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Schema-Based Instruction With Concrete and Virtual Manipulatives to Teach Problem Solving to Students With Autism

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Abstract

The current study evaluated the effects of modified schema-based instruction on the mathematical word problem solving skills of three elementary students with autism spectrum disorders and moderate intellectual disability. Participants learned to solve compare problem type with themes that related to their interests and daily experiences. In addition, researchers compared the effects of concrete and virtual manipulatives within the treatment package. Results of the multiple probes across participants with an embedded alternating treatments design showed a functional relation between modified schema-based instruction and word problem solving. Two of three participants performed more steps in the virtual condition and one participant had equal performance in both concrete and virtual conditions. When given a choice between conditions upon skill mastery, all three participants preferred the virtual condition and maintained treatment effects. Implications for practice and future research are discussed.

Keywords

schema-based instruction, mathematical word problem solving, autism spectrum disorders, manipulatives

Most mathematics educators would identify problem solving as the cornerstone of mathematical learning (National Council of Teachers of Mathematics [NCTM], 2000). Teaching problem solving allows students to learn *when* and *why*, not just *how*, to apply these skills. Students typically learn problem solving through the use of word problems. Fuchs et al. (2008) note that word problems can be a source of difficulty for many students because these problems require not only calculation, but also comprehension of linguistic information.

According to Mayer's (1985) model of problem solving, there are four sequential phases to solving a mathematical word problem, including problem translation, problem integration, solution planning, and solution execution. Each phase requires different cognitive skills for successful completion. At the problem translation phase, semantic language skills construct meaning from the problem for which a student must be able to determine "what is happening" to identify known and unknown information. The problem integration phase requires selection of integral parts of the problem (e.g., known and unknown amounts) and translating them to a mathematical structure (e.g., number sentence in equation format). Once the student has a mathematical understanding of the problem, he or she needs to devise a plan for finding the solution. The solution planning and execution phases involve choosing the correct operations and carrying out those computations to arrive at a correct answer. Because problem solving is innately a chained task of discrete behaviors, each phase of problem solving is dependent upon successful completion of the previous phase in order to yield correct execution and ultimate arrival at a correct answer (Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007).

The challenge of teaching word problem solving to students with developmental disabilities like autism spectrum disorders (ASD) and intellectual disability (ID) is twofold. First, there is a lack of research on instructional strategies for teaching word problem solving to individuals with ASD and ID. Browder, Spooner, Ahlgrim-Delzell, Harris, and Wakeman (2008) conducted a meta-analysis to determine effective practices for teaching mathematical standards to students with moderate or severe developmental disabilities

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and only found two studies (of 68 published between 1975 and 2005) that addressed the higher-level mathematical skill of problem solving (i.e., Miser, 1985; Neef, Nelles, Iwata, & Page, 2003). Fortunately, additional research on teaching grade-aligned problem solving has emerged recently. For example, mathematics stories based on familiar activities and a step-by-step training in a task analysis have been used successfully to teach adolescents with ASD and ID to identify and organize key facts and solve the problem stated in the story (Browder, Jimenez, & Trela, 2012; Browder, Trela, et al., 2012). Similarly, a task analysis and systematic prompting has been used to teach high school students with ID to use the Pythagorean Theorem (Creech-Galloway, Collins, Knight, & Bausch, 2013). Overall, the limited and emerging research base for teaching mathematical problem solving to students with ASD and/or ID supports the effectiveness of using a task analysis along with systematic prompting with feedback to teach problem solving skills.

Second, theories of problem solving rely on the ability of individuals to use metacognitive strategies to plan and execute problem solving. Students with ASD have unique characteristics that often create challenges at each step of Mayer's (1985) problem solving approach. For example, deficits in reading and language may create barriers to translating the problem, and working memory challenges can make it difficult for students with ASD to integrate the problem into a mathematical structure. Furthermore, many students with ASD who also have ID may have limited skill repertoires from which to plan and execute the problem solution if they have not had effective instruction in mathematics. Strategy instruction may help overcome the barriers to word problem solving that many students with ASD and ID often experience due to their learning characteristics. Schema-based instruction (SBI) is one type of strategy instruction that has shown effectiveness in teaching students with high incidence disabilities to solve mathematical word problems (Jitendra & Hoff, 1996). A schema is an outline or framework for solving a problem that can be represented through pictures, diagrams, number sentences, or equations (Marshall, 1995; Powell, 2011). SBI has three essential components, including (a) identification of the problem structure to determine the problem type, (b) use of visual representations of the structure to determine problem type and to organize information from the problem, and (c) explicit instruction on the schema-based problem solving method (Jitendra et al., 2015). SBI has been deemed an evidence-based practice for teaching mathematical problem solving to students with high incidence disabilities (Gersten et al., 2009; Jitendra et al., 2015) and may show potential for teaching word problem solving to students with ASD and ID with some modifications.

