Using Mathematics, Mathematical Applications, Mathematical Modelling, and Mathematical Literacy: A Theoretical Study

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Abstract
The purpose of this theoretical study is to explore the relationships between the concepts of using mathematics in the daily life, mathematical applications, mathematical modelling, and mathematical literacy. As these concepts are generally taken as independent concepts in the related literature, they are confused with each other and it becomes difficult to understand the relationships between them. This study elaborates on the mathematical processes contained in these concepts and the mathematical skills required by these processes. An attempt is made to reveal the common ideas within these concepts, the points they emphasize, and the basic skills and acquisitions expected to be introduced to individuals. In this regard, sample application and modelling problems are presented. At the end of the study, some recommendations are put forward based on the results obtained in the study and the ideas suggested in the related literature.
Keywords: using mathematics, mathematical application, mathematical modelling, mathematical literacy

Introduction

School mathematics, which deals with the questions of "What kind of individuals do we want to provide the society with?" and "What should we teach about mathematics and how should we teach it?", has two general purposes: (1) to provide the staff needed in the fields of industry and technology as well as many other areas in the daily life by giving mathematics education to a large mass in the society; (2) to prepare the prospective mathematicians who are to engage in mathematics in academic and scientific terms by shaping them as mathematicians from young ages. In brief, the task of producing new knowledge in mathematics education falls to academic mathematics whereas the task of transferring the produced knowledge to young generations falls to school mathematics. As the existing mathematical knowledge is much beyond what can be taught during school period, school mathematics teaches students basic concepts and ways of acquiring mathematical knowledge (Baki, 2014a, p. 34). Accordingly, the concept of mathematical literacy, which emphasizes individuals' skills of using and interpreting mathematics in their daily lives, can be deemed as the way mathematics is included in curricula (Baki, 2014b, p. 307). NCTM (1989, 2000) stresses the need for people who understand mathematics and can use it in their lives and states that one of the basic goals of mathematics education is to provide the society with mathematically literate individuals.

According to a content-oriented curriculum, the basic domains of school mathematics are using mathematics, numbers, algebra, geometry, statistics, and probability (Baki, 2014a; p.317). NCTM (2000) has created problem-solving, reasoning, communicating, and connecting standards within the scope of the mathematical skills intended to be introduced to students and states that these skills support learning the basic subject areas of mathematics and are improved through these learning processes. What is meant by mathematical learning here is learning and using mathematics. NCTM (2000) highlights students' noticing and understanding the relationships between mathematical ideas and using them outside the mathematical world within the scope of mathematical skills and emphasizes mathematical *applications* and *modelling* processes. Similarly, the new mathematics curriculum that has been put into practice in Turkey has mathematical modelling and problem-solving among the mathematical skills and competences it aims to develop and attaches a great importance to designing educational environments by putting these skills at the center. In this regard, the curriculum aims to make students notice that mathematics is a systematic way of thinking that generates solutions to real life problems via modelling and see that mathematics offers a very useful language to explain real world situations and make predictions about future (MEB, 2013). Hence, it can be said that the skills of using mathematics, mathematical application, mathematical modelling, and mathematical literacy, which are focused on in the present study, are among the skills which the current mathematics education aims to develop in individuals. These concepts and the related mathematical skills are explored in detail below.

1. Using Mathematics

Henn (2007) mentions two main characteristics of mathematics: (1) mathematics, just like fine arts and music, has a unique aesthetic beauty that no ordinary person can reach (2) it has an extraordinary functionality that can put all parts of life in order and allow us to understand them. The reason why mathematics is the sole and greatest phenomenon of education in the world is that it is used in a lot of extra-mathematical areas in various ways. In many countries, long periods of time are reserved for teaching mathematics within curricula. This is because mathematics offers undeniable benefits for both understanding other courses in the curricula and solving

