Teaching Algebraic Equations to Middle School Students with Intellectual Disabilities

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Abstract

The purpose of this study was to replicate similar instructional techniques of Jimenez, Browder, and Courtade (2008) using a single-subject multiple-probe across participants design to investigate the effects of task analytic instruction coupled with semi-concrete representations to teach linear algebraic equations to middle school students with intellectual disabilities. Over the past decade, instructional strategies used to teach academics to students with intellectual disabilities have seen a dramatic change. Federal laws (e.g., IDEA, 1997; 2004) and state assessments have assisted in creating a balance of functional and academic instruction with this population. Data were analyzed using visual inspection and descriptive comparison between baseline and intervention phases for each student. Results suggest a functional relationship across all participants. Generalization measures and limitations are discussed.

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Since the introduction of federal mandates such as No Child Left Behind (NCLB, 2002) and the Individuals with Disabilities Education Improvement Act (IDEIA, 2004) there have been increased efforts to build student knowledge in the area of mathematics. The United States Department of Education (2009) has invested in public schools across the nation and has made a conscious effort to establish math excellence. These measures have been taken with good reason. According to the final report provided by the National Mathematics Advisory Panel (NMAP, 2008), mathematic literacy has been a consistent problem in the United States, specifically in the content area of algebra. The NMAP has stated that algebra is “a gateway to later achievement” (p. 3), suggesting that being able to master algebraic principles leads to acquiring more advanced mathematical skills.

Teaching how to solve algebraic equations is not only critical for improving mathematical literacy for typically developing students but it is equally important for students with intellectual disabilities (ID). If in fact learning algebraic principles leads to acquisition of more advanced mathematical skills then it becomes essential that students with ID learn these skills in a way that is meaningful to them, especially since federal mandates (e.g., NCLB, 2002; IDEIA, 2004) entail access to the general curriculum and require that students make progress in content areas (e.g., Algebra). Currently, in spite of strides made to improve age-appropriate content focused on math excellence, students with ID are often taught functional math skills (Browder & Grasso, 1999; Browder, Spooner, Ahlgrim-Delzell, Harris, & Wakeman, 2008; Jimenez, Browder, & Courtade, 2008). For example, Browder, Spooner, et al. (2008) conducted a meta-analysis to determine the types of mathematical interventions/skills that have been taught to students with moderate to severe ID. The results yielded 68 experiments, of which 54 were single subject design studies. Of these interventions, 48 of the studies concentrated on functional math skills such as money, number matching, counting, and basic calculations, and only two studies focused on algebra instruction (e.g., solving story word problems, quantifying numbers; Miser, 1985; Neef, Nelles, Iwata, & Page, 2003). Functional skills are needed and should be considered for this population; however when teachers are required to meet content standards, the age-appropriate academic component should not be neglected.

Despite the lack of instruction on algebraic skills, several studies found by Browder, Spooner, et al. (2008) successfully used systematic instruction to teach functional math skills to students with ID (e.g., Akmanoglu & Batu, 2004; Colyer & Collins, 1996; McDonnell, 1987; McDonnell & Ferguson 1989). In addition to Browder, Spooner, et al. a more recent review by Spooner, Knight, Browder, and Smith (2012) also found that these systematic instruction components (e.g., task analytic instruction, prompting procedures, corrective feedback, discrete response training) could be recognized as evidenced-based practices used to teach age-appropriate academics to students with moderate to severe ID. Since there is a need for additional research for successful teaching strategies for this population, especially in math (Browder, Spooner, et al.), the use of evidence-based practices suggested by Spooner et al. (i.e., systematic instruction components) should be considered when teaching math to this population.

Several studies related to teaching algebra to students with learning disabilities (LD) and emotional and behavioral disorders (EBD) have been conducted (Mancl, Miller, & Kennedy, 2012; Witzel, 2005; Witzel, Mercer, & Miller, 2003), however a limited number of studies exist for students with ID. Findings from these studies support a framework for the systematic and explicit instruction of components of algebra that include the use of a graduated instructional sequence (e.g., concrete, semiconcrete/representational, abstract) and graphic organizers (Dexter & Hughes, 2011; Ives, 2007; Mancl et al., 2012; Miller & Hudson, 2007; Rotter, 2004; Strickland & Maccini, 2010; Witzel, 2005; Witzel et al., 2003). The graduated instructional sequence, often defined as either the concrete-representational-abstract (CRA) or concrete-semiconcrete-abstract (CSA) sequences, have been found to have large impacts on the mathematics achievement of students with disabilities (Mancl et al., 2012; Witzel, 2005; Witzel et al., 2003). In this sequence, instruction begins with the use of concrete manipulative devices, representing abstract mathematics, which students use in order to develop conceptual understanding of a mathematical principle (Witzel et al., 2003). Once mastery has been developed at the concrete level, students move into the representational or semiconcrete stage where they use drawings or structures (e.g., graphic organizers) to support their continued mastery of the mathematical principle (Strickland & Maccini, 2010). The final stage of the process, the abstract stage, removes all scaffolds and students complete the mathematics with mathematical symbols alone (Witzel et al., 2003).

