematics called projective geometry.* (Alder, 1960, p. 80)

1. Share the technique described above by Alder with the students.
2. Provide the students with copies of 3-4 different photos of plants, flowers, birds, dogs, and or horses.
3. Ask each student individually to select one photo and to try to follow the technique described above by Alder to make a painting of their selected photo.
4. Ask each student to write 2 paragraphs descripting what they did and what they learned.

D. Paying Loyalty and Respect to the Prince of Mathematics:

Carl Friedrich Gauss (1777-1855) has been described by many scholars as the prince of mathematics. Ask the students to individually:

1. Conduct library research on Carl Friedrich Gauss (1777-1855). Then prepare a presentation to introduce him to a group of visitors from outside our solar system.
2. In your presentation, talk about the significant contribution Gauss made in mathematics to humanity, and to understanding the world around us.
3. Conclude your presentation with a justification as to why he has been called "the prince of mathematics."
4. Finally end your talk with your own perspective on what you think of Carl Friedrich Gauss and why all school children should learn about his life and contribution to our understanding of the world around us.

E. Being a Well Known Artist:

Tell your students that each one of them is a recognized artist and people are coming from all over the world to look at his or her artwork. Then ask them to use one of their combinations of plane figures and or 3-dimensional figures to answer the following questions:

1. What type of geometrical shape and or form (plane figures and or 3-Dimensional figures) will you select to create an art project? Explain
2. What type of art will you choose to make? Explain
3. What have you learned from actively engaging in this authentic learning experience?

4. What informative advice would you give to someone who wants to start using geometrical shapes and forms for making art projects?

F. Fun and Challenge: How Many Coins Can You Fit in a Plane Figure?

Given that the surface areas of different types of plane figures are equal (the same), use Table 15 to predict which type of plane figure you think will hold the highest number of coins (pennies) that are placed inside? (Adapted from Cherif, Gialamas, and Adams 2003)

1. Identify how the surface area of each selected type of plane figure is measured and calculated.
2. Identify how the surface area of a circle is measured and calculated.
3. Identify how the surface area of a coin (a penny) is measured and calculated.
4. Write down your predications. Explain why you think your predication is more reasonable.
5. Explain how you are going to test your predictions to find out which type of plane figure holds the highest number of pennies in comparison to the other types of plane figures that have the same surface areas.
6. Conduct the experiment and record your findings in Table 16.
7. Do your predictions agree or disagree with your actual findings? Explain.
8. What type of conclusion or inference can you make from your findings?
9. What have you learned from engaging in this learning exercise?
10. Do you think your predictions and your actual finding will still be the same if you work with 3-D shaped object instead of 2-D figures? Explain.
11. How do you calculate the volume of a given 3-D solid object? How do you calculate the volume of a Penny?
12. If you work with 3-D shaped object instead of 2-D figure, why do you think it is important to know the volume of the solid shape as well as the volume of the Penny to successfully complete the same activity?

Table 16 - Student's Predictions and Actual Findings

	Plane Figure	Prediction	Actual Findings
1	Triangle		
2	Square		
3	Pentagon		
4	Hexagon		
5	Heptagon		
6	Octagon		
7	Nonagon		
8	Decagon		
9	Trapezoid		
10	Kite		
11	Rectangle		
12	Circle		

G. Relationships Between Surface Area and Volume in Living Organisms

Living forms are made up of one or more cells and a cell is the smallest unit of structure and function in all-living forms, including humans. Most cells are microscopic with roughly the same size regardless of the size of the living organism. For example, the average cell size in the human body is just one-fifth the thickness of a piece of paper. But why are living cells so small? And what does this have to do with geometry and geometrical shapes? The answer is that, the life of a living cell depends on the exchange of materials across its surface. To overcome the import-export challenges cross the surface, living cells must maintain a favorable surface-volume ratio based on the physical relationship between surface area and volume. In short, the greater the cell volume, the more surface area it requires. As Postlewait and Hopson (2003) explained:

> *A cell's active cytoplasm needs to take in materials to fuel activities and build cell parts, and it needs to get rid of the waste it produces as by-products – in general, the more cytoplasm, the more materials and waste. A cell imports materials and exports waste across its surface envelope or plasma membrane, a boundary, gated cell wall, and raincoat all in one. The greater the surface area of this plasma membrane, the more rapidly the cell exchanges substances with its environment. When a cell increases in size, its volume increases more rapidly than its surface area, and its import-export needs outstrip its ability to exchange these items with the surroundings. If a cell got much larger than a certain typical size (for a bacterium, under 10 micrometers; for an animal cell, 5 to 30 micrometers; for a plant cell, 35 to 80 micrometers), it couldn't meet its material and waste needs quickly enough to survive. (Note that the abbreviation um is frequently used for a micrometer, one millionth of a meter.) This is why an elephant's liver is hundreds of times bigger than a mouse's liver, but its cells are the same size. There are just millions more of them. For an analogy to the surface-area-to-volume problem, think of a pile*

of wet laundry. If left in a heap, this soggy pile takes a long time to dry because its exposed surface area is small compared with its volume. But if you hang the items on a line to dry, the surface area is large, while the volume is unchanged, and the laundry can dry much faster. (Postlewait and Hopson, 2003, p. 39).