Rockwell, Griffin, and Jones (2011) conducted the first investigation of SBI and taught a 10-year-old student with ASD to use a schematic diagram to solve three types of word problems. Rockwell et al. used direct instruction strategies including think alouds, guided practice, and independent practice to teach the four-step mnemonic "RUNS," which stood for (a) Read the problem, (b) Use a diagram, (c) Number sentence, and (d) State the answer. Using a multiple probe across behaviors design, the researchers found a functional relation between SBI and word problem solving performance for the participant. Two challenges can arise in applying SBI like that of Rockwell et al. (2011) to students who have both ASD and ID. First, prior studies supporting the effectiveness of SBI for students with ASD included participants who had both the procedural knowledge and computational abilities to solve the problems (Rockwell et al., 2011; Whitby, 2012). When students do not have the computational skills to solve problems, conceptual strategy instruction needs to be supplemented with procedural strategy instruction. Second, the majority of SBI studies used heuristics for solving problems that assumed students had the phonics skills to learn a mnemonic (e.g., "R" in RUN stands for "read"). Students with ASD and ID may lack this fluency with phonics. Modifying the traditional SBI by taking into consideration all of the above challenges and integrating the use of evidence-based practices such as a task analysis and systematic prompting for students with moderate and severe cognitive disabilities will likely show promise in supporting students with ASD and ID in learning word problem solving skills.

Saunders, Lo, and Browder (2016) implemented one of the first modified SBI studies to teach three students with ASD and moderate ID to solve group and change problem types with the support of computer-based video instruction. The modified SBI instruction included (a) a task analysis paired with pictures in each written step as a heuristic for solving the problem, which replaced the mnemonics found in other SBI interventions; (b) a color-coded graphic organizer for each problem type as concrete schemas, with manipulatives to enhance the salient features of each problem type; and (c) explicit and systematic prompting to teach each step of the task analysis addressing both conceptual and procedural knowledge of word problem solving. All three students learned to discriminate between problem types and to solve group problems; one of the students also learned to solve change problems.

In Saunders et al. (2016) study, the participants used virtual manipulatives to solve word problems during modified SBI. Use of manipulatives in word problem solving can make mathematics learning concrete for students with disabilities, particularly if they do not have fluency in mathematical facts (Bouck, Satsangi, Doughty, & Courtney, 2013; Powell et al., 2015). In a meta-analysis of the effects of concrete manipulatives versus abstract symbols alone, Carbonneau, Marley, and Selig (2012) found instruction with manipulatives produced a medium sized effect on student learning. Concrete manipulatives have been used to successfully teach the procedural component of mathematical word problem solving to students with learning disabilities (Cass, Cates, Smith, & Jackson, 2003), ASD (Bouck et al., 2013), and ID (Browder, Jimenez, & Trela, 2012; Browder, Trela, et al., 2012).

Virtual manipulatives require fewer materials, offer more flexibility for making the materials directly related to the word problem theme (e.g., pictures of cars, friends, food), and have been found beneficial for general education students (Reimer & Moyer, 2005; Suh, Moyer, & Heo, 2005). However, research on the use of virtual manipulatives for students with ASD or ID is rare and there is a lack of research on the differential effects of concrete and virtual manipulatives. Bouck et al. (2013) compared the effects of virtual and concrete manipulatives on the computational skills of three elementary-aged students with ASD. Students learned single- and double-digit addition and subtraction computation skills using concrete and virtual manipulatives in the form of base 10 blocks. Results showed that participants were able to increase accuracy and independence in solving addition and subtraction computation problems in both virtual and concrete conditions; however, all three participants demonstrated a more rapid acquisition of skills in the virtual manipulatives condition.

Considering current research limitations related to SBI and integration of virtual versus concrete manipulatives in teaching word problem solving skills to students with ASD and ID, the purpose of the current study was to extend the study by Saunders et al. (2016) by evaluating the effects of modified SBI on the mathematical word problem solving skills, focusing on compare problems, of elementary students with ASD and moderate ID. A second research purpose was to compare the effects of virtual versus concrete manipulatives within the treatment package.

Method

Participants

Three elementary students classified with both ASD and moderate ID participated in this study. Participants were selected based on the inclusion criteria, including (a) educational or medical diagnosis of autism; (b) participation in alternate assessment aligned with alternate achievement standards (AA-AAS) if in a tested grade (i.e., third grade and above); and (c) satisfactory performance on prescreening measure. The prescreening measure assessed the participants' ability to (a) receptively and expressively identify numerals up to 10, (b) make sets of numbers to 10, (c) count with one-to-one correspondence, and (d) solve one- and two-digit addition and subtraction word problems. The purpose of items (a) to (c) on the prescreening measure was to ensure the participant had sufficient early numeracy skills to access the content of the word problems, as well as to determine that manipulatives would be an appropriate instructional material. The purpose of item (d) was to ensure that the participant was not already able to solve word problems independently and was an appropriate candidate for the intervention. A participant achieved satisfactory performance on prescreening measure if he completed items (a) to (c) with 100% accuracy and item (d) with no more than 50% accuracy.