the problems encountered in life. The objective of learning mathematics is to learn mathematical concepts, skills, and strategies and use these instruments for solving the problems encountered in real world (Muller and Burkhardt, 2007). In this regard, the goal of mathematics education is to provide individuals with mathematical disposition and skills of using mathematics in an appropriate way when necessary (Altun and Memnun, 2008).
Using mathematics requires knowing mathematics, which knowledge to use, and where to use such knowledge in the first place. In the literature, the usage areas of mathematics are taken as mathematical world (mathematical universe) and real world in general. While using mathematics requires mathematical understanding and doing mathematics in the mathematical world, it requires different knowledge and skills in addition to them in real life. Baki (2014a) takes using mathematics in two parts as follows:

"First, it involves the student's understanding or making sense of the relationships, characteristics, definitions, and evidences within mathematics by using his mathematical knowledge. Secondly, it refers to the student's applying his mathematical knowledge to new situations and the solution of problems in the daily life or constructing new mathematical knowledge by using his existing knowledge and using such new knowledge in solving problems. (p. 318)"

Based on the definitions and explanations here, two main kinds of use of mathematics can be expressed as follows:

- Using mathematics to make sense of mathematics in the mathematical world, to notice mathematical concepts and the relationships between these concepts, and to do mathematics *(using mathematics in the mathematical world)*
- Using mathematics to transform the situations encountered in real world into mathematical language by noticing the living aspect of mathematics, to analyze and interpret such situations, to generate real solutions to real problems, and to make effective decisions *(using mathematics in real life)*

1.1. Skills of Using Mathematics

The skills to be needed by individuals in the processes of using mathematics whether in the mathematical world or in real world are expressed in different combinations involving basic mathematical skills (i.e. problem-solving, associating, communicating, mathematical modelling, reasoning) in the literature. De Lange (1996) lists the skills of using mathematics as using the common mathematical knowledge, using formulas in mathematical communication, making sense of and using all kinds of graphical and linear representations, using mathematical models, and reasoning on their relationships with the current situation while PISA survey takes them as communicating, mathematizing, representation, reasoning and argument, devising and using different strategies, using the mathematical language and operations, and using mathematical tools (Milli Eğitim Bakanlığı [MEB], 2011). The "National Curriculum for Math" published in the United Kingdom defines the concept of using mathematics as a process consisting of knowledge transfer, application, proving, and organizing and takes the basic skills which students are required to have for the domain of using and applying mathematics as problem-solving, representation, research, reasoning, and communicating. Within the scope of "functional mathematics" put forward by Forman and Steen (1999) in the USA as an internal curriculum for school mathematics, the skills of using mathematics are taken as thinking and reasoning, discussion, communication, modelling, problem-solving, representation, and using a symbolic-formal and technical language. Baki (2014a, p.318) summarizes the indicators of using mathematics as follows:

i) The student must be capable of speaking and writing about a mathematical activity or problem he is dealing with and making explanations by expressing himself easily and using mathematical terminology.
ii) The student must be capable of making predictions and assumptions about the mathematical activity he is working on, verifying the results he reaches, and making generalizations.
iii) The student must be capable of mathematically expressing his proving and generalizations and abstracting them by using a formal language.
iv) The student must be capable of interpreting the mathematical knowledge presented to him visually or in written…..(*)

Based on the indicators given above and the studies explained above, it can be said that effectively using mathematics in the daily life or in the mathematical world requires having basic mathematical knowledge and skills and interpreting such knowledge and skills by transferring them from the mathematical world to real world or from real world to the mathematical world as necessary.