Graphic organizers are commonly used in algebra classes to support the semiconcrete understanding of concepts being taught, as much of algebra is difficult to represent at the concrete level (Dexter & Hughes, 2011; Strickland & Maccini, 2010). A graphic organizer acts as a visual cue that includes graphic elements (e.g., color-coded boxes) that can systematically guide students through the problem solving process. Graphic organizers have been found to have large impacts on the reading comprehension of students with disabilities, and when applied to mathematics, similarly support students in developing a clear understanding of the relationships of different mathematical concepts (Dexter & Hughes, 2011; Ives, 2007). Strickland and Maccini (2010) suggested that graphic organizers may address deficits in the language of mathematics and in working memory to support students in solving multistep problems common in algebra. This framework allows students to develop strong conceptual understanding of the components of algebra, thereby increasing their understanding of the steps that should be completed to solve specific types of problems (Ives, 2007; Witzel, 2005).

To this end, Jimenez et al. (2008) conducted, what the authors suggested, the first experimental study employing a framework of systematic and explicit instruction that examined how to teach an algebra skill to students with moderate ID using the graduated instructional sequence. Jimenez et al. used a multiple-probe across participants design to evaluate the efficacy of a task analytic intervention, paired with concrete representations (e.g., Mancl, Miller, & Kennedy, 2012; Strickland & Maccini, 2012; Witzel, 2005; Witzel, Mercer, & Miller, 2003), objects or materials that can be manipulated by a student in order to aide in the acquisition of academic or functional skills, on the acquisition of algebraic equations for high school students with moderate ID in an urban self-contained setting. Data were collected on the correct independent number of steps students completed using an algebraic equation task analysis to solve for *Y*. Students in the study were given a demonstration on how to complete an algebraic equation using manipulative devices (e.g., a color coded number line, erasable markers, additional objects for counting) in order to provide them with concrete (e.g., objects to count) and semiconcrete (e.g., color-coded number line) experiences. During the intervention phase, results of the study demonstrate an increase in the steps completed to solve algebraic equations across all participants as compared to the baseline phase. Two of the three participants were able to maintain skills in follow up sessions and were able to generalize skills across different materials and settings.

The findings from Jimenez et al. (2008) provide two important implications on what algebra instruction may need to include for students with ID. First, instruction may need to incorporate manipulative devices and semi concrete representations to make abstract concepts more concrete (e.g., Witzel, et al., 2003; Witzel, 2005). According to Witzel (2005), the use of concrete or semi concrete materials during math instruction helps to increase the probability that children will learn sequential steps that are typically associated with problem solving. Secondly, instruction may need to utilize prompting procedures (e.g., time delay, a system of least-to-most prompts, a system of most-to-least prompts), repeated trials, and task analyses (e.g., Browder, Spooner, et al., 2008; Spooner et al., 2012). By taking these current implications into consideration, and knowing the need for additional research in this particular subject area, the purpose of this study was to strengthen the reliability and generality of the implications for algebra instruction suggested by Jimenez et al. The current study examined the effects of task analytic instruction with concrete and semi concrete representations, including the use of color-coded sequencing and graphic organizers, on the acquisition of simple linear algebraic equations (e.g., $5 + Y = 15$) for students with ID.

Method

Participants

Three middle school students with ID met the following requirements to be included in the study. The students: a) had an IQ score with a confidence interval that fell within the mild moderate range (i.e., < 70), b) were verbal, c) were able to write numbers legibly, d) had basic calculator skills (i.e., recognize numbers, addition, subtraction, & equals buttons), and e) participated in the state's alternate assessment. The students who participated (pseudonyms used below) met the aforementioned requirements and were also identified by their classroom teacher as having deficits in functional math skills (e.g., identifying numbers, counting in sequence, using money for purchases, telling time). Ashlynn was a 15-year-old Caucasian female. Jim was a 15-year-old Caucasian male and Ronald was a 12-year-old Caucasian male. All students received daily life skills instruction, primarily in a one-on-one format, in a self-contained setting, and were only included into the general education environment for a small percentage of the day. IQ scores for each student were 63 (*Wechsler Abbreviated Scale of Intelligence,* 1999), 56 (*Kaufman Brief Intelligence Test,* 2004) and 43 (*Wechsler Intelligence Scale for Children -IV,* 2003) respectively, with a mean of 53.

