



INSTRUCTIONAL DESIGN-BASED RESEARCH ON PROBLEM SOLVING STRATEGIES¹

Elçin Emre-Akdoğan, Ziya Argün

Abstract: The main goal of this study is to find out the effect of the instructional design method on the enhancement of problem solving abilities of students. Teaching sessions were applied to ten students who are in 11th grade, to teach them problem solving strategies which are working backwards, finding pattern, adopting a different point of view, solving a simpler analogous problem, extreme cases, make drawing, intelligent guessing and testing, accounting all possibilities, organizing data, logical reasoning. Our study based on one-on-one (teacher-experimenter and student) design experiments where we conduct teaching sessions with a small group of students to study in depth and detail. We designed sessions to teach high school students, problem-solving strategies in a ten-week long period. Before and after the application of instructional design, 12 different problems were given to students. We made interviews with students about their opinions on the effect of this design. At the end of the analysis, we observed that the instructional design improved students on choosing and applying an appropriate strategy. In addition to this, students became aware of the existence of several different problem-solving strategies and they realized that they could use more than one strategy for a problem.

Key words: Problem Solving, Problem Solving Strategies, Instructional Design-Based Research, High School Level

1. Introduction

Problem solving has an important place in the curriculum. It is an essential goal of all mathematics instructions and an integral part of all mathematical activities. Hence, being a successful problem solver has become a dominant theme in many international and national standards (American Association for the Advancement of Science (AAAS), 1993; Ministry of National Education (MoNE), 2013; National Council of Teachers of English (NCTE), 1996; National Council of Teachers of Mathematics (NCTM), 1989, 1991. Furthermore, curriculum aims that students should become a good problem solver with the enhancement of mathematical thinking ability (MoNE, 2013). Mathematical thinking ability enables students to think in a wide perspective both in mathematical problems and in real life problems. Halmos (1980) argues that students' mathematical experiences should prepare them for tackling challenges in "real" problem solving, learning during their professional life. Halmos (1980) stated:

I do believe that problems are the heart of mathematics, and I hope that as teachers, in the classroom, in seminars, and in the books and articles we write, we will emphasize them more and more, and that we will train our students to be better problem-posers and problem solvers than we are. (p.524)

¹ This study was presented as an oral presentation at 16th Conference on Problem Solving in Mathematics Education (ProMath), University of Helsinki, 27-30 May 2014, Helsinki, Finland.

At this point, mathematics teachers have a great opportunity to challenge the curiosity of their students by setting them non-routine problems, instead of spending time with routine problems that kills students' interest, hampers their intellectual development and thinking ability and misuses their opportunity (Polya, 1971). Engaging in challenging and non-routine problems help students to enhance their problem solving ability. However, it is important to recognize that problem solving ability of students depends on context, classroom setting and problems that teacher prefers. At this point, an important need arises that teachers should be provided with more well documented information about "teaching via problem solving" (Schroeder, & Lester, 1989). Therefore, the need for the well-structured instructional design on problem solving strategies has revealed. However, instructional-design research has devoted too little attention to the study of problem solving processes and problem solving has not sufficiently acknowledged in the instructional design literature (Jonassen, 2000). Few studies focus on instructional design on problem solving, for instance Smith & Ragan (1999) prescribe only general problem-solving strategies. Gagne, Briggs, & Wager (1992) propose only a brief template for applying the events of instruction on the problem solving.

After all, in our study we design an instruction on the problem solving strategies. In the context of instructional design based research on problem solving strategies, success of problem solving depends on two other main factors related to solver's ability, one is choosing an appropriate strategy and the other is applying it properly. In this study, while students are engaging in challenging and non-routine problems we observe how students choose an appropriate strategy and how they apply the strategy. In this regard, the main goal of this study is to find out the effect of the instructional design method on the enhancement of problem solving abilities of high school students.