1.2. Using Mathematics in the Mathematical World

Using mathematics in the mathematical world requires understanding mathematical concepts in connection with each other and using them in different mathematical situations. Here, the individual uses mathematics to do mathematics or understand it better. The use of the number Π in trigonometry can be showed as an example of using mathematics to understand it better. An individual can make many different inferences by using his mathematical knowledge in order to make sense of a situation. If he has inadequate mathematical knowledge, he can make an inference like the following: if a full tour on the unit circle corresponds to 2Π radians, $2\Pi = 360^o$ and thus $\Pi = 180^o$. As a result, the individual may not find wrong representations (e.g. $\alpha = 30^o + 2k\Pi$) in mathematical processes odd and may use them. On the other hand, an individual who questions the use of Π in trigonometry with well-learned mathematical knowledge will try to make sense of the situation by using his mathematical knowledge and demonstrate the relationships and significant differences between the concepts in various ways (See Figure 1)

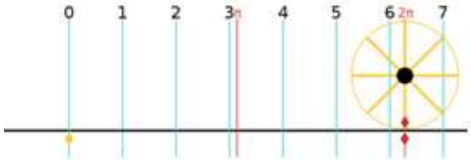


Figure 1. Example 1 for Use of Mathematics in the Mathematical World

As it can be seen in the Figure 1 above, the individual thinks that 2Π units of distance will be covered with a full rotational motion of the unit circle on the plane and this will correspond to approximately 2x3.14 = 6.28 units. Based on the knowledge that the measure of the central angle facing the angle at the length of the radius in a circle is one radian, it can be inferred that 2Π radians will correspond to 360^o. That is 6.28 (2x3.14) radians will correspond to 360^o. Assuming that $2\Pi = 360^o$ will cause a different meaning to be attributed to the number Π and thus lead to a wrong learning of mathematics. An individual who knows mathematics and can use his skills concerning these processes can achieve a more meaningful learning of mathematics by using it.

Another example of the use of mathematics in the mathematical world is Egyptians' calculating the area of circle based on the area of square. In the calculation of the area of the circle whose diameter is nine units in the example below, the diameter of the circle is divided into nine equal parts, and it is accepted that the area of the square formed with eights parts of it is equal to the area of the circle.

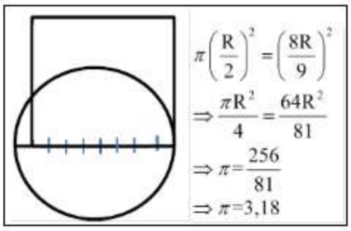


Figure 2. Example 2 for use of Mathematics in the Mathematical World

In this example, it is seen that when the diameter of the circle and the edge of the square are taken at the ratio of 8/9 and the area of the circle is compared with the area of the square in the current representation, a value that is quite close to Π is found (Baki, 2014b). Here, mathematics is used for doing mathematics rather than understanding mathematics.

1.3. Using Mathematics in Real Life-Applying Mathematics

While the purpose in using mathematics within itself is to achieve better understanding of it or to do mathematics, the purpose in using mathematics in real life is to seek solutions to the problems encountered. Although mathematics itself exists in real life, there is a distinction resulting from the difference between the processes of using mathematics. As a matter of fact, the use of mathematics within itself exists in the mathematical world whereas its use in real life requires the intersection of the mathematical world and real world. In real life situations, mathematical studies are carried out in connection with real situations, and results are interpreted in connection with real situations. The correctness and validity of the obtained results depend on the real situation and current circumstances. The goal is to produce the best solution for the problem situation the individual encounters.

In the literature, the concept of using mathematics in real life is referred to as "applying mathematics" whereas the real life problem handled is referred to as "application of mathematics". The word "application" is used for indicating any kind of situation that brings together mathematics and real life (Niss, Blum and Galbraith, 2007). Any real life situation can be deemed as an "application" and any kind of connection between mathematics and reality can be regarded as an application of mathematics (Sloyer, Blum and Huntley, 1995). King (2002) calls the process of using mathematics as "application" and describes applied mathematics as *the intellectual area that is* *characterized by use of mathematics to understand the nature and work on concrete phenomena.* An applied mathematician is a person who explains real world by using the language of mathematics, answering important questions by using mathematics, and applying mathematics to real life. Applied mathematics does not mean daily mathematics. Mathematical question is clear in daily mathematics; however, this is not the case in applied mathematics all the time. Applied mathematics problems may be from the fields of biology, finance, medicine, business, and so on and require more complex and longer processes relative to real life problems. Their processes of solution may take more than one day. Mathematical applications, which always incorporate real life situations, may differ within themselves. Pollak (1976) divides mathematical applications into sub-sections. These two categories are still in use.