Setting

The setting was in a rural public middle school in the southeastern United States. The middle school was a Title One school that served grades sixth through eighth and had a population of over 800 students. Eight students in this class met the eligibility requirements for the state's alternative assessment and a majority (i.e., 14 out of 15) received free and reduced lunch. The study took place in a self-contained classroom for students with ID during the teacher's planning period (i.e., 7:45 A.M and 8:45 A.M). During this time of the school day the students were at breakfast. Each student participating in the study was taken from the breakfast setting and taken to the classroom in order to solve the equations one-on-one with the teacher. Each data collection session was approximately 15 minutes long with a range between 5 to 20 minutes.

Experimenter

The fourth author of this study was the teacher and the primary experimenter. She has a Master's degree in special education, and over 15 years of experience working with students with disabilities as an educator, department chair, Special Olympics coordinator, and alternative assessment chair.

Data Collection Procedure

Dependent variable. The research team used the implications for algebra instruction suggested by Jimenez et al. (2008) by pairing concrete and semi concrete representations paired with systematic instruction to teach algebraic equations. This intervention identified 10-steps of solving an equation using a task analysis (see Table 1) in addition to concrete manipulative devices (e.g., number flip cards) and semi concrete representation (e.g., graphic organizer using color-coded sequencing) (see Figure 1). The dependent measure for this study was the number of correct steps the students completed independently on the 10-step task analysis to solve for *Y* in a simple algebraic equation. The researchers recorded a "+" if students completed a step independently, and a "-" was recorded if students needed any level of prompting, or omitted a step. The total number of steps students completed independently was graphed during baseline and intervention phases.

Inter-rater reliability (IRR). A second member of the research team (i.e., the first author) and the paraprofessional in the classroom served as second data collectors during baseline and intervention sessions for each student. Inter-rater reliability (IRR) was calculated using an item-by-item method, by dividing the total number of agreements by the total number of possible agreements plus disagreements and multiplying by 100. As recommended by Kratochwill et al. (2010), IRR was collected during 20% of sessions for Ashlynn resulting in a 100% agreement rate. For Jim, IRR was collected during 20% of sessions with a 100% agreement. Finally for Ronald, IRR was collected during 40% of sessions with a 95% agreement. Overall, IRR was completed for 23% of the baseline and intervention sessions and resulted in a 98.5% agreement rate.

Social validity. At the end of the study, a social validity survey was emailed to the parents of the three students to examine the practical significance. The surveys intent was to provide the parents with the procedures of the study and to assess parent and student satisfaction. The survey required the parents to read the survey's questions to their child and answer the questions using a 5-point Likert scale. The brief survey found that the students enjoyed learning about algebra and reported the intervention to be entertaining and beneficial.

Materials

Most of the materials used in this study were teacher made and/or easily accessible and included: a) an 8 X 11 laminated color-coded graphic organizer to support student understanding of the algebra concepts being taught (i.e., semi concrete representation) with Velcro removable numbers that were called "flip cards," (i.e., concrete manipulative devices) b) dry erase markers (to write their answers and make notes if so desired), c) dry erase boards that served as "scratch paper," for their notes if needed, for problem solving, d) calculators, and e) a 10-step task analysis. Each graphic organizer (see Figure 1) contained a specific laminated linear equation (e.g., $5 + y = 12$; $y - 11 = 28$) that was stapled to yellow (i.e., addition problems) and red (i.e., subtraction problems) folders. Velcro was attached to each box on the individual graphic organizers with the exception of the gray boxes that followed the expression "$Y =$" in which the students were instructed to write the answer to the equation. A total of 10 equations were made for the baseline and intervention phases of this study (i.e., 5 addition and 5 subtraction), and two additional were designed for the teaching phase.

Research Design

A single-subject multiple probe design across participants was implemented to evaluate the effectiveness of a task analysis coupled with concrete and semi concrete representations to teach middle school students with ID simple linear algebraic equations. Before the study began, the three participants were randomly assigned to see who would receive the intervention first. Baseline data followed for all of the students for three consecutive days. Since a stable baseline trend was established for all students, the first student (i.e., Ashlynn) that was randomly assigned to the intervention entered the training phase. Once Ashlynn completed the training phase, the intervention began and the researchers examined the number of independent responses made. Once Ashlynn was able to master six out of 10 steps on the task analysis for three consecutive sessions the second student, Jim, was then introduced to the training phase and then the intervention. These steps were also followed for the final participant.

