

Article

Networking Theories on Giftedness—What We Can Learn from Synthesizing Renzulli’s Domain General and Krutetskii’s Mathematics-Specific Theory

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Abstract: Giftedness is an increasingly important research topic in educational sciences and mathematics education in particular. In this paper, we contribute to further theorizing mathematical giftedness through illustrating how networking processes can be conducted and illustrating their potential benefits. The paper focuses on two theories: Renzulli’s domain-general theory on giftedness as an interplay of creativity, above-average ability, and task commitment; and Krutetskii’s mathematics-specific theory on gifted students’ abilities. In a “proof of concept”, we illustrate how the abilities offered in Krutetskii’s theory can be mapped to the three traits described by Renzulli. This is realized through a mapping process in which two raters independently mapped the abilities offered by Krutetskii to Renzulli’s traits. The results of this mapping give first insights into (a) possible mappings of Krutetskii’s abilities to Renzulli’s traits and, thus, (b) a possible domain-specific specification of Renzulli’s theory. This mapping hints at interesting potential phenomena: in Krutetskii’s theory, above-average ability appears to be the trait that predominantly is addressed, whereas creativity and especially task-commitment seem less represented. Our mapping demonstrates what a mathematics-specific specification of Renzulli’s theory can look like. Finally, we elaborate on the consequences of our findings, restrictions of our methodology, and on possible future research.

Keywords: giftedness; theories; mathematics education; networking theories; domain-general; domain-specific

1. Introduction

Giftedness and gifted behavior are topics increasingly gaining significance in mathematics education and mathematics education research [1]. While research has addressed weaker students and learning difficulties as well as their support to a greater extent, there are a smaller number of studies investigating how high-achievers can be supported in their learning [2]. However, there is a need for increased research in this area—especially because it is important to give these students the opportunity to seize their potential. They should have the possibility to unfold their abilities in the sense of Vygotsky’s zone of proximal development [3]. They must be offered situations in which they can perform on “the level of potential development through problem-solving” [3] (p. 86). In order to be able to adequately support these students in their learning, it is important to know the characteristics of gifted students’ abilities and of gifted behavior. Empirically grounded theories on giftedness and its traits are significant for conceptualizing the characteristics of such behavior.

In gifted education research, theories can be considered either domain-general or domain-specific. In *domain-general* theories, giftedness is conceptualized across domains or independently from domains.

For instance, Rost [4,5] focuses on intelligence and defines persons as gifted if their IQ scores are above 130—two standard deviations above the mean—and if they, accordingly, belong to the smartest 2% of their peer group. According to Rost, these individuals can excel in many domains. This so-called “intellectual” conception of giftedness dates back to Terman [6] who equated giftedness with high IQ. Another example of a domain-general theory is Renzulli’s [7,8] three-ring model, where gifted behavior in general is considered being characterized by an interplay of three traits: creativity, above-average ability, and task-commitment. *Domain-specific* theories, on the other hand, address single domains in particular, drawing on the assumption that gifted behavior has different characteristics in different domains. In mathematics education, one of the most profound theories was presented by Krutetskii [9]. This theory is considered domain-specific because it comprises a set of abilities characterizing mathematically gifted students—without making predictions on whether mathematically gifted students can excel in music, literature, or other domains. Both general and domain-specific theories on giftedness are established in research on giftedness.

One of the major shortcomings of theory use in gifted education research is that connections, similarities, and differences of theories are rarely under investigation or made explicit. This is especially true for connections between domain-general and domain-specific theories. Ideally, we think that domain-specific theories should position themselves in the field of domain-general theories on giftedness and make these relations explicit. This can be realized either by clarifying and complementing the range and significance of those theories regarding their specific domain; for example, a domain-general assumption about the problem solving ability of gifted students can be explicated by traits such as pattern-recognition for mathematics specific giftedness, or this can be realized by clarifying why domain-general assumptions on giftedness cannot be applied to their specific domain. However, such connections are rarely explicated. In particular, we see that Renzulli’s ring model is often applied in mathematics education research. However, the question of how this theory relates to existing domain-specific theories (e.g., Krutetskii’s) has not sufficiently been clarified in mathematics education research. Whether the traits of mathematically gifted behavior meet the three domain-general traits (creativity, above-average ability, and task-commitment), to what extent they do so, and what these three domain-general traits specifically mean in the field of mathematics education, has not yet been clarified.

In this paper, we illustrate what a theory networking process of a domain-specific and a domain-general theory can look like. In detail, two raters mapped gifted students’ abilities (as offered by Krutetskii) to the three traits of gifted behavior (as offered by Renzulli). Via this “proof of concept”, we aim to illustrate the feasibility of the networking process and its possible outcomes. Such a comparison and integration of domain-general and domain-specific theories offers the opportunity to develop a more fine-grained theoretical account of mathematical giftedness that takes into account the existing knowledge in different areas of research. Besides the results of the mapping and the corresponding conclusions, the paper more generally shows how a process of networking theories can contribute to theory development in the domain of research on giftedness.

2. Theoretical Background

2.1. Theories

The term *theory* is not easy to define—as Mason and Waywood state:

“Theory is a value-laden term with a long and convoluted history. (. . .) What is common in the use of the word ‘theory’ is the human enterprise of making sense, in providing answers to peoples’ questions about why, how and what. How that sense-making arises is itself the subject of theorising.” [10] (pp. 1055 f.)

In the domain of giftedness, theory deals, for example, with the questions of how giftedness and gifted behavior can be conceptualized and what characterizes gifted behavior. Theory in this field aims

to make sense of the phenomenon of giftedness. The process of increasingly understanding giftedness in its facets is a process of progressively theorizing.

There is a broad variety of understandings of the term theory in the field of mathematics education—as Bikner-Ahsbabs et al. point out:

“There is *no shared unique definition* of theory or theoretical approach among mathematics education researchers (see Assude et al. 2008). The large diversity already starts with the heterogeneity of what is called a theoretical approach or a theory by various researchers and different scholarly traditions.” [11] (p. 5)

However, researchers seem to roughly agree that theories are networks of concepts, assumptions, principles, and terminology characterizing certain phenomena, and/or explaining phenomena, and making predications [12,13].