i) Applied mathematics involves beginning with a situation that emerges in real life or in fields other than mathematics, making mathematical comments or forming models, subjecting such models to mathematical manipulations, and applying the results to the original solution. These “other fields” may include biology, social sciences, business, medicine, and so on.

ii) Applied mathematics is what is done by those people who apply mathematics when necessary in order to deal with the situations they encounter in their lives.

Kemme (1985) makes the following distinction between i and ii: applied mathematics vs. users’ mathematics (cited: De Lange, 1996). Real applications (i) (classical applied mathematics) contain important applications that are suitable for certain expert students in higher grades. Users’ mathematics (ii) can be defined as mathematics used in the daily life and must be part of the curriculum at every level of mathematics education. The following well-known application problem can be taken for real applications (King, 2002):

A golf ball which is hit with a gold club is ascending at an angle given relative to the ground and at a known velocity. How far will it go?

If the unknown velocity at any moment during the motion of the ball is represented with “v”, the following differential equation can be written:

dv/dt = g …….(D)

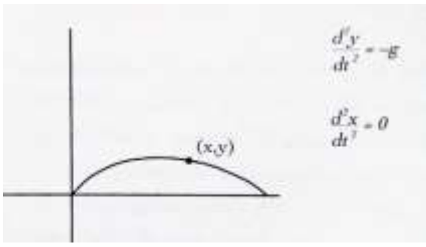


Figure 3. The motion of a golf ball

This equation means the following: “The temporal change rate of velocity is equal to g, which refers to gravitational acceleration.” When the mathematical knowledge acquired here is transferred to real world (i.e. interpreted in real world), the below-mentioned real results are reached:

- “m” does not exist in the equation (D). Hence, the mass of the ball does not have any effect on its motion.
- The golf ball moves along a curve on the plane of motion, not along a straight line. Thus, the position of the ball at any moment is determined with the x and y coordinates showed in the Figure 3. The equation (D) is a vector equation. Two differential equations related to the (x, y) coordinates of the ball can be written instead of it. This being the case, while the ball is moving, the vector both stretches and rotates. The equation (D) defines this motion.
- Lastly, it is seen that the problem dealing with the motion of the ball is reduced to the analysis of the equation (D). How far the ball will go can be found out through solution of this equation.

King (2002) refers to the equation (D) as a *mathematical model* and schematizes such mathematical application process as indicated in the Figure 4 below. In the Figure 4, the golf problem corresponds to the colored part in the real world rectangle. The first step in the solution process is to build a mathematical model for the real world problem. King (2002) describes this process as *making an abstract copy of the real world phenomenon in the mathematical world.* This model corresponds to the colored area in the mathematical world rectangle in the Figure 4. The process of building the mathematical model (i.e. leaving the real world) is showed with the curved arrow seen in the Figure 4 and called as “abstraction”.

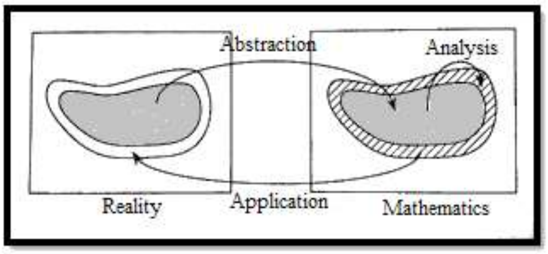


Figure 4. Applied Mathematics Process (King, 2002)

Hereafter, the mathematical model formed in the mathematical world, as distinct from real world, is employed, and new mathematical knowledge is produced. This process can be regarded as the analysis stage. In the golf ball example, the analysis process tells us how far the ball will go. In the scheme, this new knowledge about the model is showed by fattening the area in the mathematical world. The next step is to "apply" new mathematical facts (shaded area) to real world phenomena. This new knowledge is displayed with the enlarged real world area in the Figure 4. In the golf ball example, this area incorporates new "real phenomena" such as how far the ball will go, how high the ball will ascend, and the exact position of the ball at any moment. The example below can be taken as an example of the second kind of mathematical applications, which are referred to as users' mathematics (ii).