Research Procedure

Baseline phase. Baseline data were collected on all three students for three consecutive days. The researchers felt that it would have been unethical to collect baseline data for additional sessions due to the frustration and difficulties students experienced when required to complete the algebraic equation without training. During the first three baseline sessions each student was given a calculator, a dry erase board, dry erase markers, and the same equations (i.e., $5 + y = 12$; $y - 2 = 14$; $10 + y = 30$) following the data collection routine described previously. For example, on the first day, Ashylnn was given $5 + y = 12$ to complete. After her attempt, Jim and Ronald were obtained at different periods during the allotted time. Additional probe data were deemed appropriate to collect on Ronald to examine a possible threat of treatment diffusion since many data collection sessions had passed (i.e., approx. 15 sessions) before he was able to enter the training and intervention phases.

Training phase. Before students were introduced to the intervention phase, an errorless learning procedure was used to assure that students had a chance to complete the desired responses successfully. The researchers used a 0-second delay method to teach each student the steps on the task analysis in order to complete two algebraic equations (i.e., one addition and one subtraction equation). During the training phase the teacher followed a written script (i.e., procedural fidelity checklist) of the task analysis to teach students an addition equation (e.g., $5 + Y = 12$) and a subtraction equation (e.g., $Y - 2 = 14$). The script followed the task analysis and consisted of the following constructs: a) what the teacher says (e.g., we are going to complete an algebra equation), b) material presentation (e.g., make sure all of the manipulative devices and the graphic organizer were available), c) instructing student responses (e.g., the steps required to complete the algebraic equation) and d) prompting (i.e., errorless learning during the teaching phase; least to most prompts during the intervention). The students were instructed how and when they will use their calculators to help them complete the algebraic equations. Additionally a dry erase board and marker was given to the students (to serve as "scratch paper") in case they wanted to write any additional information pertaining to the problem.

Intervention phase. After the students were introduced to the teaching phase, intervention data were collected on the following session (i.e., the next day) and continued for the duration of the study. Similar to baseline, the intervention data were collected on the number of steps completed on the task analysis. Calculators, dry erase boards, and dry erase markers along with the laminated equations were given to the students. During the intervention, a system of least to most prompts was introduced with a 3-second delay. A 5-second delay between prompts was used with Ronald because it was noted that he needed more time to process and respond to information. At the beginning of the intervention session students were given an algebraic problem with their number flip cards (e.g., concrete representation) and graphic organizer (i.e., semi concrete representation). They were first given the opportunity to complete the steps independently. If a step was missed or omitted the researcher would give an indirect verbal prompt (e.g., "What is the first thing we do when we have to complete an algebra equation?"). If needed, a direct verbal prompt was then provided (e.g., "Read the equation aloud"). Further assistance included modeling (e.g., "I am going to flip this number to the same color. Now you try.") followed by, if necessary, a physical prompt (e.g., "Let me help you flip this number."). Besides the additional response time Ronald was given, all students were provided the same level of prompts.

Generalization phase. During the generalization phase the researchers chose to make adaptations to the provided materials as the students became more proficient on the steps of the task analysis. The researchers wanted to evaluate if the task analysis and the semi concrete representation could have served as a scaffolding technique in order to eventually promote complete independence and problem completion. During this phase, portions of the semi concrete representation materials in conjunction with the task analysis were faded away. For example, the researchers systematically made adaptations to the materials by: a) removing the Velcro from the equations (e.g., transition from concrete to semi concrete), b) removing the Velcro and omitting the color boxes by using gray boxes only, c) removing the Velcro omitting the grey boxes by using white boxes only (e.g., transition from semi concrete to abstract), and d) deleting some of the steps of the task analysis in order to make the problem solving process more "natural."

Procedural fidelity. A second member of the research team (i.e., the first author) as well as the paraprofessional in the classroom collected procedural fidelity. Procedural fidelity assessed whether the teacher researcher followed the designed task analysis and to assure that each step was being completed in the original order using a least to most prompting system (i.e., indirect and direct verbal, model, and physical prompts). Procedural fidelity occurred for 19% for Ashlynn, 33% for Jim, and 27%, for Ronald. Procedural fidelity was collected during baseline, teaching, and intervention phases resulting in 100% completion across all three students.

Results

Student performance data for independent steps completed on the task analysis are displayed in Figure 2. All three students showed an increase in the number of correct responses on the steps of the task analysis throughout the duration of the intervention. An increase is demonstrated when comparing the number of steps the students' completed during the baseline phase (*M*=1; range from 0 to 2) to the number of steps completed during the intervention phase (*M*=7.2; range from 2 to 9).