In this paper, we address theories on giftedness. One of the most important theories on giftedness in the field of mathematics is Krutetskii’s [9] theory about gifted students’ mathematical abilities. His theory is based on a pioneering investigation, where he considered the question of what characterizes mathematical abilities during school age. By analyzing an extensive amount of data, he found answers to this question—via a research process of theorizing. This process led to a theory on gifted students’ mathematical abilities: Krutetskii found certain traits being constitutive for gifted students’ mathematical abilities during school age, which each concern one of four categories (obtaining, processing, and retaining mathematical information, as well as a general synthetic component, see Section 2.3.2). Another example of theories on giftedness is Renzulli’s [8] ring model of giftedness; a domain-general theory that is often used in educational research. Renzulli’s theory is applicable in different domains, and will be—besides Krutetskii’s theory—considered within the focus of this paper (Section 2.3.1).

2.2. Networking of Theories

In mathematics and gifted education, there are a variety of theoretical approaches, reflecting different research paradigms. As Bikner-Ahsbabs et al. state, “Differences exist in the ways to conceptualize and question mathematical activities and educational processes, in the type of results they can provide, but also in their scopes and backgrounds” [11] (p. 6).

Steen [14] argues that the diversity of theoretical approaches in mathematics education may indicate lacking maturity (see also [11] (p. 8)). Even though a variety of theories can be useful, for instance, for grasping complex phenomena or for promoting debate and helping the field grow, co-existing unconnected approaches may also cause communicational problems and hinder an integration of findings and, hence, the development of the research area. Bikner-Ahsbabs et al. conclude, “That is why we emphasize that the diversity of theoretical approaches can *only* become fruitful *if* connections between them are *actively established*” [11] (p. 8). In this sense, it is important to intentionally build up connections between theories: a networking of theories, which means “research practices that aim at creating a dialogue and establishing relationships between parts of theoretical approaches” [15] (p. 118).

Networking processes can have different aims, depending on how far theories are supposed to be connected or intertwined. One can, for instance, aim to compare theories and their characteristics, with a lower ambition to point out connections. On the other extreme, one can aim to unify theories. In this paper, we aim to synthesize two theoretical approaches. Synthesizing has a higher degree of integration than, for instance, comparing or coordinating theories; however, it pays attention to the theories’ various inheritances and backgrounds. It does not aim to unify them. With this synthesis, we aim to contribute to theory development in the domain of giftedness. We aim to illustrate what such a synthesis process can look like; in particular, how it can be conducted and what its outcomes can look like. More specifically, we directly contribute to the scientific discussion in the domain of mathematical giftedness.

2.3. Perspectives on Giftedness: Domain-General or Domain-Specific

The debate on domain-generality of giftedness dates back to the outgoing 19th century and beginning of the 20th century. It focuses on the question of whether giftedness characterizes persons and their behavior generally across domains, or whether it applies to certain domains in particular. Early conceptualizations of giftedness, for example, by Galton viewed giftedness as domain-general [16]. Alongside the development of multi-factorial intelligence models, domain-related manifestations of giftedness have increasingly gained popularity [16]. Researchers promoting domain-specific conceptualizations of giftedness argue that in our modern society, knowledge and science are highly specialized—as are contributions to a scientific field. This specificity in scientific fields hints at a specificity of giftedness in these fields as well [17]. We think that these considerations have to be taken into account and taken seriously in our modern, high-technology society and economy. Further, even though such a trend towards domain-specificity over time can be perceived, both kinds of theories are still significant in contemporary educational research.

In the following, we illustrate domain-general and domain-specific theories on giftedness with one example each. With this selection, we do not strive for generality of our theoretical approach but for a well-elaborated example of the mapping process. The domain-general theory that we discuss is Renzulli's [7,8,18] theory on gifted behavior (see Section 2.3.1). It is used for conceptualizing giftedness and its characteristics, and is often applied in domain-specific research in mathematics education. Even though Renzulli's theory is frequently applied and referred to in educational research, especially in mathematics education, there were no systematical investigations conducted yet addressing the questions of how Renzulli's theory matches the findings on giftedness in mathematics education and if it is compatible to these findings. The domain-specific theory presented is Krutetskii's [9] theory on mathematically gifted students' abilities (see Section 2.3.2). This is the most notable study in mathematics education research on giftedness regarding the number of participants, the covered time period, and the usage of multiple research methods. Therefore, it is often referred to when giftedness is addressed in mathematics education research.

2.3.1. Renzulli's Domain-General Ring Model

Dating back to the 1970s, Renzulli started to develop a theory of giftedness naturally taking into account the contemporary state of research on giftedness and that aimed towards being "operational, i.e., useful to school personnel, and defensible in terms of research findings" [7] (p. 180). Aside from establishing an operational definition of giftedness, Renzulli's aim was a definition that (1) is derived from the best available studies; that (2) provides guidance for selection and development of instruments for identification; and (3) provides direction for developing practices for gifted youngsters as learners with special needs [7].

In his seminal article, Renzulli analyzed and compared definitions of giftedness as well as related research. He focused on gifted persons as "persons who have achieved recognition because of their unique accomplishments and creative contributions" [7] (p. 181). He summarized his literature analyses declaring that gifted persons "possess a relatively well-defined set of three interlocking clusters of traits. These clusters consist of above-average though not necessarily superior general ability, task commitment, and creativity" [7] (p. 181). These three traits that characterize giftedness are summarized in Table 1 [7,8].

Renzulli stressed that "no single cluster 'makes giftedness'. Rather, it is the interaction among the three clusters that research has shown to be the necessary ingredient for creative/productive accomplishment" [7] (p. 183).

Table 1. Behavioral manifestations of giftedness according to Renzulli’s three-ring definition.

Abbreviation	Description
C	<i>Creativity</i> refers to flexibility and originality of thought as well as to curiosity, the willingness to take risks, and the sensitiveness to aesthetic aspects. Renzulli states that “In this model the term <i>creative</i> refers to someone who is recognized for his or her creative accomplishments or persons who have a facility for generating many interesting and feasible ideas.” [8] (p. 72)
A	<i>Above-average ability</i> comprises both general and specific ability: “General ability refers to the capacity to process information, integrate experiences that result in appropriate and adaptive responses in new situations, and engage in abstract thinking. Verbal and numerical reasoning, spatial relations, memory, and word fluency are examples of general ability. Specific ability is the capacity to acquire knowledge, skill, or competence to perform in a specialized area. For example, the skills of an archaeologist or mathematician would be considered specific ability skills.” [8] (p. 71)
T	<i>Task commitment</i> is described as a “refined or focused form of motivation [. . .]. Whereas motivation is usually defined in terms of a general energized process that triggers responses in organisms, task commitment represents energy brought to bear on a particular problem (task) or specific performance area.” [8] (p. 72) Renzulli furthermore describes this cluster by referring to perseverance, endurance, hard work, or confidence in one’s ability.