This explains why a large organism can survive – because it has more cells than a small organism, but the cells are roughly the same size (Postlewait and Hopson, 2003, Hardin and Bertoni, 2016).

Given that all the surface-volume ratios of various 3-dimensional solid shapes are equal, which shape and form, if they are living cells, is the most efficient for material exchange? Biologists have already answered these questions by showing that the primary means of solving cells' surface-volume problem is though altering cell shape and content.

A long, thin cell, such as a nerve cell that reaches from a giraffe's spine down to its hoof, can have the same volume as a round or cube-shaped cell, but a greatly expanded surface area. An egg yolk survives even though it is large and roundish because the active cytoplasm is flattened into a thin sheet just below the outer membrane, while the metabolically sluggish yolk inside consists mainly of storage lipids and protein. Oranges and grapefruits solve the problem in a similar way: each miniature sack inside a citrus fruit section is an elongated, spindle-shaped single cell with a very thin layer of cytoplasm surrounding a droplet of sugary juice. (Postlewait and Hopson, 2003, p. 39).

Thus, living cells strive for two essential things, having the right size and the preferred shape for survival and to function economically. Living cells achieve this by maintaining favorable surface-volume ratios based on the physical relationship between surface area and volume.

6.2. Part Four – Stage II: General Questions for the Reinforcement of Students' Understanding of the Intended Learning Concepts

1. Which of the following figures is your favorite and which one is the least favorite? Explain. (Regular polygon, triangle, rhombus, kite, quadrilateral, parallelogram, hexagon, rectangle, square, trapezoid).
2. Which is the best plane figure for placing corner-by-corner to create a perfect circle? Explain, and then use the selected plane figure to draw and create a perfect circle.
3. You have 6 individual cubes. The side of the first one is 10 cm; the second is 4 ft, the third is 14 yards, the fourth is 20 mm; the fifth is 24 in, and the sixth is 2 m. Calculate the volume of each of them.
4. The moon is approximately 240,000 miles away from the earth. Scientists (astronomers) found out this fact long time ago before the first human landed on the moon.
   a. When did humans land on the moon for the first time?
   b. How did the first human land on the moon?
   c. How do you think scientists and mathematicians were able to calculate the distance between the moon and the earth without actually going there?
5. Study Table 17 below and then use column 4 to pick and match names with a description.

Table 17 - Study, Pick, and Match

Name		Description of the Plane and or Solid shapes		Match a Number to a Letter
1	Cone	a	A cone has a circular base connected to a vertex.	
2	Cube	d	A cube is a prism with squares for sides and faces.	
3	Circle	c	Set of all points in a plane that are the same distance from another point.	
4	Cylinder	b	A cylinder has two equal circular bases that are parallel.	
5	Prism	e	It is a figure made of two parallel faces that are polygons of the same shape and sides that are parallelograms.	
6	Pyramid	f	A figure with a base that is a polygon and triangular sides.	
7	Sphere	g	A figure with a curved surface in which all points on the surface are equal distance from the center.	
8	Triangular prism	h	A prism with triangular faces.	

7. Final Remark:

As soon as humans discovered a need to identify and discriminate in order to facilitate their life and living, they invented a way to count which in turn developed into arithmetic. Since humans were also living in physical space, they needed to know how to measure their space and in turn invented geometry. However, humans were not stationary and moved around for various reasons, which lead them to discover the need for knowing distances and direction. This led to the invention of trigonometry, which relates distance to direction. To save

time of repeated counting and calculation, humans invented algebra. To think accurately about motion and change humans invented calculus. As you can see, one of the unique things about humans is the ability to think, relate things to each other, and generate inferences, new ideas, and new inventions. Consequently, whenever people start a new kind of work, the work generates new challenges, which motivate them to search for and invent new branches of mathematics as the solution. Thus we can conclude as Alder (1960) stated:

> *Mathematics grew up with civilization and the need for people to meet challenges and understand the world in which they live. In doing so, and through thousands of years, people have discovered along the way the unity and usefulness of mathematics to arts, science, engineering, finance, computers, games of chance, and many known and unknown aspects of life and living. Geometry and Geometrical shapes and forms have been part of these amazing cognitive processes.*

Learning by doing, practice and experience has being perceived to be the best type of lasting learning. Regardless of what learning style a given person naturally prefers, there is a wide understanding among educators that most adults, adolescents, and children learn best by experiencing a blend of activities that promote three learning domains: cognitive, affective, and behavioral. The Development Awareness Approach of Teaching and Learning provides this in an instructional platform. It is a guided discovery strategy for learning that is rooted in constructivism which holds the perspective that individuals actively generate their own representations of the world, which in turn influences their behavior (Young and Marks-Maran 1998; Kelly, 1955).