All participants were male, diagnosed with ASD and moderate ID, and received all academic instruction in a classroom for students with ASD. Typical classroom mathematics instruction involved small group or one-on-one instruction using teacher-created materials or worksheets from Unique Learning Systems (N2y, 2014), a curriculum prescribed by the district.

Andrew was an 11-year-old Caucasian male in the fourth grade. His full-scale IQ was 58 according to the Leiter International Performance Scale–Revised (Roid & Miller, 2002). Andrew was diagnosed with ASD by a medical physician. He was able to speak in three- to four-word phrases when directly asked a question by an adult. Andrew did not respond to peers' or siblings' initiations of conversation. He had a very soft speaking voice, which made him difficult to understand. Andrew was compliant with adult directions and completed assigned work quickly. His mathematics individualized education program (IEP) goals included independently identifying the operation in word problems, setting up calculation problems, and choosing a tool to solve the equation. In the prescreening measure, Andrew had satisfactory early numeracy skills (items a-c); however, he was unable to solve any of the word problems.

Nathan was a 9-year-old Hispanic male in the third grade. Nathan's full scale IQ was 46 based on the Standford-Binet Intelligence Scale, Fifth edition (Roid, 2003); results of the Adaptive Behavior Assessment System–Second edition (ABAS-II; Harrison & Oakland, 2003) showed that his adaptive behavior standard scores were 57 (teacher rating) and 53 (parent rating). Nathan's diagnosis of ASD was based on the Childhood Autism Rating Scale, second edition (CARS; Schopler, Reichler, & Renner, 1988). Although Spanish was the primary language spoken in his home, Nathan only communicated and followed directions given in English in the school setting and also received speech services in English. Nathan had limited spoken language, and did not initiate conversations or independently make requests to meet his needs. He would answer yes/no questions, but when given two choices he often repeated the latter option offered. Nathan enjoyed his schoolwork and was eager to please adults. According to his IEP, Nathan had mathematics goals of solving two-digit addition and subtraction problems and solving word problems by setting up the correct number sentence and choosing the correct operation. Nathan participated in Saunders et al. (2016) study to learn group and change problem types, but had not been exposed to the compare problem type. Nathan's performance on the prescreening measure confirmed he had adequate early numeracy skills (Items a–c) but was unable to correctly solve any of the word problems.

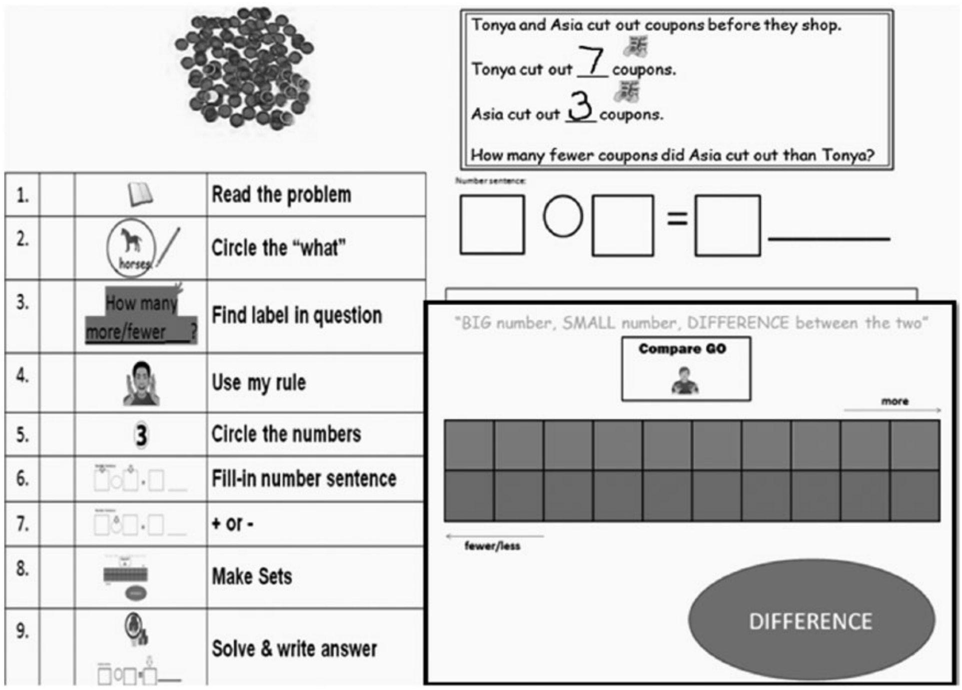

Figure 1. Example of concrete student materials: word problem solving mat with a compare word problem and the graphic organizer placed on top (bottom right), manipulatives (top left), and student self-instruction sheet (bottom left).