Even though Renzulli uses the term “giftedness” in his earlier works, he makes explicit in his later works [8,18] that he addresses “gifted behavior” rather than “gifted individuals”. He highlights the significance of empirical observation for fostering (and conducting research on) “[i]ndividuals [. . .] capable of developing this composite set of traits [above-average ability, high levels of task commitment, and high levels of creativity] and applying them to any potentially valuable area of human performance” [8] (p. 69). Renzulli additionally explains his preference to focus on behavioral characteristics over a focus on a general attribution of giftedness [18]. He points out that such a focus is especially helpful for teachers as well as for researchers. For example, the statement “Elaine is a gifted third grader” is considered less helpful than “Elaine is a third grader who reads at the adult level and who has a fascination for biographies about women of scientific accomplishment” [18] (p. 82). Following Renzulli’s idea to develop an operational theory applicable in practice, his theory has been used in research related to school and enrichment projects for students [19].

For his definition of giftedness to be operational, Renzulli states that it should be applicable to different socially useful performance areas. Each of “the three clusters [. . .] can be brought to bear on a multitude of specific performance areas” [7] (p. 183). For applying these clusters to specific performance areas (e.g., mathematics) and being able to recognize superior performance in these areas, Renzulli’s theory on giftedness needs to be further specified domain-specifically and related to existing findings in these domains.

2.3.2. Krutetskii’s Domain-Specific Theory on Mathematically Gifted Students’ Abilities

In the field of mathematics education, several studies have been conducted with the aim of identifying components of mathematical giftedness or mathematically gifted behavior.

The most notable study to date has been coordinated by the Russian psychologist Krutetskii [9]. Over a period of 12 years, from 1955–1966, Krutetskii and his team worked with more than 200 gifted and normal (“mathematically capable” and “relatively incapable”) students from grade 2 to 10. With qualitative and quantitative methods of observing problem solving processes, conducting interviews, questioning parents, teachers, mathematicians, and mathematics educators, as well as conducting biographical research of prominent mathematicians and physicists, Krutetskii identified qualities of mind that distinguish a person who is able to excel at mathematics [9] (pp. 81 ff.). Krutetskii explicitly speaks of “gifted” or “very gifted pupils” [9] (p. 334) that have certain abilities such as generalizing mathematical ideas to a great extent. When writing about “capable, average, and incapable pupils” [9], this suggests a view in which a person is seen as gifted (or not). However, he focuses on

their behavior and activities—and, thus, observable characteristics—when operationalizing giftedness; for instance, solving problems or reversing thoughts.

Krutetskii's empirical work led to a set of mathematical abilities that his research had revealed to be significant for mathematical giftedness—a theory on mathematically gifted students' abilities (Table 2, see also Section 4.1).

Table 2. General outline of the structure of mathematical abilities [9] (pp. 350 f.).

1.	Obtaining Mathematical Information:
(a)	The ability for formalized perception of mathematical material, for grasping the formal structure of a problem.
2.	Processing Mathematical Information:
(a)	The ability for logical thought in the sphere of quantitative and spatial relationships, number and letter symbols; the ability to think in mathematical symbols.
(b)	The ability for rapid and broad generalization of mathematical objects, relations, and operations.
(c)	The ability to curtail the process of mathematical reasoning and the system of corresponding operations; the ability to think in curtailed structures.
(d)	Flexibility of mental processes in mathematical activity.
(e)	Striving for clarity, simplicity, economy, and rationality of solutions.
(f)	The ability for rapid and free reconstruction of the direction of a mental process, switching from a direct to a reverse train of thought (reversibility of the mental process in mathematical reasoning).
3.	Retaining Mathematical Information:
(a)	Mathematical memory [memory of mathematical generalizations] (generalized memory for mathematical relationships, type characteristics, schemes of arguments and proofs, methods of problem solving, and principles of approach).
4.	General Synthetic Component:
(a)	Mathematical cast of mind [striving to make the phenomena of the environment mathematical, constantly urging to pay attention to the mathematical aspect of phenomena, noticing spatial and quantitative relationships, bonds, and functional dependencies everywhere].

Krutetskii pointed out that these abilities are closely interrelated and influence one another. In addition to these abilities, Krutetskii identified abilities “whose presence is not obligatory (although useful) in this structure [of mathematical giftedness]” [9] (p. 351). These are

- a swiftness of mental processes,
- computational abilities,
- a memory for symbols, numbers, and formulas, an ability for spatial concepts, and
- an ability to visualize abstract mathematical relationships and dependencies.

Krutetskii summarizes his findings stating, “the detailed outline of the structure can be represented in a different, extremely concise formula: mathematical giftedness is characterized by generalized, curtailed, and flexible thinking in the realm of mathematical relationships and number and letter symbols, and by a mathematical cast of mind.” [9] (p. 352)

He also reflects on the specificity of mathematical abilities, whether they are domain-general or -specific [9] (pp. 353 ff.). He summarizes his research in the following way:

“Certain features of a pupil's mental activity can characterize his mathematical activity alone—can appear only in the realm of the spatial and numerical relationships expressed in number and letter symbols, without characterizing other forms of his activity and without correlating with corresponding manifestations in other areas. Thus, mental abilities that are general by nature (such as the ability to generalize) in a number of cases can appear as specific abilities (the ability to generalize mathematical objects, relations, and operations). There appears to be every basis for speaking of special, specific abilities, and not of general abilities that are only refracted in a unique way in mathematical activity.” [9] (p. 360)

2.3.3. Comparing Renzulli's and Krutetskii's Perspectives on Giftedness

Both Renzulli and Krutetskii take a process-related view on giftedness. Even though both, to some extent, focus on giftedness of persons, both stress the importance of observable behavior and come up with an operational definition of giftedness. Renzulli explicitly talks about gifted behavior instead of giftedness and Krutetskii focuses on problem solving processes. Such similar perspectives are a prerequisite for the networking process presented in this paper [20].