Ashlynn. During instructional sessions, Ashlynn increased the number of correct responses from baseline (M=1.3; range from 1 to 2) to intervention (M=7.9; range from 5 to 9). She required assistance during initial instructional sessions; however, Ashlynn quickly mastered 60% of the steps by the second intervention session. She was able to complete 90% of the steps by the 8th session of the intervention phase. It was found that Ashlynn always omitted step 2 (i.e., match the given variables and function with the corresponding colored box), which prevented her from completing 100% of the desired steps.

Jim. During instructional sessions, Jim increased the number of correct responses from baseline (M=.67; range from 0 to 1) to intervention (M=7.4; range from 3 to 9). During the intervention phase, Jim needed a number of prompts during the first three sessions in order to complete the equation. More specifically, due to his communication difficulties, Jim was given additional prompts when required to read the problem aloud (i.e., step 1) as well as to confirm if they matched (i.e., step 10). As a result, the primary researcher often provided Jim with a prompt to help him confirm that the answers matched (i.e., 10th step). Despite his difficulties, Jim was able to meet the research criteria and correctly completed six or more steps of the task analysis independently for more than three consecutive sessions.

Ronald. During instructional sessions, Ronald increased the number of correct responses from baseline (M=0) to intervention (M=5.2; range from 2 to 8). Due to time constraints, Ronald was only in the intervention for 4 sessions, and the school district's spring break required a 10-day lapse in data collection between his second and third intervention session. Although Ronald did not get to spend as much time in the intervention phase, he started showing a slow increase in trend by completing 70% of the steps independently on his last session.

Generalization/Adaptations

As previously mentioned, the researchers wanted to evaluate if the task analysis and the pairing of concrete and semi concrete representation could have served as a scaffolding technique in order to eventually promote complete independence and problem completion. At the start of the study it was determined that the components of the graphic organizer would be faded by: a) removing the Velcro from the equations, b) removing the Velcro and omitting the color boxes by using gray boxes only, c) removing the Velcro omitting the grey boxes by using white boxes only, and d) deleting some of the steps of the task analysis in order to make the problem solving process more "typical" and efficient. After implementing the adaptations it was determined that the researchers would omit the 3rd generalization step (i.e., removing the Velcro omitting the grey boxes by using white boxes only) because of the students' difficulty completing the steps after the color boxes were faded during the 2nd step (i.e., using grey boxes). Table 2 shows the percentage of required steps completed during the generalization/adaptation stage.

It is important to note, due to time constraints and student absences, that every student was not given an equal amount of time using the intervention adaptations. Ashlynn was provided with the most instruction showing that 77% of the required adaptation steps were completed across steps 1, 2, and 4. Again, it is important to note that it was determined that adaptation step 3 was omitted due to Ashlynn's decrease from the no Velcro step (i.e., step 1; 90%) to the gray box only step (i.e., step 2; 40%). Jim was only given adaptations 1 and 4 and completed 55% of the required steps. Jim was not given the 2nd adaptation (i.e., gray boxes only) due to his absences during this planned step. Ronald was only given adaptation 1 and 2, due to time constraints, showing a required completion mean of 50%. Similar to Ashlynn it was found that Ronald decreased from step 1(i.e., 90%) to step 2 (i.e., 10%).

Discussion

Similar to Jimenez et al., (2008), the results of this research suggest that students with mild and moderate ID are able to demonstrate the acquisition of new math skills that are grade and age-appropriate when provided systematic instruction through the use of task analyses, prompting, and pairings of concrete and semi concrete representations (i.e., graphic organizers). This study adds to the small body of literature supporting the use of standards-based instruction in math for this population of students (e.g., Browder, Jimenez, & Trela, 2012; Browder, Spooner, et al., 2008; Jimenez et al., 2008) by demonstrating gains in acquisition of mathematical content. While all students performed well during the intervention phase, compared to baseline, it is important to note that Ashlynn did not have to complete all the steps in the task analysis in order to answer the equation. Ashlynn often omitted step 2 which required her to match the given variables and function with the corresponding colored box. On the other hand Jim and Ronald needed all the steps to complete the equations successfully. Despite the omission of step 2, on behalf of Ashlynn, this study verifies the usefulness of the intervention package. It is also important to note that there are possibly two main elements to the intervention package that may have contributed to overall student success.