Caleb was a 7-year-old Caucasian male in the second grade. His cognitive scale was 55 according to the Developmental Profile-3 (DP-3; Alpern, 2007), and full scale IQ was 62 according to the Leiter International Performance Scale–Revised (Roid & Miller, 2002). Caleb was diagnosed with ASD based on the Gilliam Autism Rating Scale–Second edition (GARS-2; Gilliam, 2006). His adaptive behavior standard scores based on the ABAS-II (Harrison & Oakland, 2003) were 57 from teacher report and 53 from parent report. Caleb's verbal abilities were his strength. Caleb was extremely routine oriented and enjoyed completing academic tasks, but he became upset when the schedule was abruptly changed and he demonstrated stereotypy that impeded his access to the general curriculum. Caleb's mathematics IEP goals were to solve a simple word problem, tell time using analog and digital clocks, and use coins correctly. Caleb participated in the same previous study as Nathan, with instruction in group and change problem types, but not compare. Items a to c of the prescreening measure indicated Caleb had adequate early numeracy skills but was unable to solve any word problems.

Setting

This study took place in a public elementary school in an urban school district in the southeast United States. The school had approximately 1,100 students enrolled in kindergarten through fifth grade. The school had a diverse student population made up of 4% Asian, 34% African American, 24% Hispanic, and 2% multiracial students; 44.5% of the school's population was labeled as economically disadvantaged. Students with disabilities made up 8% of the school's population. Intervention sessions were conducted daily in a small tutorial room with individual student tables and desks on the same hallway as the participants' classrooms.

Materials

Materials for the study included (a) a problem solving mat; (b) a graphic organizer (schema diagram), either in the form of laminated paper or virtually displayed on an iPad 3; (c) a laminated student self-instruction sheet (i.e., task analysis) that contained pictures alongside text for each step of solving the word problem; and (d) compare-type word problems. Figure 1 displays an example of the problem solving mat with concrete materials. Depending on the condition, participants were either provided with a laminated graphic organizer and plastic round manipulatives or an iPad 3 with virtual blue circles they dragged onto an identical graphic organizer displayed on the iPad 3. The SMARTnotebook© application was used to display the graphic organizer and create the virtual manipulatives. All word problems followed a predictable format. The first line of text set the context for the word problem. The second line contained the first "what" (the referent) with a picture above it and the amount. The third line of text contained the second "what" (the compared concept) with a picture over it and the amount. The final sentence of the word problem was the problem statement. Experts in the fields of elementary mathematics and SBI reviewed and validated all materials.

Mathematical Content

The targeted mathematical content for this study was word problem solving for the compare problem type, as each participant had IEP goals related to solving word problems. A compare problem depicts the comparison of the quantity of two different items (e.g., three apples and one banana), or two quantities of the same item (e.g., five apples and two apples). The compare problems in this study were always arranged so that the difference was the unknown quantity (e.g., how many more apples than bananas). Solving compare word problems addresses the NCTM standards of Numbers and Operations as well as Algebra (NCTM, 2000), and matches to grade-aligned Common Core State Standards in Mathematics for each of the participants (Common Core State Standards Initiative, 2015). Specifically, for Andrew, the intervention attended to "Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations" (4.OA.3). For Nathan, the standard of "Solve two-step word problems using the four operations . . . " (3.OA.D.8) was targeted. For Caleb, the intervention addressed two standards, "Add and subtract within 1000 using concrete models or drawings . . . " (2.NBT.B.7) and "Use addition and subtraction within 100 to solve one-and two-step word problems involving situations of . . . comparing" (2.OA.A.1).

Dependent Variable

The dependent variable was mathematical word problem solving, measured by the total number of points a participant received by independently performing the nine steps of the task analysis. The steps of the task analysis were (1) read the problem, (2) circle the whats, (3) find label in question, (4) use my rule, (5) circle the numbers, (6) fill in number sentence, (7) + or −, (8) make sets, and (9) solve and write answer. Steps 1 to 8 were worth 1 point each, and the final step "solve and write answer" was worth 2 separate points for being two distinct behaviors. A total of 30 points were possible in each session across three word problems. Criterion for mastery and changing from the intervention to choice phase for a student was his achieving a score of at least 8 out of 10 points, which must include independent correct responses on Steps 8 and 9, for at least two out of three problems for 2 consecutive days.

Interobserver Agreement and Procedural Fidelity

Interobserver agreement (IOA) and procedural fidelity data were collected across all experimental conditions with 40% of the baseline and 75% of the intervention sessions for Andrew, 50% of the baseline and 35% of the intervention sessions for Nathan, and 42% of the baseline and 33% of the intervention sessions for Caleb, either in vivo or via permanent product (video) observations. IOA data collection exceeded the standard of a minimum of 20% IOA set forth by What Works Clearinghouse (WWC; Kratochwill et al., 2010). IOA was evaluated using an item-by-item method and calculated by dividing the total agreed items by the total agreed and disagreed items and multiplied by 100 (Kazdin, 1982). Mean IOA for Andrew was 100% in baseline and 94% in intervention (range 89%–100%), for Nathan 90% in baseline (range 80%–100%) and 99% in intervention (range 96%–100%), and for Caleb 96% in baseline (range 88%–100%) and 97% in intervention (range 92%–100%).