A tap whose rubber joint is broken drops water once every three seconds. Calculate the amount of water to be wasted by the tap in one week if it is not repaired. Would the wasted water be enough to fill a glass in one week? Or would it be enough to fill a carboy or a bathtub?

Figure 5. Mathematical Application Example

The solution of a real life problem is in question in the application example seen in the Figure 5 (MEB, 2014). The solution process of this problem involves similar stages to the golf problem above. Thus, it can be explained with the Figure 4. Firstly, a model of the current situation is formed in the mathematical world, and the amount of water dropped by the tap is expressed as a function of the time (BS(t)) as indicated below.

$$BS\,(t)=\begin{cases} t/3\,, & t\equiv 0 \pmod 3 \\ (t-1)/3, & t\equiv 1 \pmod 3 \\ (t-2)/3, & t\equiv 2 \pmod 3 \end{cases}$$

Then the mathematical data available are organized for one-week period to reach new mathematical knowledge. Such new knowledge produces the number of drops from the tap in one week.

$$t= 7x24x60x60$$
$$BS(t)= 7x24x60x20 = 201600$$

Although work in the mathematical world seems to be finished, it is necessary to consider the mathematical data available in real world and then study it with mathematical tools in the mathematical world again in order to answer the question asking whether 201,600 drops of water will fill a carboy in real life. In this stage, the individual can make predictions and assumptions, demonstrate the correctness/incorrectness of his predictions and assumptions, and produce the most appropriate solution for the problem. To know whether 201,600 drops of water will fill a carboy, one has to answer the question of how many drops a carboy of water contains. Answering this question requires studying in the mathematical world in connection with real world.

If one tea spoon holds approximately 20 drops of water
and if four tea spoons are approximately 5 ml,
80 drops of water mean ≈ 5 ml.
One water glass holds ≈ 250 ml ≈ 50x80=4000 drops of water
Then
201,600 drops of water fill ≈ 50 water glasses.

1 water glass	0.25 lt
1 carboy	19 lt
1 carboy	19÷0.25=76 glasses of water

In the last stage, one has to transfer new mathematical knowledge to real world and interpret it there.

If one carboy holds 76 glasses of water, the tap mentioned in the problem will not fill one carboy in one week.

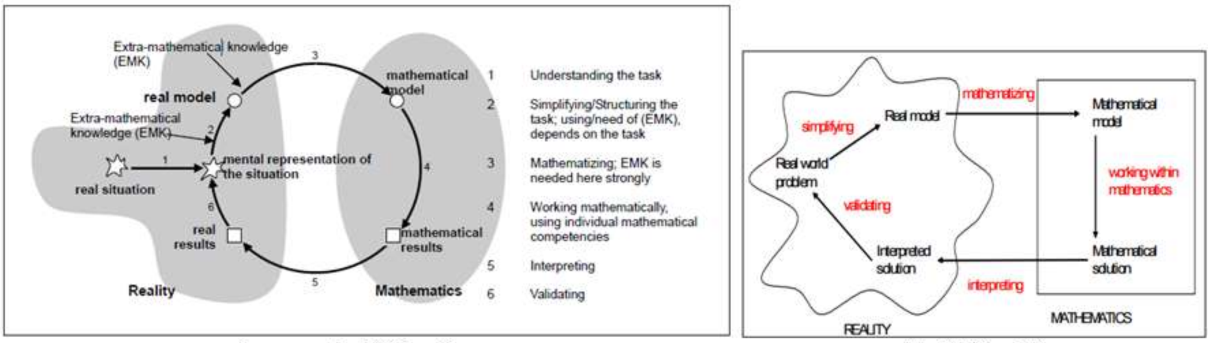
Two kinds of application problems discussed above can be compared as follows:

1. Both application processes require using a mathematical model.
2. The second application is a kind of problem situation in which more individuals can produce a solution in real life in comparison to the first application.
3. The mathematics used in the mathematical world in the first application is more comprehensive and complex than the mathematics used in the second application.
4. In the first application, one does not need real world while working within the mathematical world. He just interprets what the mathematical data available corresponds to in real world in the last stage of the process. In the second application, on the other hand, the solution process of the problem requires acting by establishing a connection between real world and the mathematical world as a whole.

The first kind of applications described above generally require advanced mathematical knowledge and use of mathematical techniques. In these kinds of applications, the problem is mostly defined within a theoretical framework and is mainly studied in the mathematical world. However, the process involves using a mathematical model as well. In the second kind of applications, on the other hand, the solution of the problem mostly depends on the use of the formed mathematical model in accordance with reality as the mathematics studied here involves simpler and daily situations. In these kinds of applications, focus is on the mathematical processes needed to complete the modelling circle in a successful way, and thus these kinds of applications refer to the modelling process (Niss et al., 2007).

2. Mathematical Modelling

Haines and Crouch (2007) define mathematical modelling as a cyclical process in which real life problems are expressed and solved by use of the mathematical language, and solutions are tested through interpretations based on reality. Verschaffel, Greer, and De Corte (2002) describe mathematical modelling as the explanation of real life situations and the relationships within these situations by use of mathematics. Mathematical modelling is a general term that draws attention in a lot of disciplines apart from mathematics and involves real life-related, open-ended, and applied problem-solving practices at every level of education. There are a lot of theoretical models aimed at defining the mathematical modelling process (Blum and Leib, 2007; Borromeo Ferri, 2006; Galbraith and Stillman, 2006; Lesh and Doerr, 2003; Maab, 2006; Verschaffel, Greer and De Corte, 2002). Though these theoretical models differ from one another in terms of the stages they include and the transitions between them as well as the explanations regarding such stages and transitions (see Figure 6), they all agree that the modelling process has a cyclical structure (Zbiek and Conner, 2006). In general, the following stages are stated to exist in the mathematical modelling process: Real life situation, the picture the student has in his mind in regard to the real situation (situation model), real model, mathematical model, mathematical results, and real results (Blum and Leib, 2007; Borromeo Ferri, 2006; Maab, 2006). The cognitive behaviors displayed by students during these stages and the transitions between them constitute the modelling process (Figure 6).



Borromeo and Ferri (2006, p. 92) Maaß (2006, p.115)

Figure 6. Modelling Process

Modelling competencies involve the knowledge, skills, and capabilities that are needed to conduct the modelling process properly, eagerness to engage in this process, and metacognitive skills that are required for an effective modelling process (Maaß, 2006). Schoenfeld (1992) describes the modelling competence as follows: *Knowledge* of concepts and skills; *strategy* for how to use such knowledge in the modelling process; the individual's *metacognitive control* in the problem-solving process; *potential* for mathematical thinking; and *belief* in mathematics as a strong tool. Although the components here are independent from each other, they are used by the individual in connection with each other in the modelling process.

Modelling skills, on the other hand, include such technical skills required for completing any modelling

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process as understanding the real life situation, establishing a model, and performing mathematical operations on the model. In this regard, it can be said that modelling competencies include modelling skills and additionally involve eagerness to display these skills in line with a purpose (Kaiser, 2007). Blum and Kaiser (1997, p. 9) divided mathematical modelling skills into the following subcomponents in parallel with modelling steps.