Regarding domain-general vs. domain-specificity, Renzulli develops a general theory with the aim of using it for gifted behavior in different domains. However, he clearly states that gifted behavior can and has to be operationalized in a domain-specific way, which becomes apparent in his description of the ring "above-average ability". Krutetskii emphasizes the necessity to operationalize giftedness domain-specifically. He presents abilities of students who excel at one specific domain: mathematics. These traits can be used to operationalize Renzulli's rings for the domain of mathematics.

2.4. Research Aim and Research Question

When considering the characterizations and operationalizations of (mathematical) giftedness presented above, we found that there is an opportunity for networking theories. In our view, an integration of the domain-general and the domain-specific theories contributes to theory development [20].

For such an endeavor, it is crucial to carefully consider the theoretical foundations of the involved theories and to ensure that they do not contradict each other [20]. For this purpose, we conducted thorough readings of Renzulli's and Krutetskii's theories: this gave us deeper insight into the fundamental assumptions of the theories and confirmed our first assumption that the theories are, despite their differences, compatible. Both theories draw on an understanding of giftedness as a multi-faceted concept. This is to say that they do not consider one single facet, such as an IQ score (such as Rost, [5]), to be decisive for giftedness, but a variety of traits or abilities. Furthermore, both theories have a process view on giftedness and focus on gifted behavior rather than on giftedness in general.

Renzulli, for example, states:

"The term *gifted* is used in our lexicon only as an adjective, and even then, it is used as a developmental perspective. Thus, for example, we speak and write about the development of gifted behaviors in specific areas of learning and human expression rather than giftedness as a state of being. If we use the g-word, it is to label the service rather than the student. This orientation has allowed many special-needs students opportunities to develop high levels of creative and productive accomplishments that otherwise would have been denied through traditional special program models." [18] (p. 81)

Even though Krutetskii has more of a person-view and tries to characterize gifted students' abilities, his research focuses on the abilities necessary in students' problem solving processes. His theory, accordingly, aims to grasp these processes. Krutetskii points out:

"[In o]ur study of mathematical ability [. . .] we proceeded from the notion that the most fruitful approach to the study of the complex problem of ability would be a combination of a number of methods, with one dominating. [. . .] The basic material was obtained by experimental research. [. . .] The experimental method of investigating mathematical ability was a qualitative and quantitative analysis of the solution of special experimental mathematical problems by pupils with various abilities in mathematics." [9] (p. 81)

Both theories have a focus on what students actually do, how they deal with problems they encounter, and which traits of their behavior can be considered "gifted". Based on this similar, multi-faceted process view of the two theories, we argue that an integration of these theories is feasible.

Our aim is therefore to conduct the networking process called “integration” [20]—a process of theory development that goes beyond understanding empirical phenomena and that can contribute to a better understanding of how giftedness, or rather gifted behavior, can be characterized in mathematics.

Therefore, we aim to map the traits of gifted behavior outlined in Krutetskii’s domain-specific theory to the traits described in Renzulli’s domain-general theory (Figure 1). We investigate how they meet each other; whether, for instance, the traits described by Krutetskii concern all three rings described by Renzulli, or whether they concern predominantly one of the rings. Thus, our first research question addresses the mapping process:

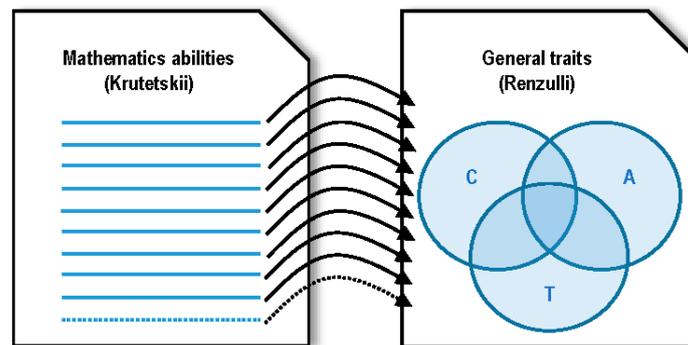


Figure 1. Research Question 1: Mapping mathematically gifted students’ abilities to the traits Creativity, Above-Average Ability, and Task-Commitment (process-view).

- (1) How can the traits of mathematically gifted behavior offered by Krutetskii be mapped to the three clusters of gifted behavior in Renzulli’s ring model?

Whereas the first research question focuses on how the domain-specific traits can be mapped to the domain-general rings, the second question focuses on the *product* of this process; on the picture that emerges for each of Renzulli’s rings in a domain-specific perspective (Figure 2). Based on the results from research question 1, we investigate how the three rings offered by Renzulli can be operationalized and specified for the domain of mathematics on behalf on Krutetskii’s theory.

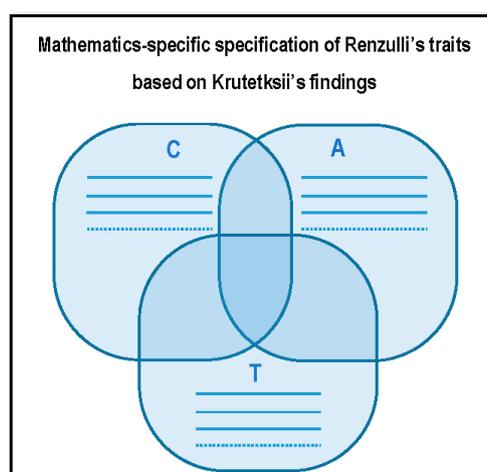


Figure 2. Research Question 2: Domain-specific specification of the three traits Creativity, Above-Average Ability, and Task-Commitment (product-view).

- (2) How can Renzulli’s rings be specified in a domain-specific perspective in mathematics using Krutetskii’s theory?

The results of research question 2 enable us to envision the outcome of our theory integration process: a fine-grained domain-specific theory on gifted behavior in terms of creativity, above-average ability, and task-commitment.

3. Methods

3.1. The Raters

The mapping process is conducted by two raters in a “proof of concept”, with the aim of illustrating the mapping process, its feasibility, first trends, and possible outcomes. Accordingly, the mapping does not (yet) aim to meet the requirements of saturation and validation of its results. Such an endeavor should, however, be an objective for the next step of future research (see Section 5). The fact that the outcomes presented here rely only on a small number of raters has to be taken into consideration in the interpretation and discussion of the results.