Kolb (1984), has argued that "Learning is the process whereby knowledge is created through the transformation of experience." (p. 38) Effective learning is seen when a person progresses through a cycle of four stages: (1) having a concrete experience followed by (2) observation of and reflection on that experience which leads to (3) the formation of abstract concepts (analysis) and generalizations (conclusions) which are then (4) used to test hypothesis in future situations, resulting in new experiences (ŞAHİN, 2016).

In learning by doing, practicing, and experience, during the learning process, individuals cognitively search for and build information based on old knowledge and experiences and in turn actively generate their own representations of the world, which ultimately influences their behavior and how they interact with the world around them (Young and Marks-Maran 1998). In doing so, the people link new and old knowledge and that they make meaning of their world through these constructs. The underlying assumption is that people do not just respond to the world around them, they act upon it (Shuell 1986).

This learning approach supports a link between theory and practice and between reflection and action (Staniforth, and Cherif, 1986; Cherif, 1988; 1993; Horton et al. 1990), that marries "knowing what" with "knowing how" for an individualized construct of knowing. By restructuring of the knowledge a learner already has using drawing, writing, and demonstrating, they will not only learn new approaches of viewing the world around them, but will also discover and create new insight and perspectives.

Acknowledgment:
The authors are indebted to insightful comments and suggestions from those teachers who tried these activities in their classes and those faculty who try the same thing with their student-teachers in their teaching method classes. These teachers and faculty and held several discussions with us on the focus, objectives, and effectiveness of the approach and the activities. We are also grateful most of all to the students who provided us with the ground-level feedback that helped us improve the learning activities.

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Journal of Education and Practice
ISSN 2222-1735 (Paper) ISSN 2222-288X (Online)
Vol.8, No.13, 2017

Appendix 1:

Pictorial Examples of Geometrical Shape and Design from Nature and Human-made World

https://www.pinterest.com/emmatuzz/sacred-geometry-in-nature/.
https://www.pinterest.com/emmatuzz/sacred-geometry-in-nature/

- http://www.wilkinsoneyre.com/thoughts/essays/movement-and-geometry
- http://www.designhome.ae/the-growth-of-the-green-building-movement-in-the-middle-east/

Appendix 2:

A. Closed Figures With Many Sides

Polygon is the name that is used to describe a closed figure with straight sides. A polygon is characterized by having the number of angles the same as the number of sides. Table 18 shows examples of different types of polygons. Re-activate your memory by providing the needed examples in table 18 bellow.

Table 18 - different types of polygons.

To Find the …	**Calculate**	**Example**
Area of a quadrilateral	Length x Width	
Area of a triangle	½ x Base x Height	
Area of a parallelogram	Base x Height	
Area of an irregular shape	Separate the shape into two or more smaller figures. Then find the area of each part and then add them together.	
Area of a circle	$A=\pi\cdot r^2$ **or** $A=\pi\cdot r\cdot r$ where A is the area, and r is the radius and π = 3.14.	
Circumference of a circle	C = 2 π r where C is the area, and r is the radius and π = 3.14.	

Table 19

The Relationship Between the Number of Sides and the Number of Angles in Closed Figure (polygon)

The Number of sides in a polygon (closed figure) is the same as the number of sides			
Number of Sides	**Sum of Angles in Degrees**	**Name of the Closed Figure**	**Observation**
3	180˚	Triangle	
4	2 x 180˚ = 360˚	Quadrilateral	
5	3 x 180˚ = 540˚	Pentagon	
6	4 x 180˚ = 720˚	Hexagon	
8	6 x 180˚ = 1080˚	Octagon	
10	2 x 180˚ = 1140˚	Decagon	
N sides	(N-2) x 180˚ =		

As we all know, the distance across a circle, through its center, is known as the diameter of the circle while the distance around the circle is called its circumference. Mathematicians have discovered that the circumference of a given circle is about three times as long as the diameter of the same circle. The mathematicians have concluded that the circumference of any circle is a fixed number times its diameter. However, because this fixed number cannot be written exactly as a fraction or decimal, we mathematicians agreed on the use of the Greek letter π (pi) to stand for it. It is almost equal to 3 1/7, or 3.14.

B. Can you figure the value of pi even of you don't have a measuring tool?

The answer is yes. Irving Adler (1960) provided the following fun and easy way to identify the value of a pi as follow:

> *Strange as it may seem, there is a way of calculating the value of π by dropping a stick on the floor. The floor has to be made of planks of the same width. Use a thin stick, such as a tooth-pick, that is as long as the planks are wide. Simply drop the stick many times [the more the better]. Keep count of the number of times you drop it and the number of times it falls on a crack. Double the number of times you drop the stick and then divide by the number of times it fell on a crack. The result is your value of π. For example, if you drop the stick 100 times, and it falls on a crack only 62 times, divide 200 by 62. The result is about 3.2. This is not a very accurate value of π. The more times your drop the stick, the more accurate a value you will get. When you drop the stick, whether or not it crosses a crack depends on where its centers falls, and how it is turned around its center. When a stick turns around its center, it moves around a circle. That is why π, which is related to measuring a circle, is also related to the chance that the stick will cross a crack.* (p. 26)

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