First, the intervention package included the use of concrete and semi concrete representations, which for example, have been used by Mancl, Miller, & Kennedy (2012), Witzel (2003), Witzel et al. (2005), and Jimenez et al. (2008) to teach abstract mathematical concepts to students with and without disabilities. Concrete and semi concrete materials, as stated earlier, help to simplify the process of learning algebraic equations, increasing the likelihood that students can remember critical steps that are involved in the problem solving process. However, unlike Witzel's studies and Jimenez et al., this study used a hybrid of concrete and semi concrete representations to teach equations to students with ID. For example, the current study incorporated the use of concrete (e.g., numerical flip cards) and semi concrete representations (i.e., color-coded graphic organizers). The incorporation of graphic organizers, during academic instruction, is often used to teach mathematical skills to students with learning disabilities and emotional behavior disorders because of its positive empirical value (Ives, 2007; Maccini & Gagnon, 2006). The use of this hybrid model of concrete and semi concrete representations (i.e., flip cards, graphic organizers) may be beneficial for helping students with mild and moderate ID acquire the necessary skills for learning to solve algebraic equations because it provides both visual and physical supports, extending the literature on using such a model for this specific population.

Secondly, an important aspect of the intervention package was the use of a task analysis paired with a least-to-most prompting procedure. According to Spooner et al. (2012) systematic instructional procedures such as the use of a task analyses have been used to teach a variety of academic skills (e.g., literacy, math, & science) to students with ID. In fact, recent research in literacy has suggested that TAs can be monumental in teaching emergent literacy skills to students with IDD or ASD (Browder, Trela, & Jimenez, 2007; Spooner, Rivera, Browder, Baker, & Salas, 2009). These skills can include print concepts (e.g., opening a book, tuning a page, text pointing), alphabet knowledge, and phonological awareness (Browder et al. 2009; Browder, Ahlgrim-Delzell, Courtade, Gibbs, & Flowers, 2008; Browder & Spooner, 2006; Justice & Kaderavek, 2002; Spooner et al., 2009). Since later conventional forms of literacy (e.g., story problems in math) are influenced upon the success of students' ability to master early literacy skills it is important to teach emergent literacy to students with ID and ASD. In a recent study by Browder et al. (2012) students with ID were taught math units that aligned with the national math standards. The math standards were taught using story-based math problems pairing key math vocabulary with semi concrete symbols (i.e., graphic organizers) and manipulative devices to assist with student comprehension. Similar to the current study, Browder et al. (2012) found that the use of multiple graphic organizers paired with task analytic instruction increased the students' acquisition of math skills.

A unique aspect of the current study was the attempt to fade the use of the concrete representations. Although the use of graphic organizers and TAs have been found effective for this population across academic (e.g., Browder et al., 2012) and functional skills (e.g., Gaule, Nietupski, & Certo, 1985), the current study attempted to fade the organizers as well as the steps of the TA to promote student independence. This study attempted to examine the use of systematic procedures to fade aspects of the hybrid intervention. Past research has examined the use of fading when teaching reinforcement (Kelley, Lerman, Fisher, Roane, & Zangrillo, 2011), self-management (Rock & Thead, 2007), as well as sight word instruction (Didden, de Graaff, Nelemans, Vooren, & Lancioni, 2006) and some have found the difficulty in fading visual aids (Sigafoos et al., 2007). Research by Zisimopoulos (2010) found that students with moderate ID were able to acquire, maintain, and generalize basic multiplication facts when concrete and semi concrete representations were faded. The study by Zisimopoulos, introduced multiplication facts paired with visual representations that were systematically faded across sessions. Upon completion of the intervention both students were able to generalize and maintain most of the math facts without the pictures. Although all of the adaptations made for this study did not show large gains, some progress was found as the intervention package was faded (see Table 2). Future research may consider using fading techniques in order to promote student independence and decrease student dependence on treatment packages.

Limitations and Future Research

Several limitations to the study should be considered and used to advance the research in this particular subject area. First, although the students were randomly assigned to the intervention phase, by chance, the students with higher IQs, communication skills, and processing abilities were introduced to the treatment first. Though it did not appear to show a decrease in trend during the intervention phase, it should be noted that the students' individual differences might have impacted the gains on the desired task. Future research may need to account for the individual differences of the students and stratify the selection of students to the intervention to assure that there is an equal representation of abilities entering the treatment phases.

The second limitation of the current study is that the instruction was taught in a self-contained environment that was not during normal instruction time (e.g., breakfast). Although, the researchers made sure the students had eaten breakfast and finished their morning routine before training, the students may have not been mentally ready to complete math outside their normal daily routine. Future research may suggest a more typical environment to implement similar strategies.

A third limitation was time constraint. Since it was late in the school year, the researchers had to take into account school breaks and end of year assessments. Furthermore, the study ended as the third student started to show positive gains in instruction. Future research should plan enough time, taking into consideration school functions, and breaks in order to collect a similar amount of data across all students. A final limitation to the study was that the generalization phase, also described as the adaptation phase, was not equally implemented across all of the students. Due to student absences, lack of time, and a failed conceptual design of one of the aspects it was difficult to capture a good understanding of the adaptations that could be made to insure independence. It was the researchers' intentions to slowly modify the graphic organizer in order for the students to complete the task without concrete assistance. However, although this is a limitation, some required skills were found across all students during this phase. Future research, again, should take time constraints into consideration in order to examine the completed research design.