The two raters are the two authors of this paper. Both raters are senior researchers at the university level within mathematics education research and have been conducting research in the domain of mathematical giftedness, interest, and creativity for several years. Their studies involve classroom studies investigating task design for mathematically gifted students [21], curriculum design for mathematically interested students [22], and detailed analyses of gifted students’ problem solving processes. Both raters were familiar with Renzulli’s and Krutetskii’s works before this investigation. Both raters have completed teacher education and are mathematics school teachers. They each have an experience of six years as in-service school teachers, while they have taught courses for mathematically gifted and/or interested students for three years each.

3.2. The Mapping Process

For considering the first research question, we set out from Renzulli’s [7,8] ring model using the three traits of gifted behavior—above-average ability, creativity, and task commitment—as a basis for the networking process. Specifically, we used them to organize the findings on mathematically gifted students’ abilities. We then focused on the abilities listed by Krutetskii. We first considered each of them (Table 2) in detail, also by deeper readings of Krutetskii’s studies and descriptions. Subsequently, we mapped each of these traits to either one, two, or three of the rings offered by Renzulli. For example, Krutetskii’s trait “the ability to think in mathematical symbols” was mapped to Renzulli’s ring above-average ability. This was done separately by two raters.

In a second step, the interrater agreement was investigated by comparing the two raters’ mappings. In a third step, we discussed the cases in which the mapping differed, with the aim to grasp possible different interpretations of Renzulli’s traits and/or the abilities provided by Krutetskii, drawbacks on the feasibility of the mapping process, and in order to, if possible, come to a consensual agreement through discussion and further readings.

For answering the second question, we evaluated the outcomes of the mapping with special regard to the three rings and discussed to what extent the rings each were addressed.

4. The Mapping and its Results

Table 3 presents the mapping obtained by the two raters. In the left column, the traits of mathematical gifted behavior are outlined while the second and third column indicate how rater 1 and 2 mapped the traits to Renzulli’s rings. The fourth column displays whether the mappings immediately matched. For example, rater 1 mapped trait 1 (a) to above-average ability (A) and creativity (C) while rater 2 mapped it to above-average ability but not to creativity. So, there was an agreement that 1 (a) applies to above-average ability and that it does not apply to task-commitment. However, there was a disagreement concerning the relation to creativity, so this issue was the subject of further discussion in a second step, which aimed to clarify and establish consensual agreement. The last column shows the consensual decisions that were reached after the discussion of the critical cases.

Table 3. Matching of Krutetskii’s domain-specific traits of giftedness to Renzulli’s general traits.

Krutetskii’s Domain-Specific Traits	Mapping to Renzulli’s Rings			Match?	Consensual Decision
	Rater 1	Rater 2			
1. Obtaining mathematical information					
(a) The ability for formalized perception of mathematical material, for grasping the formal structure of a problem.	A	A	A yes	✓	A yes
	C		C y/n	x	C no
			T no	✓	T no
2. Processing mathematical information					
(a) The ability for logical thought in the sphere of quantitative and spatial relationships, number and letter symbols; the ability to think in mathematical symbols.	A	A	A yes	✓	A yes
			C no	✓	C no
			T no	✓	T no
(b) The ability for rapid and broad generalization of mathematical objects, relations, and operations.	A C	A	A yes	✓	A yes
			C y/n	x	C yes
			T no	✓	T no
(c) The ability to curtail the process of mathematical reasoning and the system of corresponding operations; the ability to think in curtailed structures.	A	A	A yes	✓	A yes
			C no	✓	C no
			T no	✓	T no
(d) Flexibility of mental processes in mathematical activity.	C	A C	A n/y	x	A no
			C yes	✓	C yes
			T no	✓	T no
(e) Striving for clarity, simplicity, economy (“elegance”), and rationality of solutions.	A C T	C? T	A y/n	x	A no
			C yes	✓	C yes
			T yes	✓	T yes
(f) The ability for rapid and free reconstruction of the direction of a mental process, switching from a direct to a reverse train of thought (reversibility of the mental process in mathematical reasoning).	A C	A C	A yes	✓	A yes
			C yes	✓	C yes
			T no	✓	T no
3. Retaining mathematical information					
(a) Mathematical memory [memory of mathematical generalizations] (generalized memory for mathematical relationships, type characteristics, schemes of arguments and proofs, methods of problem solving, and principles of approach).	A C?	A	A yes	✓	A yes
			C y/n	x	C no
			T no	✓	T no
4. General synthetic component					
(a) Mathematical cast of mind [striving to make the phenomena of the environment mathematical, constantly urging to pay attention to the mathematical aspect of phenomena, noticing spatial and quantitative relationships, bonds, and functional dependencies everywhere].	T A C	A C T	A yes	✓	A yes
			C yes	✓	C yes
			T yes	✓	T yes
			Matches: 22		
			Non-matches: 5		

Notes: A = Above-average ability; C = Creativity; T = Task-commitment; ✓ = ratings match; x = ratings do not match.

4.1. Results from the Mapping

4.1.1. Interrater Agreement and Disagreement

The two raters agreed in 22 out of 27 possible matches, resulting in an interrater agreement of 81.5%. The following traits were mapped identically by both raters: 2 (a), 2 (c), 2 (f), and 4 (a). In the other cases—1 (a), 2 (b), 2 (d), 2 (e), and 3 (a)—there were different mappings initially. Note that no more than one category (C, A, or T) showed disagreements in all of these cases.

Disagreement emerged twice on the relevance of *above-average ability* (2 (d) and 2 (e)) and three times on the relevance of *creativity* (1 (a), 2 (b), and 3 (a)). In their discussions on the disagreement cases, the raters elaborated on how and why they perceived that the mathematics traits each were related to above-average ability/creativity. This gave them the opportunity to also reflect on the concepts of above-average ability/creativity and their interrelation.

In the following, we elaborate on all cases (agreements and disagreements of the mapping process) in the order in which Krutetskii presented the traits. We outline the reasoning backing the raters’ mappings for each trait, regardless of the initial agreement.