A second observer used a procedural fidelity checklist to document the degree to which the intervention was implemented consistently as designed. The 11-item fidelity checklist addressed the degree to which the instructor followed the alternation sequence of virtual and concrete manipulatives, as well as the implementation of instructional elements. To calculate the procedural fidelity, the number of elements correctly implemented was divided by the total number of procedural elements and then multiplied by 100 (Billingsley, White, & Munson, 1980). Results of procedural fidelity were 99% with a range of 98% to 100%.

Experimental Design

A multiple probe across participants with an embedded alternating treatments design was used. All participants began baseline simultaneously. The participant with the lowest and most stable level of performance after at least five baseline sessions received the intervention first, whereas the other two participants remained in the baseline condition. After the first participant (Andrew) showed a clear accelerating trend or improved level of a minimum of three data points, the second participant with low but stable baseline data path entered the intervention. Nathan was chosen to enter intervention next because he was agitated in baseline from knowing he was not getting the answers right and expressing the desire for instruction and assistance. In contrast, Caleb did not seem to be affected by continued baseline probes.

Beginning in baseline, the random alternation between virtual and concrete conditions followed a predetermined sequence for each participant with no more than two consecutive sessions in each condition. Participants moved from intervention to the choice phase, which also served as a maintenance measure, once they received at least 8 (out of 10) points for two out of three problems for 2 consecutive days; received points must include correct responses on making sets (Step 8) as well as solving the problem and writing the correct answer (Step 9).

Table 1. Expected Student Response for Each Step of Self-Instruction Sheet.

Step	Expected student response
1. Read the problem	Read problem or asked for problem to be read vocally or though gestures
2. Circle the "whats"	Circled items being compared
3. Find label in question	Read the question sentence again by locating question mark and finding the label (i.e., the word that comes after "how many"); put the word into the number sentence above the underline
4. Use my rule	Stated the rule for the compare problem type ("big number, small number, difference between the two") and/or used hand motions
5. Find how many	Circled the numbers in word problem
6. Fill-in number sentence	Completed the number sentence using circled numbers from word problem
7. + or −	Put subtraction symbol in the circle symbol in the number sentence to solve compare problem type
8. Make sets	Used manipulatives to represent bigger number in top 10 frame (colored in green) and smaller number in bottom 10 frame (colored in red)
9. Solve and write answer	Pushed counters from top 10 frame that did not have a match in bottom 10 frame to the circle labeled "difference"; wrote the number of manipulatives in "difference" circle in the final box of the number sentence

Procedures

Baseline. During baseline, participants received daily math instruction for 30 to 45 min from their respective special education teachers without a formal curriculum that focused on the extensions of the Common Core and students' IEP goals (e.g., computation) using teacher-created materials or worksheets from Unique Learning Systems (N2y, 2014). Classroom teachers agreed to not provide instruction on word problem solving during the course of the current study.

During data collection sessions, participants were presented with the student self-instruction sheet, problem solving mat, either concrete or virtual manipulatives and graphic organizer, and the first word problem. The first author served as the primary researcher and instructor, and asked the participant to "solve the word problem." The instructor read the word problem aloud if the participant requested, and provided praise for on-task behavior with no specific feedback or prompting related to the participant's word problem solving. This procedure continued until the participant attempted all three word problems. At the end of each baseline session, the participants were given 3 min to play age-appropriate games on the iPad that were not related to math (e.g., racing games) as reinforcement for completing the task.

Intervention. The first author (i.e., instructor) implemented all interventions with each participant individually. The instructor was a doctoral student in special education and a former classroom teacher of students with ASD with over 7 years of experience working with students with ASD. She had a previous relationship with two participants (Nathan and Caleb) from a prior research study. Lessons in each intervention session followed a model-lead-test format with embedded system of least prompts. The self-instruction sheet consisted of the steps of the task analysis in written format with picture supports for emerging or non-readers and a box to check off each step, and served as the heuristic traditionally used in SBI. The instructor modeled each of the steps during a 3-day training period with active student participation, during which no data were taken. Each session during this training period lasted approximately 15 to 20 min. The participant was taught to follow the student self-instruction sheet and check off each step as it was completed. See Table 1 for the expected participant responses for each step.

Following the three-session training period, the instructor provided least intrusive prompting as needed to assist the participants in solving the three word problems. The least intrusive prompting hierarchy consisted of three levels (verbal, specific verbal, and a model prompt). The prompts for each step differed slightly in content when it came to the specific verbal prompt depending on the step. Generally, the verbal prompt directed the participant to look to their self-instruction sheet to see what was next (e.g., "This step says circle the whats."). The specific verbal prompt directed the participant to the key actions or stimuli of that step (e.g., "This step says circle the whats. Remember, the whats have pictures over them."). In the model prompt, the instructor would do a full model of the correct action required for the step paired with an explicit think aloud procedure, followed by requesting the participant to repeat the action. In the case of circling the whats, the instructor would say "I am going to circle the whats. Here is the first what. I am going to circle it. Can you find the second what?" The instructor required the student to complete the behavior before moving to the next step.