Conclusion

The purpose of the following study was to examine the effects of the use of a task analysis with the incorporation of concrete and semi-concrete representations to teach middle school students with ID how to solve simple linear algebraic equations. All three students, despite variability in IQ range, age, and grade level, were successfully able to solve basic linear equations. Results from the study help to support research from Jimenez et al. (2008), Witzel, (2005), and Witzel, et al. (2003) about the importance of utilizing concrete and semi concrete representations as aides in explaining abstract concepts while also incorporating explicit and systematic instruction to students with disabilities. Moreover, it demonstrated the usefulness of graphic organizers for such populations. This study showed positive gains, however there is a need for additional research in this area that may examine the use of these strategies across grade levels, additional math content, and/or the use of technology to provide additional supports. As noted before, there is a lack of data driven research in algebra for this population (Browder, Spooner, et al., 2008). Due to the increase of accountability measures and the need for teachers to access the general education curriculum, more studies need to examine age-appropriate strategies that can be used with students with ID to promote access.

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Table 1

Task Analysis of Steps to Solve an Algebraic Equation

Student:				Student will be able to follow the steps to answer an algebraic equation independently.				
Steps (below)	Date	Worksheet Number _						
1. Student will *read the equation aloud** (if necessary with assistance) $5+Y=8$								
2. Student will *match the given variables and function with the corresponding colored box.*								
3. Student will *move the function and flip it to the corresponding color on the opposite side of the equation.*								
4. Student will move *the number variable to the opposite side of the equation.*								
5. Student *will type in calculator the new equation.* (*e.g.*, 8-5=)								
6. Student will *write the answer in the gray text box.*								
7. Student will *check answer by placing the original equation on the corresponding boxes* and will *write in answer for Y.*								
8. Student will *type in calculator, equation with answer for Y.*								
9. Student will *write in answer from calculator*.								
10. Student will *confirm if answer in the corresponding white text box matches original sum.*								

(+) Independent correct; (V) Verbal; (M) Model; (P) Physical; (0) no response; (-) Error

* Note: The verbal prompts provided to the students are in bold.

Table 2

Adaptations made Post-Intervention to the Graphic Organizer, Percentages of Required Steps Completed, and Means

Student	Adaptation	Mean Percentage of Required Steps Completed	Mean Percentage Of Steps for Each Student
Ashylnn	No Velcro	90%	
	No Velcro Gray Box Only	40%	
	*No Velcro White Boxes Only	*N/A	
	**No Velcro without Steps 2,7,8,9,10	100%	
			77%
Jim	No Velcro	70%	
	No Velcro Gray Box Only	N/A	
	*No Velcro White Boxes Only	*N/A	
	**No Velcro without Steps 2,7,8,9,10	40%	
			55%
Ronald	No Velcro	90%	
	No Velcro Gray Box Only	10%	
	*No Velcro White Boxes Only	*N/A	
	**No Velcro without Steps 2,7,8,9,10	N/A	
			50%

*It was determined that this step was not feasible due to the percentage decrease during the gray box only step

**This step was added to assess accuracy and fluency. It was determined that these were the steps needed in order to get the answer correct.