1st Step: To understand the problem and to establish a model based on reality

- To make assumptions for the problem and to simplify the situation
- To identify the quantities influencing the situation, to name them, and to determine key variables
- To find out the relationships between the variables
- To differentiate between the information that is to be used in solution and the information that is not to be used in solution by considering what is given in the problem

2nd Step: To establish a mathematical model by using the real model

- To mathematically express the interrelated quantities and the relationships between them
- To simplify the relevant quantities and the relationships between them if necessary and to decrease their number and complexity
- To use appropriate mathematical notations and to graphically represent situations

3rd Step: To answer the mathematical question by using the formed mathematical model

- To use appropriate problem-solving strategies (e.g. division of the problem into parts, approaching the problem from a different perspective, varying the quantities)
- To use mathematical knowledge to solve the problem

4th Step: To interpret the obtained mathematical results in real world

- To interpret mathematical results in extra-mathematical contexts
- To generalize the results obtained for a specific situation
- To express and/or discuss mathematical solutions by using an appropriate mathematical language

5th Step: To validate the solution

- To critically analyze and check the obtained solutions
- To review some parts of the established model or to start the modelling process again if the solutions are inconsistent with the problem situation
- To reflect on other ways of solving the problem or to develop the existing solutions in different ways
- To question the model in general

There is no strict order to be followed for the steps of the modelling process. The individual's mathematical knowledge and skill to use mathematics can be regarded as the individual factors influential on the differentiation of the modelling process. The structure and characteristics of the problem are also among the factors affecting the modelling steps. The following problem can be taken as a mathematical modeling problem.

The table on the following page gives some of the planets' average distances D from the sun and their period P of revolution around the sun in years. The distances have been normalized so that Earth is one unit from the sun. Thus, Jupiter's distance of 5.2 means that Jupiter's distance from the sun is 5.2 times farther than Earth's. *

Planet	*D*	*P*
Earth	1	1
Jupiter	5.2	11.9
Saturn	9.54	29.5
Uranus	19.2	84.0

(a) Plot the points (D , P) for these planets. Would a straight line or an exponential curve fi t these points best?
(b) Plot the points (ln D , ln P) for these planets. Do these points appear to lie on a line?
(c) Determine a linear equation that approximates the data points, with x = ln D and y = ln P. Use the fi rst and last data points (rounded to 2 decimal places). Graph your line and the data on the same coordinate axes.
(d) Use the linear equation to predict the period of the planet Pluto if its distance is 39.5. Compare your answer with the true value of 248.5 years.

Figure 7. A Mathematical Modelling Problem Example

The situation contained in the problem above (Figure 7) is a real situation. The solution process requires mathematically expressing (modelling) the relationship between the variables of distance and period in the problem.

Based on the steps suggested by Blum and Kaiser (1997, p. 9), the solution process of the problem can be explained as follows:

1. *To understand the problem and to establish a model based on reality:* In this process, it is first necessary to make sense of the table contained in the problem and to identify he relationship between the values x=lnD and y=lnP that exist in real world.

$$\ln(5.2) = 1,64 \quad \ln(11,9) = 2,47$$
$$\ln(9,54) = 2,25 \quad \ln(29,5) = 3,38$$
$$\ln(19,2) \sim 2,95 \quad \ln(84) \sim 4,43$$
$$(2,47/1,64) \sim (3,38/2,25) \sim (4,43/2,95) \sim 1,50$$

2. *To establish a mathematical model by using the real model:* In this process, it is necessary to mathematically express (model) the relationship between the variables.

$$\boxed{y = \frac{3x}{2}}$$

3. *To answer the mathematical question by using the formed mathematical model:* In this process, it is necessary to decide on what kind of a path one has to follow in order to find out the period of the planet and to perform the following mathematical operations.

$$\ln(D) \sim \ln(39,5) \sim 3,67$$
$$y = \ln(P) = \frac{3 \times 3,67}{2} = 5,50$$
$$P = e^{5,5} = (2,71)^{5,5}$$