Intervention sessions typically lasted 10 to 15 min, with the length of time decreasing as participants became more fluent in problem solving. The instructor took data on the number of steps the participant was able to complete independently. Prompt levels were recorded for instructional purposes; however, only independent correct responses are graphed for data analysis. This method of assessment was a type of multiple-opportunity probe in that the student was given the opportunity to perform a step without help for purposes of data collection and then given prompting as needed to complete the step to set up the next response. Due to the chained nature of solving a word problem, each step is dependent on the correct execution of the one before, so the instructor must either prompt or set up each step to determine if later responses in the chain have been mastered. In addition, one single-opportunity probe was conducted in the concrete and virtual condition each, where no prompting or feedback was provided to determine if the student could perform the entire sequence without prompt.

Choice and maintenance. During the choice phase, the instructor presented both sets of materials (iPad graphic organizer and concrete graphic organizer) to the participant at the beginning of the session and asked which he would prefer to use that day. Once the participant indicated his choice, either by pointing or stating his answer aloud, the non-selected items were removed and the session began. Three novel problems were presented following baseline procedures (i.e., no specific feedback or prompt for problem solving–related behaviors, praise for on-task behavior) and the chosen condition was used for all three problems.

Social Validity

Both the special education teachers and the participants completed consumer satisfaction questionnaires for a social validity measure. The instructor interviewed the participants who were asked seven questions and were told they could say "yes" or "no," or point to the smiling or frowning face on the questionnaire to respond to statements related to the outcomes of the intervention. The teachers were asked to rate 12 items related to components and outcomes of the intervention on a 6-point Likert scale labeled completely disagree, mostly disagree, slightly disagree, slightly agree, mostly agree, and completely agree. In an open-ended question, teachers also were asked to provide feedback on the instructional methods and materials that were used.

Results

Scores Received on Performing Task Analysis Steps

Figure 2 shows the number of points each participant received for independent performance on the task analysis steps in both concrete and virtual conditions. All participants demonstrated a stable baseline receiving no more than 10 total points on the task analysis steps across the three problems and an immediate increase in level in both conditions upon entering intervention. Andrew scored 0 points consistently during baseline, whereas Nathan and Caleb demonstrated stable responding during baseline with mean points of 8.6 (range 8–9) and 6 (no range), respectively. Nathan's and Caleb's baseline performance was likely the result of participating in a previous study focusing on learning how to solve group and change word problem types using modified SBI, which included a tasks analysis with only three steps being applicable to all three problem types, including read the problem, circle the whats, and fill in number sentence (Saunders et al., 2016). Because Nathan and Caleb were not exposed to compare problem type prior to the study, they did not score correctly on the task analysis steps that were specific to the compare problems during baseline.

After the training period and with the support of least intrusive prompting during the intervention condition, all participants showed an immediate increase in level and/or ascending trend for both concrete and virtual conditions. Andrew met mastery criteria for solving compare word problems in six intervention sessions. Nathan met mastery criteria for solving the compare word problems in seven intervention sessions and scored slightly higher in the concrete condition. Caleb met mastery in eight intervention sessions with no difference in rate of learning based on concrete and virtual conditions.

Results of the single-opportunity probes (represented in triangle data points in Figure 2) in intervention during which no prompting or feedback was provided showed all participants scored similarly when compared with the multiple-opportunity probe data (circular data points in Figure 2) for both concrete and virtual conditions. The only exception was for Andrew whose performance from the single-opportunity probe in the concrete condition was lower than his performance from the multiple-opportunity probes in the same condition. During the choice and maintenance condition, all participants preferred virtual manipulatives and maintained high level of performance in solving compare word problems.

Social Validity

The teachers rated most items as mostly agree and completely agree, indicating they felt that the intervention contributed to their students' early numeracy and word problem solving skills. In responding to the open-ended question, Andrew's teacher commented that she would prefer to use concrete manipulatives to introduce new material for their kinesthetic property, but that virtual manipulatives seem to be more motivating for her students when there were no technical problems. Caleb's teacher stated that she observed Caleb to exhibit a much higher level of engagement when he was completing tasks associated with the study compared with his typical performance. She was impressed that Caleb used the materials provided to complete the word problem with minimal assistance. All participants reported