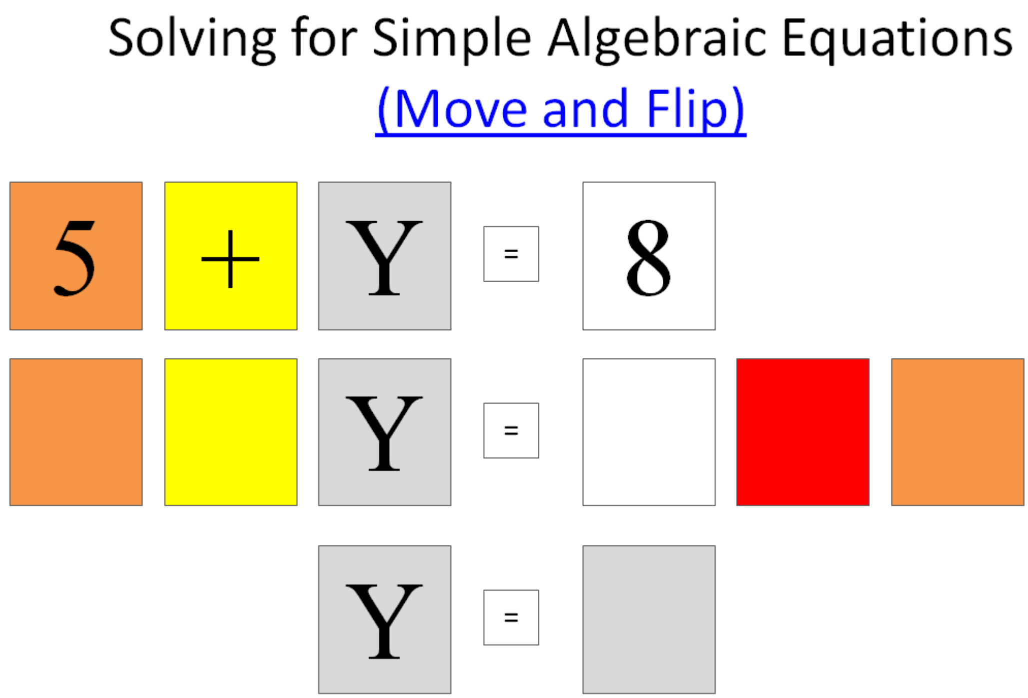

Figure 1. Example of concrete representation.

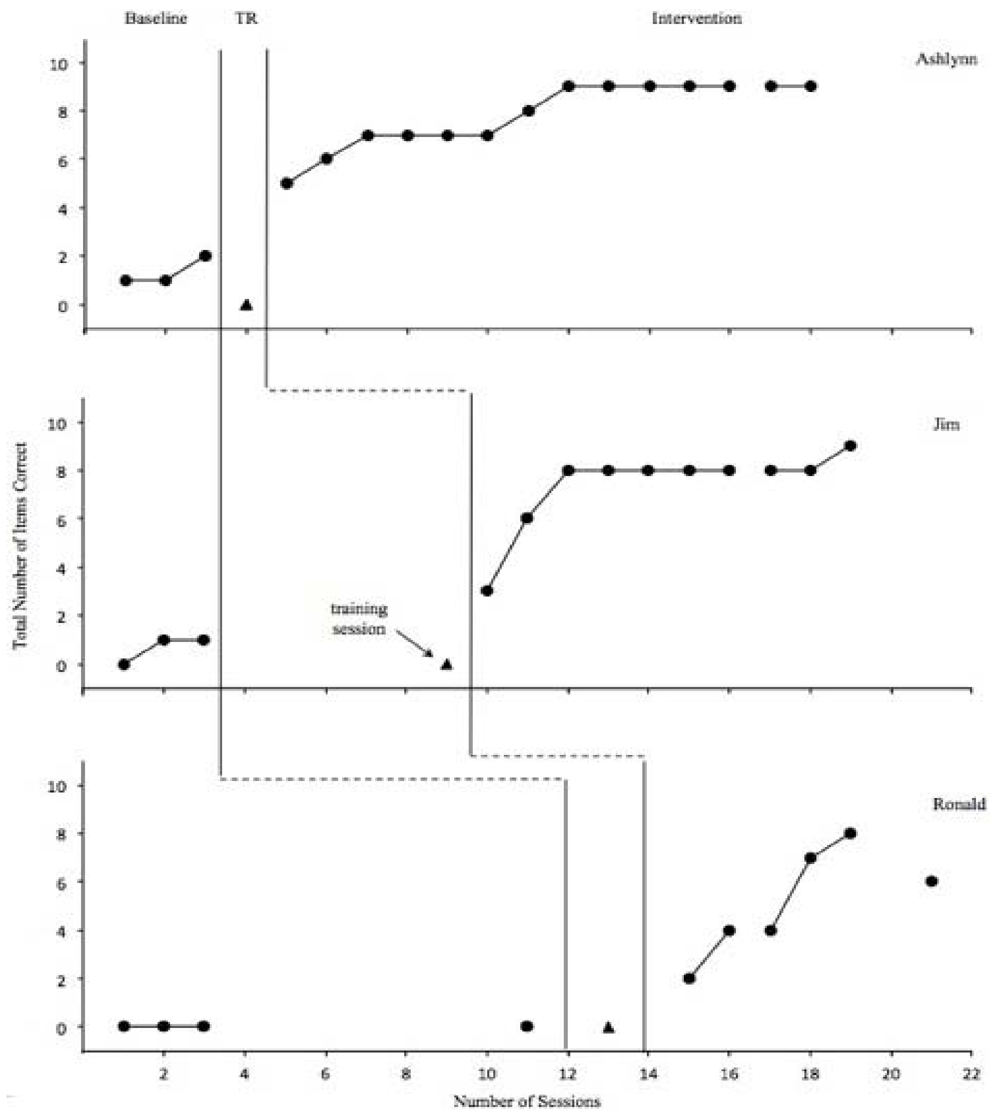

Figure 2. Results across student participants.