2. Theoretical Framework

2.1. Problem Solving

There is a consensus on the definition of problem solving, considering the importance of teaching problem solving in school mathematics. If the problem is defined by Piaget's term of "disequilibrium" on the center, it can be described as a situation or an object, which disrupts an individual's present balance thus making him uncomfortable (Baki & Bell, 1997). According to John Dewey "problem" is almost everything that confuses human mind, challenges it and ultimately obscures faith. Polya (1965) stated that, a problem occurs when there is no obvious way of accomplishing your goal.

If you do not have a memorized algorithm, this shortcoming will cause a problem for you. For instance, a problem arises if you are asked to find the sum of $6+8=$ ___ and you do not have that fact memorized (Carpenter, 1980). This problem correspond to the definition of a problem wanting to get from the given state to the goal state but lacking a direct route to the goal while problem solving refers to the process of moving from the given state to the goal state of a problem. Mayer (1983) collected together thinking or problem solving as a series of mental operations that are directed toward some goal. Similarly, Hayes (1981) described problem solving as "finding an appropriate way to cross a gap." Two major parts of problem solving are (a) representing the problem and (b) searching for a means to solve the problem (Mayer, 1983).

Polya (1981) has given us a classification of problems from a pedagogical perspective:

1. One rule under your nose-the type of problem to be solved by mechanical application of a rule that has just been presented or discussed.
2. Application with some choice- a problem that can be solved by application of a rule or procedure given earlier in class so that the solver has to use some judgment.
3. Choice of combination- a problem that requires the solver to combine two or more rules or examples given in class.
4. Approaching research level- a problem that also requires a novel combination of rules or examples but that has many ramifications and requires a high degree of independence and the use of plausible reasoning.

Polya (1981) argues that both the degree of difficulty and the educational value (with respect to level of teaching students to think) increase as one goes from type 1 to type 4. John Dewey (1976) is usually credited with having said that “children are learning by doing”, but as Papert (1975) notes, the appropriate dictum- which he credits to Dewey, Montessori, and Piaget is that “children learn by doing and by thinking about what they do” (p.219).

2.2. Problem Solving Instruction

Carpenter (1980) and Lester (1980) have noted that most children approach problems in an impulsive way, attending primarily to surface features of the problem statement in order to decide what action to take. The child’s goal is to do something-anything. Most school instructions seem to reinforce their impulsivity rather than encouraging children to focus on a problem deeply and reflect on what the problem statement says. Because the children see the problem as a school task instead of an intellectual challenge that is worth accepting. They grab at answers to escape from the task as fast as possible. Successful problem solving instruction often needs to transform the terms of the school situation that previous instruction has negotiated and reinforced (Killpatrick, 1985). In his book “Mathematical Discovery”, Polya (1965) makes a similar plea:

In mathematics, know-how is much more important than mere possession of information. ...What is know-how in mathematics? The ability to solve problems-not merely routine problems but problems requiring some degree of independence, judgment, originality, creativity. Therefore, the first and foremost duty of the high school in teaching mathematics is to emphasize methodological work in problem solving. (p. xii)

Lester (1980) classified problem solving instruction research into four categories:

1. Instruction to develop master thinking strategies (e.g., originality and creativity training);
2. Instruction in the use of specific “tool skills” (e.g., making a table, organizing data, writing an equation);
3. Instruction in the use of specific heuristics (e.g., looking for a pattern, working backward);
4. Instruction in the use of general heuristics (e.g., means-end analysis, planning).

None of these four categories has shown to be clearly superior to one another. However good problem solving instruction probably involves some combination of Lester’s classification of problem solving instructions in the use of all categories. On the other hand, Lester point out the fact that regardless of instruction used experience in solving a wide variety of problems over an extended period seems to be essential (Lester, 1980).