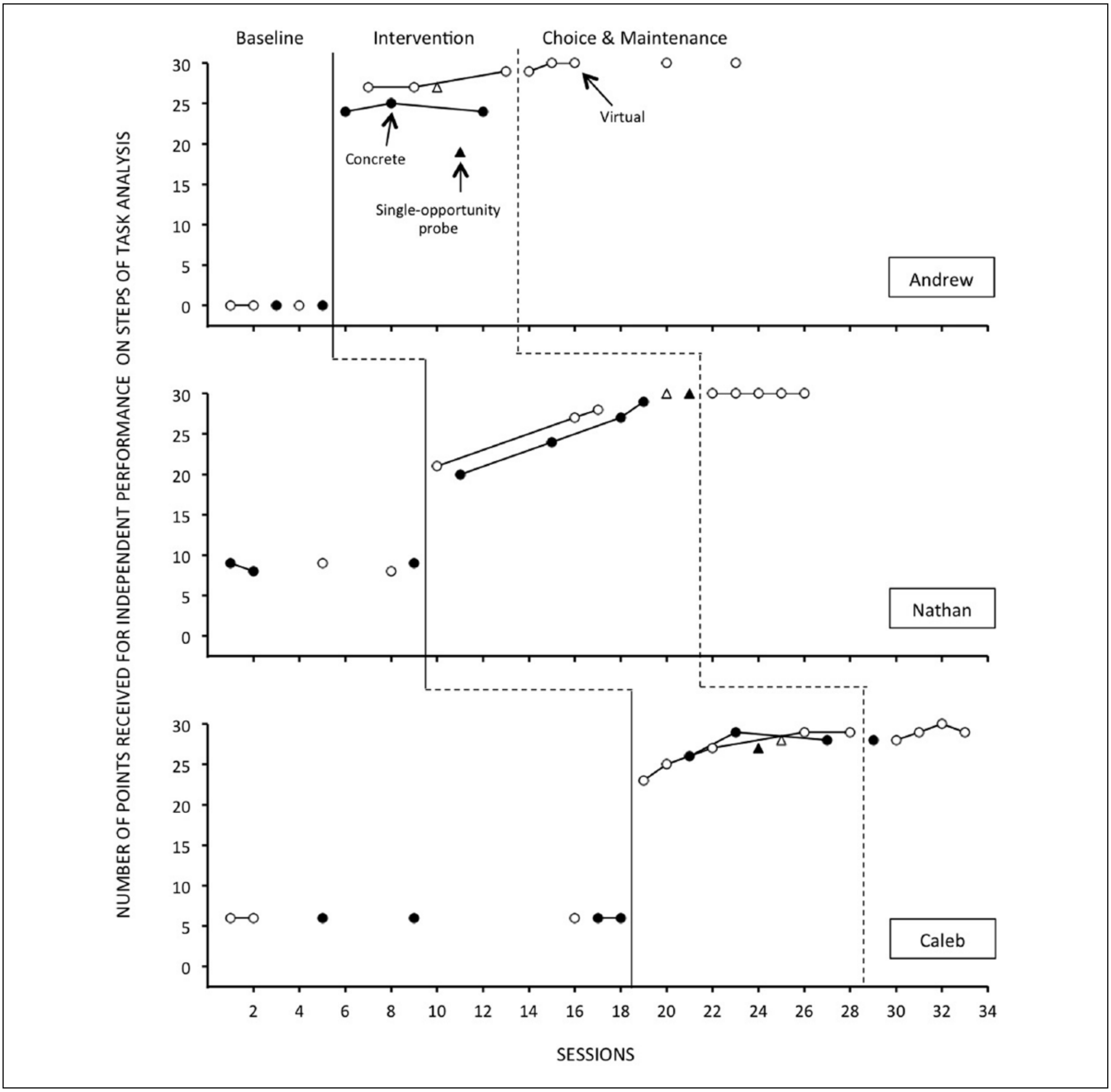

Figure 2. Number of points received from independently completing steps of task analysis by participants.

that they liked the math lessons, and the materials used including those in both concrete and virtual conditions. They also indicated the self-instruction sheet helped them solve the word problems.

Discussion

The purpose of the study was to examine the effects of modified SBI on the word problem solving skills of three students with ASD and moderate ID with a focus on the compare word problem type, and comparing the differential effects of concrete versus virtual manipulatives. Results of the study showed that all three participants improved their performance after the intervention and two participants showed higher acquisition of task analytic steps in the virtual condition. A functional relation between modified SBI and mathematics problem solving exists with three demonstrations of intervention effects at three different points in time.

Although mathematical problem solving is a core skill that is required for students to gain a deeper understanding of mathematical concepts, students with ASD and moderate ID tend to struggle with problem solving because of the cognitive and linguistic demands. Because of this, students with ASD and moderate ID must be explicitly taught mathematical problem solving. Not only must these students develop conceptual understanding of the mathematical processes represented by the story grammar within word problems, but they also need procedural instruction and supports. The current study adds to the literature on teaching grade-aligned mathematics to students with severe disabilities, including ASD. This study demonstrated that modified SBI is effective in teaching mathematical word problem solving to three elementary students with ASD and moderate ID by supplementing the essential components of SBI with evidence-based practices for teaching mathematics to students with severe disabilities, including the use of a task analysis and systematic promoting (Browder et al., 2008). This study also extends Saunders et al. (2016) study findings that students with ASD and moderate ID can successfully learn how to solve compare word problems.

