

# An Adaptive Model of Student Performance Using Inverse Bayes

Charles Lang

Harvard Graduate School of Education, USA

charles\_lang@mail.harvard.edu

**ABSTRACT:** This article proposes a coherent framework for the use of Inverse Bayesian estimation to summarize and make predictions about student behaviour in adaptive educational settings. The Inverse Bayes Filter utilizes Bayes theorem to estimate the relative impact of contextual factors and internal student factors on student performance using time series data across a range of possible dimensions. The Inverse Bayesian algorithm treats the student as a Bayesian learner; her partial credit score or confidence is proportional to both her prior knowledge and how she interprets her environment. Once the algorithm has weighted internal and external factors, this information is used to make a prediction about the student's next attempt.

**KEYWORDS:** Inverse Bayes, adaptive, electronic tutor, online assessment, student certainty, student confidence

## 1 MOTIVATION

This work is motivated by the desire to make actionable inferences from high dimension, time series data in an iterative fashion. As online assessment becomes more sophisticated and the scale of data collection on student behaviour increases, adaptive engines will be required that can process a large number of variables on the fly in a way that makes intuitive sense to educators and students.

## 2 THEORETICAL BACKGROUND

Bayes Theorem has been used to model the human mind since at least the 1920s with successful modelling approaches such as Decision Theory, Bayesian Knowledge Tracing (BKT), and Cognitive Bayesianism having evolved over the last 90 years (Corbett & Anderson, 1994; Perfors, Tenenbaum, Griffiths, & Xu, 2011; Schlaifer & Raiffa, 1961). The foundational idea that links these methods is the use of Bayesian inference to estimate the properties of the latent factors that drive human behaviour (Chater & Oaksford, 2008). The inverse Bayesian Filter, under study here, builds on these models, but instead of using prior probabilities and likelihoods to estimate the posterior probability of student behaviour, it attempts to split the probability of a student behaviour into its likelihood and prior based on a posterior probability. Within this framework, the prior represents performance factors internal to the student, such as knowledge and skills, whereas the likelihood represents contextual or external factors important to student performance (Cronbach & Snow, 1981). The main goal of this approach is to develop likelihoods and priors that are particular to each student; in other words, to allow parameters to adjust freely to individual students' behavioural idiosyncrasies. If successful, this could

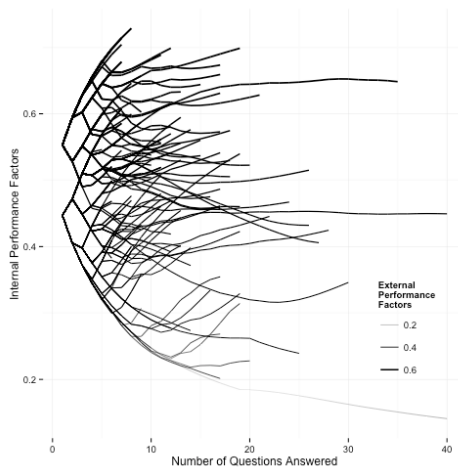
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further the development of adaptive engines within educational software. To determine whether this is a viable strategy I have sought to answer two questions: 1) How does an Inverse Bayesian model compare to BKT in terms of predicting the next action by a student in a cognitive tutor? and 2) Do the patterns of priors and likelihood make substantive sense?

### 3 METHODS

Data was generously provided from the ASSISTments online math tutoring system by Professor Neil Heffernan of Worcester Polytechnic Institute (Feng & Heffernan, 2007; www.assistments.org). The data set consisted of 448, 12–14 year olds and their answers to math problems concerning Pythagorean Theorem. Three variables were analyzed using the Inverse Bayesian model: correct/incorrect responses, student confidence, and the partial credit metric developed by Wang and Heffernan that incorporates the number of hints and attempts a student utilizes (2011).

### 4 RESULTS



**Figure 1: Weight of internal performance factors plotted against number of items answered, where the size of the line indicates external performance factors.**

**Table 1: Root Mean Square Error for Inverse Bayes predictions of the cumulative average of correct/incorrect scores, student confidence and partial credit scores.**

Cumulative Average Correct/Incorrect	Student Confidence	Partial Credit
0.275	0.210	0.419

Prediction performance of the Inverse Bayes approach is variable across several input metrics, with student confidence showing the best performance (Table 1). The performance factors that the algorithm resolves generally act as would be expected: students who know more also find the context more conducive to demonstrating that knowledge while students who know less find the context more difficult (Figure 1).

## 5 CONTRIBUTION TO LEARNING ANALYTICS

I see learning analytics as a chance to expand the metrics used within education. Rather than this being a confusing or limiting proposition, I think this plurality of measurements can allow deeper and more diverse understandings of learning. I hope to contribute to this advancement in student learning and to our understanding of what that learning is.

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