For example, a 1983 survey of college mathematics departments (Schoenfeld, 1983) revealed the following categories of goals for courses that were described:

- to train students to "think creatively" and/or "develop their problem solving ability" (usually with a focus on heuristic strategies);
- to prepare students for problem competitions such as the Putnam examinations or national or international Olympiads;
- to provide potential teachers with instruction in a narrow band of heuristic strategies;
- to learn standard techniques in particular domains, most frequently in mathematical modeling;
- to provide a new approach to remedial mathematics (basic skills) or to try to induce "critical thinking" or "analytical reasoning" skills.

2.3. Problem Solving Strategies

In the literature we investigated different type of strategies with different labeling such as working backwards, finding pattern, adopting a different point of view, solving a simpler analogous problem, extreme cases, make drawing, intelligent guessing and testing, accounting all possibilities, organizing data, logical reasoning, using models, act it out, simulation, experiment, classical solving, use variable, write an equation, write a number sentence, using prior knowledge (Posaimeter & Krulik, 1998). We decided that instructions of “using models, act it out, simulation, experiment” strategies need more time and different type of instruction comparing to other problem solving strategies. Besides, researchers labeled different names for the same strategies, so we collected them under the ten strategy which we used in instructional design based research. These strategies are working backwards, finding pattern, adopting a different point of view, solving a simpler analogous problem, extreme cases, make drawing, intelligent guessing and testing, accounting all possibilities, organizing data, logical reasoning.

3. Methodology

Our study based on one-on-one (teacher-experimenter and student) design experiments where we conduct teaching sessions with a small group of students to study in depth and detail (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). Teaching sessions were applied to ten students who are in 11th grade, to teach them problem solving strategies which are working backwards, finding pattern, adopting a different point of view, solving a simpler analogous problem, extreme cases, make drawing, intelligent guessing and testing, accounting all possibilities, organizing data, logical reasoning (Posaimeter & Krulik, 1998). Based on the observations in the classroom before the teaching sessions, these ten students were chosen due to their curiosity, questioning, and awareness on mathematics and problem solving. As we stated that we would conduct a research on problem solving strategies, these ten students were voluntary to participate in this research. We used school’s seminar hall for the teaching sessions. The seminar hall was preferred due to its possessions such as projector, computer, white board and its feasibility to allow a comfortable communication environment with the students.

3.1. Instructional Design

We designed sessions for ten-week to teach ten high school students ten problem-solving strategies. Each week students learnt single type of problem solving strategy but all worked on different problems. We designed lesson plans that include everyday life practices for each teaching session. For lessons, we prepared power point slides to instruct each strategy. During the instructional design, each week students worked on five problems, three of them during the teaching session and two of them as an assignment, which can be solved with the help of the strategy that is aimed to be instructed. However, we did not restrict students to solve the problems only with one strategy. Rather, during the experiments, we encouraged students to solve problems using different strategies. We solved problems with the students, using their ideas during the teaching sessions. The idea here is solving problems together as a class while teacher serving as a “moderator” or an orchestrator of ideas. The teacher's responsibility was not to generate solutions, but rather to help the students make the best of recourses they have (Schoenfeld, 1983). Before and after the application of instructional design, 12 different problems were given to students. We asked students to solve eleventh and twelfth problem in pre-test and post-test using more than one strategy. After the instructional design, we made interviews with the students. The aim of the interviews were to reveal opinions of students about the problems that they worked on, strategies that they learned newly and in general process of instructional design.

3.2. Data Analysis

We analyzed the solutions of the students by comparing each student’s solution before and after the teaching sessions in terms of the ability to choose and apply the appropriate problem solving strategy. In deciding the appropriate strategy/strategies for each problem, we got feedback from experts in mathematics education who are one PhD student in mathematics education, one high school mathematics teacher and six scholars in mathematics education department in a University. Those

experts solved 12 problems that we asked students, by applying different problem solving strategies. Based on solutions of experts, we determine more than one strategy for each problem, that we called them appropriate problem solving strategy (Table 1).