4.1.2. Mapping Krutetskii's Traits to the Three Traits Creativity, Above-Average Ability, and Task-Commitment (Research Question 1)

Trait 1 (a)—formalized perception of mathematical material—deals with students' ability to identify certain characteristics, missing or superfluous information, as well as solution approaches of problems "on the spot" [9] (p. 232). This was matched to Renzulli's above-average ability by both raters as the ability to formalize, to sort out irrelevant information, is characteristic for high ability in mathematics. Krutetskii [9] (pp. 232 ff.) also mentions the ability to formulate questions, for instance, in the context of under-determined problems. Rater 1 initially mapped this trait to creativity, because formalizing, for instance, geometrical patterns and modelling them in different ways can involve creative thinking. Also, the posing of questions can be related to creativity in many cases. However, the formalized perception of mathematical material does not necessarily involve creativity in these senses. Thus, the raters consensually agreed not to map it to creativity.

Trait 2 (a)—the ability for logical thoughts and to think in mathematical symbols—was matched with Renzulli's ring above-average ability by both raters, because it is a genuine characteristic of high mathematical ability. The raters also agreed that this trait does not necessarily involve creativity and task commitment, so both raters did not match it to these traits.

Trait 2 (b)—the ability for rapid and broad generalization of mathematical objects—is considered on two levels by Krutetskii: "(1) as a person's ability to see something general and known to him in what is particular and concrete (subsuming a particular case under a known general concept), and (2) the ability to see something general and still unknown to him in what is isolated and particular (to deduce the general from particular cases, to form a concept)" [9] (p. 237). Both raters identified this trait as above-average ability, as the ability to generalize characterizes high ability in mathematics. However, searching for and identifying patterns, generalizing, and specializing are also abilities which have been described by Pólya [23] and other authors as heuristics for problem solving which are closely connected to creativity [24–26]. This connection was initially seen only by rater 1 but was agreed upon by both raters after a discussion.

For trait 2 (c)—the ability to curtail mathematical reasoning—Krutetskii provides the following example: given a series of statements A to F that can be logically concluded from one another, "the capable pupil 'sees' that F follows directly from A; he realizes that F follows *immediately* from A. This is not clear, however, to the average pupil (other conditions being equal); he does not 'see' it, does not realize it. To arrive at F from A, he must travel the sometimes rather complicated journey of actualizing interrelated associations" [9] (p. 275). This trait was matched solely to above-average ability by both raters. It does not necessarily involve creative problem-solving and task-commitment.

Trait 2 (d)—flexibility of mental processes—is explained by Krutetskii [9] (pp. 275 ff.) as switching from one method of solving a problem to another. The concept of mental flexibility is closely related to the concept of creativity [27] as both raters acknowledged. This trait could also be seen as related to Renzulli's ring above-average ability (as rater 2 initially did). However, Krutetskii emphasized that "shak[ing] off a fixed or unsuccessful method of solution" [9] (p. 278) had nothing to do with the difficulty of the approach: non-gifted students stuck to the first approach they found and could not switch it, no matter whether the approach was easy or not. Therefore, consensually this trait was not rated as above-average ability, but as only creativity-related.

Trait 2 (e)—striving for clarity, economy, and rationality—is described by Krutetskii as follows: "[...] capable pupils were usually not satisfied with the first solution they found. They did not stop working on a problem, but ascertained whether it was possible to improve the solution or to do the problem more simply. They evidently felt satisfaction only when the solution they had found was economical, rational, and 'elegant'" [9] (pp. 285 f.). Krutetskii (pp. 285 f.) even speaks about "emotional reactions" of gifted students to "'elegant solutions'" that they found themselves or that were presented to them. This trait was identified as belonging to creativity and task-commitment by both raters. Rater 1 also thought about this trait as belonging to above-average ability. However, this thought was discarded consensually, as Renzulli's description of creativity (as being "sensitive to aesthetic

characteristics" [18] (p. 70) and task-commitment (as a "refined or focused form of motivation" [7] (pp. 182 f.) as well as "setting high standards for one's work" [18] (p. 70) fully captures all aspects of this trait that do not belong to creativity.

Trait 2 (f)—reversibility in the process of mathematical reasoning—expresses itself, for example, in "the transition from a direct to a converse theorem" [9] (p. 287) or in "[taking] a 'sharp turn' [. . .] in the thought, from moving in one direction to moving in reverse" (ibid.). This trait is described as "one of the manifestations of flexible thinking" (ibid.) by Krutetskii and was, thus, independently rated as both related to above-average ability and creativity by both raters. As this trait refers to reasoning but not to dedicated work, it was not rated being related to task-commitment by the raters.

Trait 3 (a)—mathematical memory—is described by Krutetskii [9] (pp. 295 f.) as remembering concrete data (and, at the same time, identifying and not remembering superfluous data) as well as memorizing "generalized methods of solving [a problem], reasoning schemes, the basic lines of a proof, and logical patterns". This has been identified as belonging to above-average ability by both raters. The raters initially disagreed whether this trait also belongs to creativity. It was discussed whether the moment in which the methods of problem-solving are applied is a creative moment, where different aspects of knowledge are getting connected. However, Krutetskii's descriptions focus on generalized mathematical memory, not primarily on its application. Therefore, the raters consensually agreed on not mapping it to creativity.

Trait 4 (a)—a mathematical cast of mind—was elaborated by Krutetskii as follows: "It is expressed in a striving to make the phenomena of the environment mathematical, in a constant urge to pay attention to the mathematical aspect of phenomena, to notice spatial and quantitative relationships, bonds, and functional dependencies everywhere—in short, to see the world 'through mathematical eyes'" [9] (p. 302). This wide-ranging trait was matched to all three of Renzulli's rings by both raters: Krutetskii explains this trait as a "tendency to interpret environmental phenomena on the level of logical and mathematical categories, to perceive many phenomena through the prism of logical and mathematical relationships, and to distinguish mathematical aspect when perceiving many phenomena in an activity" [9] (p. 302) which was interpreted as above-average ability. He also describes this trait as dedication to mathematics—for example, by composing tables of astronomical data [9] (p. 303)—(a sign for task commitment) as well as a constant tendency to pose mathematical questions (a sign for creativity).

In summary, these findings demonstrate that the idea of matching is possible; all of Krutetskii's traits could be matched to Renzulli's rings. However, the independent matching did not provide perfect interrater agreement. To clarify the disagreement cases, we had to take a very close look into Krutetskii's findings, the mathematical problems he used, the examples he presented as well as Renzulli's descriptions of above-average ability, creativity, and task-commitment. With this detailed look, it was possible to reach consensus in all cases.