The multicomponent nature of the intervention included modified SBI, a graphic organizer to visualize the comparison to be made and organize information from the problem, a student self-instruction sheet in a task analysis format as a heuristic for solving the problem, and systematic prompting to complete each step of the task analysis. The modified SBI provided in the intervention gave explicit instruction to participants through each of Mayer's (1985) phases of problem solving. The student self-instruction sheet provided support in students' executive functioning; it guided participants through the four steps of problem solving and broke the steps into measurable, observable behaviors. The problem translation phase, when semantic language skills construct meaning from the problem, was represented by the first four steps of the task analysis in which participants made meaning of the word problem by identifying the referent (the "whats"), locating the comparative phrase, and stating the rule. In Steps 5 through 7 of the task analysis, the students systematically selected the integral parts of the problem and translated them into a mathematics sentence. Finally, the solution planning and execution stages of problem solving occurred in Steps 8 and 9. The checking-off task further afforded students opportunities to be self-independent when solving a word problem.

The graphic organizer and student self-instruction sheet facilitated the conceptual understanding of the action from the word problem as well as the procedures to follow with the manipulatives in order to arrive at the solution similar to the way a mnemonic has been used in prior SBI research (Jitendra & Hoff, 1996; Powell et al., 2015; Rockwell et al., 2011). The use of schematic diagrams (e.g., graphic organizer) that represents the action of the problem is essential to SBI (Jitendra & Hoff, 1996). Although many prior studies on mathematics with students with ASD and ID have used task analytic instruction and prompting (Browder et al., 2008), only one prior study combined this systematic instruction with a modified SBI to support the needs of students with ASD and moderate ID to solve word problems (Saunders et al., 2016). The current study built on Saunders et al. (2016) to implement the modified SBI approach for students who also needed manipulatives because of limited numeracy skills.

In the current study, researchers included both virtual and concrete manipulatives to compare their effects. Although both virtual and concrete manipulatives and graphic organizers were shown to be effective supports, an increased rate of independence in the virtual condition for two participants and preference among all three participants for the virtual condition supports the use of virtual manipulatives to teach mathematics to students with ASD and moderate ID. Specifically, the difference in the rate of learning in the two conditions for two participants supports the findings of the previous investigation into the differential effects of virtual and concrete manipulatives for students with ASD (Bouck et al., 2013). This study extends the Bouck et al. (2013) findings, as the manipulatives were used by students who have not mastered basic math facts and were completing the computations in the context of a word problem. Students with both ASD and moderate ID often will be learning basic computation concurrently with problem solving. The use of manipulatives offers a feasible way to support students with ASD and moderate ID who may have limited computation skills.

Limitations and Future Research

The components of the intervention, including the materials, warrant caution when drawing conclusions regarding the extent of the participants' mastery of compare problem type. The word problems presented to each student were written in a predictable format which included four lines of text, picture supports over the "whats," and no irrelevant information. The numbers in each word problem were always arranged so that the larger number was first, preventing participants from having to determine the placement of the numbers in the number sentence based on place value. Finally, this intervention only targeted one problem type (i.e., compare), and therefore did not require participants to discriminate between problem types (i.e., group, change, compare). Each of these limitations could be addressed in future research by fading the supports within the written problem and varying the problem type.

In addition, future research is needed on teaching word problem solving to students with ASD and moderate or severe ID. This population presents unique challenges in accessing the semantics of written problems, organizing a mathematical operation, and executing the solution. The current study, along with Saunders et al. (2016), provides a possible direction for teaching problem solving to students with ASD and moderate or severe ID by modifying SBI that has been used for students with high incidence disabilities (Jitendra & Hoff, 1996). Future research is needed to analyze which variables in this multi-component intervention contribute to the outcomes (e.g., comparing performance with and without the student self-instruction sheet). Further, more research on the potential benefits of virtual manipulatives is needed. In general, there are limited examples of computer-assisted instruction in the area of mathematics for students with ASD and ID. Future research may address other technology features such as highlighting key information or embedding supports (e.g., a "help" button) for mathematical problem solving for students with ASD and ID. Finally, future investigations should include generalization of students' mathematical problem solving and enhanced social validation. Generalization training may be used to help students manage word problems written in a variety of formats, ones that include extraneous information, and that fade stimulus prompts (e.g., pictures over the key nouns). Social validation of these outcomes might include more than the teachers' responses to a Likert scale questionnaire. An open-ended teacher interview about the students' mathematical learning and observations of the students' mathematical performance outside the research sessions could offer further information on the students' understanding of problem solving.

Implications for Practice

Word problem solving is an important mathematical skill for all students, including those with ASD and ID who have significant support needs. For students who have not mastered basic math facts, manipulatives can provide concrete representations of the action of a word problem when used with a graphic organizer. The results of this study also showed that both virtual and concrete manipulatives are effective. Virtual manipulatives may reduce off-task behavior with materials (e.g., stimming and stacking) while maintaining the value and purpose of the concrete representation. Through the systematic and explicit instruction provided in modified SBI, educators can give students access to critical thinking skills and an opportunity to apply their early numeracy skills in real-world situations. Educators can teach students to follow a task analysis for solving mathematical word problems, by presenting it in a mode that is accessible to students, whether using pictures, words, or a combination of the two. Finally, educators can use the self-instruction sheet paired with modeling think-alouds to not only increase students' mathematics skills but also reduce dependency on adult supports.

Authors' Note

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