Table 1. Appropriate strategies for problems in pre-test and post-test

Problem 1.	Working backwards, Classical method
Problem 2.	Finding pattern
Problem 3.	Adopting a Different Point of View, Classical method
Problem 4.	Solving a Simpler Analogous Problem, Classical method
Problem 5.	Intelligent Guessing and Testing, Extreme cases
Problem 6.	Make drawing
Problem 7.	Intelligent Guessing and Testing, Solving a Simpler Analogous Problem, Classical method
Problem 8.	Accounting All Possibilities
Problem 9.	Organizing Data, Act it out
Problem 10.	Logical Reasoning, Classical method
Problem 11.	Adopting a Different Point of View, Accounting All Possibilities, Make Drawing, Using prior-knowledge
Problem 12.	Accounting All Possibilities, Make Drawing

4. Findings

We analyzed the solutions of the students by comparing each student's solution pre-test and post-test in terms of the ability to choose and apply the appropriate problem solving strategy. The strategies that students choose and apply in pre-test and post-test have shown in Table 2. Since we only listed appropriate strategies that students used in the Table 2, we do not mention the cases where students do not use any strategy or do not solve the problem.

Table 2. Appropriate strategies that students choose and apply in pre-test and post-test

Problem	Appropriate Strategies in Pre-test		Appropriate Strategies in Post-test	
	Strategies	Number of Students	Strategies	Number of Students
Problem 1	Working Backwards	4	Working Backwards	10
	Classical Method	6		
Problem 2	No answered	10	Finding Pattern	10
Problem 3	Adopting a Different Point of View	4	Adopting a Different Point of View	8
	Classical Method	4	Classical Method	2
Problem 4	Solving a Simpler Analogous Problem	1	Solving a Simpler Analogous Problem	10
	Classical Method	5		
Problem 5	Intelligent Guessing and Testing	3	Intelligent Guessing and Testing	7
	Classical Method	6		

Problem 6	Make Drawing	10	Make Drawing	10
Problem 7	Intelligent Guessing and Testing	1	Intelligent Guessing and Testing	5
	Solving a Simpler Analogous Problem	3	Solving a Simpler Analogous Problem	2
	Classical Method	5		
Problem 8	Accounting All Possibilities	6	Accounting All Possibilities	8
	Classical Method	3		
Problem 9	Organizing Data	7	Organizing Data	10
Problem 10	Logical Reasoning	1	Logical Reasoning	3
	Classical Method	9	Classical Method	7
Problem 11	Adopting a Different Point of View	6	Using prior-knowledge	4
	Accounting All Possibilities	2	Accounting All Possibilities	3
	Using prior-knowledge	5	Adopting a Different Point of View	6
			Make Drawing	3
Problem 12	Accounting All Possibilities	10	Accounting All Possibilities	10

According to Table 2 we can conclude that after the instructional design, students tend to use problem solving strategies different from the ones they chose before the instructional design. For example, Seval (pseudonym) did not use finding pattern strategy while solving problems in the pre-test. She did use finding pattern strategy in the post-test. Seval explained this as the following:

Although I am not a kind of person who likes to solve problems, it is much more enjoyable now and I find it amusing. However, I have not understood finding pattern for my whole life. Yet I do not think I would be able to find more of them. I think many things occurred. I managed to solve pattern problems which I could not before.

Seval used an inappropriate strategy that is classical method while solving fifth problem in pre-test (Figure 1). You can see fifth problem below:

5th Problem. 12...6 is a number which has seven digits. This number is equal to multiplication of three consecutive numbers. Determine the missing parts of the seven digit number.

$$x \cdot (x+1) \cdot (x+2) = 12 \dots 6$$

Figure 1. An example for the inappropriate strategy that Seval used while solving fifth problem in the pre-test.