4.2. Domain-Specific Specification of the Three Traits Creativity, Above-Average Ability, and Task-Commitment (Research Question 2)

Based on the mapping process and the results for research question 1 (Section 4.1), we are able to specify Renzulli's three rings—creativity, above-average ability, and task-commitment—domain-specifically in the domain of mathematics (Figure 3).

It appears that the trait above-average ability is the one most addressed by Krutetskii's theory: Mathematical above-average ability in the context of mathematics learning comprises the abilities to formally perceive and mathematize material and problems, to think logically and in mathematical symbols, to think in terms of curtailed structures, and to have a generalized memory for mathematical information. Furthermore, the abilities of generalization and reversibility of thought address above-average ability as well as mathematical creativity.

Krutetskii's theory addressed creativity to a smaller extent. Based on the results of our mapping, we can say that mathematical creativity comprises a flexibility of mental processes in mathematical

activity as well as abilities of generalization and reversibility of thought, where the latter two abilities address above-average ability as well. Further, it comprises students' striving for clarity, simplicity, and elegance of solutions.

Task-commitment was addressed two times, but never appeared to be addressed solely: based on our findings, we see that task-commitment is involved when students strive for clarity, simplicity, economy, and rationality of solutions and in the so-called "mathematical cast of mind". In the latter case, all three traits are involved (as explained in Section 4.1).

We can summarize that the results of the mapping indicate that Krutetskii's abilities can specify the rings in Renzulli's model for the domain of mathematics. However, the rings seem to be addressed to a different extent. The abilities described in Krutetskii's theory predominantly address above-average ability. Seven out of nine of Krutetskii's abilities were actually mapped to this ring. This indicates a strong focus on above-average ability in this mathematics-specific theory on giftedness. Creativity was addressed in five out of nine abilities described by Krutetskii. So, more than half of the abilities described by Krutetskii seem to involve (to a different extent) some creative behavior. Finally, task-commitment was addressed twice, each combined with creativity or creativity and above-average ability.

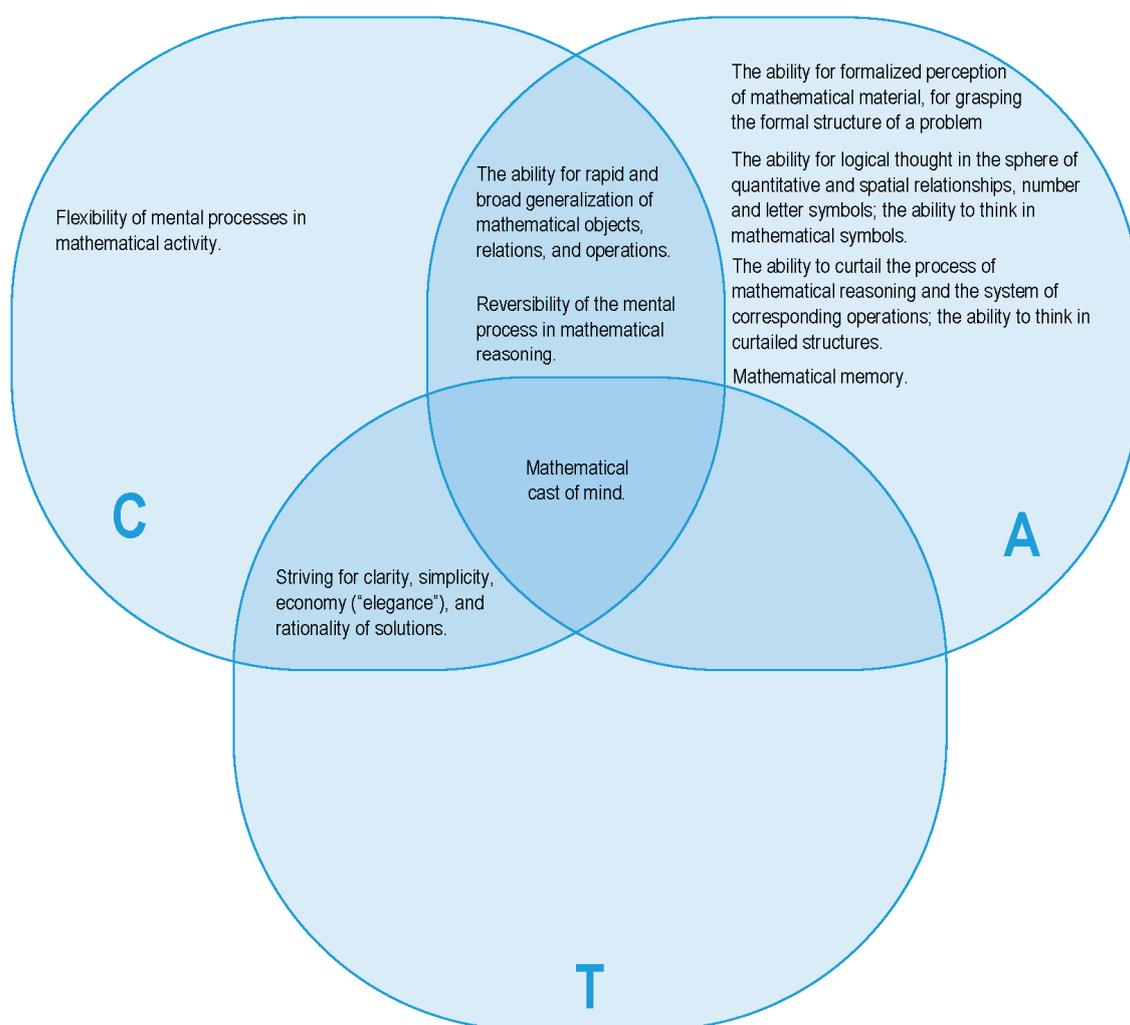


Figure 3. Domain-specific specification of the three traits Creativity, Above-Average Ability, and Task-Commitment.

5. Discussion

The crucial aim of this paper is to contribute to the ongoing research and theory discussion on giftedness, and mathematical giftedness in particular. We see this paper as a contribution to the efforts to theorize mathematical giftedness. Since theories of giftedness no longer only focus on intelligence as a single factor [28], various conceptions of giftedness have arisen in the last few decades. Thus, as Singer et al. state, “More research is needed on theoretical frameworks or models for explaining mathematical giftedness” [29] (p. 8). One of the major shortcomings of theory use in gifted education research is that connections, similarities, and differences of theories are rarely under investigation or made explicit. Therefore, our purpose was to make explicit connections between existing and frequently used theories on giftedness.