After the instructional design, we observed that Seval applied intelligence guessing and testing as an appropriate strategy while solving fifth problem in the post-test (Figure 2).

$$\begin{aligned}
 102 \cdot 103 \cdot 104 &= 1072624 \quad \times \\
 103 \cdot 104 \cdot 105 &= 1124760 \quad \times \\
 104 \cdot 105 \cdot 106 &= 1157520 \quad \times \\
 105 \cdot 106 \cdot 107 &= 1190910 \quad \times \\
 106 \cdot 107 \cdot 108 &= 1224936 \quad \checkmark
 \end{aligned}$$

Figure 2. An example for the appropriate strategy of intelligent guessing and testing which Seval used in solving fifth problem in the post-test.

We asked students to solve the problem below (eleventh problem in pre-test and post-test) using more than one strategy:

11th Problem. There are 18 team in the Turkish Super League. How many matches could be played in the first period of the league?

Seval was able to solve the problem in pre-test with a single strategy (using prior knowledge) although she was asked to solve problem by using more than one strategy. After the design experiments, she was able to apply three different strategies (using prior-knowledge, accounting all possibilities, adopting a different point of view) to solve the problem (Figure 3).

The figure shows three handwritten solutions for the 11th problem. The first solution uses combinatorics: $C(18, 2) = \frac{18!}{2!16!} = 9 \cdot 17 = 153$. The second solution uses an arithmetic series: $x = 17 + 16 + \dots + 1 = \frac{(1+17) \cdot 17}{2} = 153$. The third solution uses direct counting of matches between teams, listing 17 matches for the first team, 16 for the second, and so on, leading to the same sum of 153. The solutions are annotated with Turkish text explaining the logic and the final answer is circled as 153.

Figure 3. An example for the solutions that Seval used in eleventh problem of the post-test, by applying three different strategies (Using prior-knowledge, Accounting all possibilities, Adopting a different point of view).

When solutions of students in pre-test and post-test are compared, we can conclude that instructional design for problem solving strategies has a positive impact on the enhancement of choosing and applying the appropriate strategies. We observed from the solutions and interviews that, prior to the instructional design, the students did not take into consideration of the appropriateness of the strategy they chose and applied in pre-test and tend to use the classical strategy or the strategies they are familiar with. However, after the design experiments students could choose the appropriate strategies among the various strategies they have learned and apply.

Here are some excerpts from opinion of a student about instructional design. Seval explained her approach on problem solving before the instructional design as follows: “In the beginning I used to construct equations. I mean that I generally solve problems by forming equations. Now I realized that I constructed equations in a 50 % ratio. So it is the way it is.”

Another participant who named Selçuk explained his opinion on the process of instructional design:

As we continued to learn strategies each day, we perceived that all problems can be solved through different ways; that each problem doesn't have a single solution but more than one. In the end we learned to solve the problems through more than one way.

5. Conclusion

International and national standards and programs emphasize the significance of problem solving in mathematics education (AAAS, 1993; NCSS, 1997; NCTE, 1996; NCTM, 1989, 1991; MoNE, 2013). Problem solving is an essential goal of mathematics instructions and an integral part of all mathematical activities. In this study we focus on how students choose an appropriate strategy and how they apply the strategy, while students are engaging in challenging and 12 non-routine problems. In this regard, we aim to reveal the effect of the instructional design method on the enhancement of problem solving abilities of high school students. At the end of the analysis, we concluded that the instructional design on problem solving strategies has a positive effect on choosing and applying an appropriate strategy. During the instructional design, we observed that the participants became aware of the existence of several problem solving strategies which promote their mathematical thinking. Instructional design enabled students to use the strategies they have learnt to solve problems. At the end of the instructional design, students were able to use more than one strategy. We find out that instructional design on problem-solving strategies does have an impact on problem-solving performance of students (Schoenfeld, 1979). Contrary to working on routine problems, non-routine problems help students to enhance their ability on problem solving and mathematical thinking. In this regard, solving non-routine problems, in particular, solving problems by using more than one strategy has a vital role on mathematics education. Thus, teachers should realize the importance of solving problems with more than one strategy and lead their students by teaching several strategies. In addition to that, we can conclude that this improvement is not enough to become a good problem solver. Ten teaching sessions is very short time to improve this kind of skills and therefore these kinds of activities should be carried out during the application of whole curriculum (Lester, 1980). Lester also stated that experience in solving a wide variety of problems over an extended period seems to be essential (Lester, 1980).

Annex

Problem 1. Cemre, bought some rabbits from a rabbit farm and the number of rabbits increased by 10 % in April. 10 rabbits were born in May and Cemre sold $\frac{1}{3}$ of all the rabbits. 20 more rabbits were born in June and Cemre sold half of the existing rabbits. If 5 rabbits are born in July, Cemre will have had 55 rabbits. Accordingly, how many rabbit does Cemre have in the beginning of April?

Problem 2. We have a function machine that is capable of doing only four basic operations (addition, subtraction, multiplication, division). A table below shows numbers that are put into the machine and the out numbers produced. According to this table, which number will come out when we put 9 into the machine?

Input(x)	Output
1	1
2	9
3	29
4	67
5	129
6	221

Problem 3. If it is 10: 40 a.m. right now; what time will it be in 143999999995 minutes from now?

Problem 4. The number $(3333333334)^2$ has 20 digits. What is the sum of digits in this number?

Problem 5. $12 \dots 6$ is a number with seven digits. Since this number is equal to the multiplication of three consecutive numbers, then determine the missing digits in the number above.

Problem 6. Erkin has a toy train that runs on plastic rounded tracks. There are six phone booths around the tracks, that are placed at an equal distance from each other. The train covers the distance between the first and third phone booths in 12 seconds. How long will it take the train to complete a round trip if it goes at the same pace?

Problem 7. At a reception, every 2 guests will be served 1 plate of fried chicken, every 3 guests will be served 1 plate of rice pilaf, and every 4 guests will be served 1 plate of fruits. Since a total 65 plates of food were served, how many guests are there at the reception?

Problem 8. When my grandfather, who says his current age is that of a prime number, reaches an age that is the next prime number, the difference between the next prime number and the first prime number will be equal to the difference between his current age and the last age that is also a prime number. How old is my grandfather?

Problem 9. Ali and Gönül will play a tennis match at a local tennis club. The player who takes three sets will win the match. Since we know that one of the players took two consecutive sets, how many different results are possible between them at the end of the match?

Problem 10. If $\frac{1}{x+5} = 4$, which number is the equal of $\frac{1}{x+6}$?

Problem 11. There are 18 team in the Turkish Super League. How many matches could be played in the first period of the league?

Problem 12. There is a construction of a bridge in the town. While Beyazıt is working under the bridge for construction, he could only see the legs of people and/or animals that are passing the bridge. While there is a group composed of children and sheep are passing the bridge, Beyazıt has only seen 20 legs. How many sheep or children might be passing the bridge?

References

- [1] American Association for the Advancement of Science (AAAS) (1993), *Benchmarks for science literacy*, New York: Oxford University Press.
- [2] Baki, A. & Bell, A. (1997), *Ortaöğretim Matematik Öğretimi*, Y.Ö.K yayınları.
- [3] Carpenter, T. P. (1980), Heuristic strategies used to solve addition and subtraction problems. In R. Karplus (Ed.), *Proceedings of the Fourth International Conference for the Psychology of Mathematics Education*, Berkeley: University of California.
- [4] Cobb, P., Confrey, J., diSessa, A., Lehrer, R. & Schauble, L. (2003), Design experiments in educational research, *Educational Researcher*, 32, 1, 9–13.
- [5] Dewey, J. (1964), John Dewey on education. In R.Archambault (Ed.), Chicago: University of Chicago Press. (Original work published 1904)
- [6] Gagné, R.M., Briggs, L.J., & Wager, W.W. (1992), *Principles of instructional design (4th Ed.)*, New York: Harcourt, Brace, & Jovanovich.
- [7] Halmos, P. (1980), The heart of mathematics. *American Mathematical Monthly*, 87, 519-524.
- [8] Hayes, J.R. (1981), *The complete problem solver*, Philadelphia: The Franklin Institute Pres.
- [9] Jonassen, D. (2000), Toward a design theory of problem solving, *Educational Technology Research and Development*, 48, 4, 63-85.
- [10] Kilpatrick, J. (1985), A Retrospective Account of the Past Twenty-five Years of Research on Teaching Mathematical Problem Solving. In A. E. Silver (Ed.), *Teaching and Learning Mathematical*