As Singer et al. state, “There are numerous definitions of a gifted child.” [29] (p. 2)—both in general and in the domain of mathematics. The shortcomings are especially relevant for connections between domain-general and domain-specific theories on giftedness, as there is a “gap between the general models of giftedness and the research about giftedness in mathematics” [30] (p. 5). Even though there are theories such as Tannenbaum’s “sea star model of giftedness” [31] that combine domain-general and domain-specific conceptualizations of giftedness, still connections between existing general and domain-specific theories are—in our view—not sufficiently dealt with. For this purpose, we conducted a networking process, in which we synthesized two particular theories: Renzulli’s domain-general three ring conception on giftedness and Krutetskii’s theory on mathematically gifted students’ abilities. With this endeavor, we first wanted to investigate the feasibility of networking processes between a particular domain-general and a mathematics-specific theory on giftedness. Second, we aimed to develop a mathematics-specific specification of Renzulli’s theory that combines findings of both theories and can be used by researchers and educators in the field of mathematics education.

For being able to conduct such a networking process and for specifically integrating the research results offered by Krutetskii into the three-ring model offered by Renzulli, we paid attention to their theoretical assumptions and the question of whether these are compatible with one another. Even though the theories have different foundations to a certain extent, we found that both theories have a multi-faceted perspective on giftedness and provide operational definitions of giftedness by considering traits or abilities described as activities.

We found that the networking process between Krutetskii’s and Renzulli’s theories was feasible: in a mapping process, in which two raters independently mapped each ability described by Krutetskii to the traits offered by Renzulli, both raters were able to map all abilities. The interrater agreement was 81.5 %, which can be considered fairly high. In the disagreement cases, the raters were able to find a consensual agreement in a further step, in which both raters’ readings and interpretations of the theories were discussed. We consider this approach to be suitable for conducting networking processes between domain-general and domain-specific theories on giftedness. We think that such processes would be beneficial also in other domains, for instance, music education, STEM-education (Science, Technology, Engineering, and Mathematics) in general, or sports education. However, the differences in the two raters’ mappings as well as their discussions indicated that especially the concept of creativity is conceptualized differently in the literature on giftedness from psychology and mathematics education—leading to different interpretations and, thus, mappings. We see that there is a need to further theorize the term creativity in the domain of mathematics education research.

Our networking process enabled us to illustrate what a more fine-grained specification of Renzulli’s three ring model in mathematics education can look like. Even though the current paper is to be understood as a proof of concept, still we see that it can be the springboard for developing a coherent theory that comprises theoretical elements from both a domain-general and domain-specific perspective. In particular, we found that Krutetskii’s abilities mainly address one of Renzulli’s rings: above-average ability. This reflects a huge significance of above-average ability in Krutetskii’s theory. The question arises of why there seems to be such a strong focus on this aspect. We think that

the research focus on other traits, especially on creativity, has increased especially in recent years in mathematics education research—after Krutetskii’s seminal investigations. Creativity has more and more moved into the focus of educational contexts due to its significance in our increasingly interconnected high-technology society and economy with its need for future entrepreneurs, engineers, and scientists [32]. Research in mathematics education has especially addressed mathematical creativity in the context of giftedness in recent years (e.g., [33,34]). We think that further networking processes could take these theoretical approaches into account and, if possible, integrate them in the mathematics-specific specification of Renzulli’s theory that we started with in this paper. However, our results also suggest that creativity appears to be addressed in different abilities described by Krutetskii. This indicates that even in the 1970s, Krutetskii’s conception of giftedness addressed creativity to a certain extent, even though this was not explicitly pointed out or labelled. Finally, we found that even task commitment was addressed in Krutetskii’s theory, although to a very restricted extent and always in connection to other traits. This leads us to the question of whether it is actually possible to view mathematics-specific task-commitment independently from creativity and above-average ability, or whether it should—in a domain-specific specification—always be considered in connection to these other traits. After all, we see that task-commitment has been neglected by research on mathematical giftedness and should be dealt with more extensively in future research.

We must acknowledge that the methodological approach—the rating process—presented in this paper has certain limitations. Even though the independent raters showed high interrater agreement, the number of raters was small. As shown in Section 4.1.2, the mapping process requires deep insights into both Renzulli’s and Krutetskii’s theories. Therefore, it cannot easily be extended to or be reproduced by unexperienced raters not familiar with the theories by simply applying the rules of a coding manual. However, the results of the mapping process have been presented and discussed in the Topic Study Group on giftedness at the 2016 edition of the quadrennial ICME (International Congress on Mathematical Education) [35]. The general results of the mapping process were welcomed by the participants of the Topic Study Group. The discussion revealed, furthermore, that even expert researchers on mathematical creativity cannot easily contribute to the ratings in a meaningful way unless they are familiar with both theories. This supports our argument that a mapping as conducted in this paper requires extensive background knowledge and/or background reading of the raters.

We hope that our approach, our findings, the considerations, and open questions can lift the discussion on theories and theory development in the domain of giftedness, a domain which—as many other domains—will benefit from explicit further theoretical work and networking processes. We think that our approach contributed to the “aim at creating a dialogue and establishing relationships between parts of theoretical approaches” [15] (p. 118). We did not intend to extend Renzulli’s model, as Mönks [36], for instance, did when adding environmental aspects to Renzulli’s model; rather, we related the elements or aspects of giftedness mentioned in two theories to one another. However, Mönks lifts an important aspect that is increasingly perceived as significant in the context of giftedness in educational settings: this is, how Kaufman and Sternberg phrased it, “contextual variables, such as enculturation and socialization” [16] (p. 87). We think that future research should focus on networking processes that also take into account students’ environments and backgrounds in order to do justice to the students (p. 87).

Even though every theory has its own values and opportunities to offer, we think that networking processes as illustrated in this paper can make our scientific field more coherent or can at least point towards differences, contrasts, or contradictions. Networking processes can help researchers to cope with the “overwhelming” number and variety of theories (cf. [16] (p. 79)) and makes us aware of how theories stand in comparison to each other. In the long run, this will contribute to scientific quality and sustainability. We hope that future researchers will follow the path that we have suggested in our paper.

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