Problem Solving: Multiple Research Perspectives (pp.1-16), Hillsdale, NJ: Lawrence Erlbaum Associates.

[11] Lester, F.K. (1980), Research on mathematical problem solving. In R.J. Shumway (Ed.), *Research in mathematics education*, Reston, VA: National Council of Teachers of Mathematics.

[12] Mayer, R.E. (1983), *Thinking, problem solving, and cognition*, San Francisco: W.H. Freeman&Co.

[13] Ministry of National Education (MoNE). (2013), Talim ve Terbiye Kurulu Başkanlığı, *Ortaöğretim matematik dersi (9-12.sınıflar) öğretim programı [Board of Education, Secondary school mathematics curriculum (9-12th grades)]*, Ankara: MEB Basımevi.

[14] National Council for Social Studies (NCSS). (1997), *Expect Excellence: Curriculum Standards for Social Studies*, National Council for Social Studies.

[15] National Council of Teachers of English (NCTE). (1996), *Standards for the English Language Arts*, Urbana, IL: National Council of Teachers of English.

[16] National Council of Teachers of Mathematics (NCTM). (1989), *Curriculum and Evaluation Standards for School Mathematics*, Reston, VA: NCTM.

[17] National Council of Teachers of Mathematics (NCTM). (1991), *Curriculum and evaluation standards for school mathematics*, Reston, VA: NCTM.

[18] Papert, S. (1975), Teaching children thinking, *Journal of Structural Learning*, 4, 219-29.

[19] Polya, G. (1965), *Mathematical discovery: On understanding, learning and teaching problem solving* (Vol.2), New York: John Wiley & Sons.

[20] Polya, G. (1981), *Mathematical discovery: On understanding, learning and teaching problem solving* (Vol.2), New York: John Wiley & Sons.

[21] Polya, G. (1971), *How to Solve it: A New Aspect of Mathematical Method*, New Jersey: Princeton University Press.

[22] Posaimenter, A. S., Krulik, S. (1998), *Problem Solving Strategies for Efficient and Elegant Solutions: A Resource for the Mathematics Teacher*, California: Corwin Press.

[23] Schoenfeld, A. (1979), Explicit Heuristic Training as a Variable in Problem-Solving Performance, *Journal for Research in Mathematics Education*, 10, 3, 173–187.

[24] Schoenfeld, A. (1983), *Problem Solving in the Mathematics Curriculum: A report, recommendations, and an annotated Bibliography*, The Mathematical Association of America Committee on the Teaching of Undergraduate Mathematics.

[25] Schroeder, T. L. & Lester, F. K. (1989), Developing understanding in mathematics via problem solving. In P. R. Trafton (Ed.), *New directions for elementary school mathematics* (pp. 31-56), Reston, VA: National Council of Teachers of Mathematics.

[26] Smith, P.L., & Ragan, T.J. (1999), *Instructional design 2nd Ed.*, Columbus, OH: Merrill.

Authors

Elçin Emre-Akdoğan, Gazi Educational Faculty, Gazi University, Ankara, Turkey, e-mail: elcinemre@gazi.edu.tr

Ziya Argün, Gazi Educational Faculty, Gazi University, Ankara, Turkey, e-mail: ziya@gazi.edu.tr