4. *To interpret the obtained mathematical results in real world:* In this process, it is necessary to interpret the obtained mathematical result (5,50) in line with real situation and shape the solution process based on it. As a matter of fact, though there is no one-to-one relationship between the variables of distance to Earth and period contained in the problem, there is a mathematical relationship between the images of these values under the function of "y=ln(x)". Lastly, the approximate value of the obtained value ($(2,71)^{5,5}$) can be calculated.

$$(2,71)^{5,5} = (2,71)^{5} \cdot (2,71)^{1/2} = 146,16 \times 1,64 \sim 240$$

5. *To validate the solution:* In this process, what has been done so far and the models established are validated. The following operations can be performed to validate that the period of the planet Pluto, whose distance to Earth is 39.5, will approximate 240 according to the mathematical result obtained.

$$\frac{\ln(240)}{\ln(39,5)} \sim 3/2$$
$$\frac{5,48}{3,67} = 1,49 \sim 1,50$$

In this step, the solution process can also be validated by using the value given in the problem as indicated below.

$$\ln(248,5) = 5,51 \sim 5,50$$

When the solution processes of the application problem in the Figure 5 and the modelling problem in the Figure 7 are compared, it is seen that they involve similar processes. At this point, these two concepts must be taken in association with one another, and the differences between them must be showed.

2.1. Modelling and applications

Though the difference between mathematical modelling and mathematical applications is differently expressed by different researchers, similar points are emphasized by them. According to Galbraith (1999), although the problem situation and the mathematics contained in a typical mathematical application are interrelated, they are distinguishable: the problem situation is not needed after the mathematics required for solving the problem is applied in mathematical applications; however, an absolute understanding and exploration of the situation requires choosing and using the relevant mathematics and interpreting it in accordance with real situation in modelling problems (Ang, 2009). This view clearly manifests itself in the application problem about golf ball provided above and is stated to be a characteristic of the solution process. Niss et al. (2007) note that modelling refers to the processes existing in transition from real situation to mathematics whereas application involves the objects existing in transition from mathematics to real situation (See Figure 8).

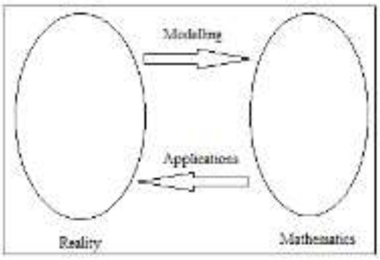


Figure 8. The Application-Modelling Relationship

It is emphasized in the same study that modelling problems involve looking at the mathematical world from outside mathematics and seeking an answer to the question of "Where can I find the mathematics that will help me solve this problem?" while application problems involve looking at real world from the mathematical world and seeking an answer to the question of "Where can I use mathematical knowledge?". The views mentioned here highlight a common idea. In general terms, mathematical application processes can be represented by finding the images of the objects existing in the mathematical world within real world while mathematical modelling processes can be represented by finding the images of real world objects within the mathematical world.

Ortlieb (2004) compares modelling and application processes in a different way. According to Ortlieb (2004), applied mathematics involves turning real life situation, which is the basic component of modelling, into a mathematical problem, and "a real model produced from mathematical models", which exists in modelling processes, does not exist in application processes. Ortlieb (2004) points out, "In applied mathematics one does not distinguish a real model from a mathematical model, but regards the transition from real life situation into a mathematical problem as a core of modelling" (Kaiser 2005, 101). Pollak (1979, 203) presents the difference between application and modelling while explaining the mathematical modelling process (Figure 6). According to the Figure 6 below, in the modelling process, real life situations are first handled in the classical applied mathematics, which exists within the mathematical world. The real life knowledge interpreted in this world turns into classical mathematical knowledge. The knowledge in hand has to be made relevant to reality for it to be interpreted in daily life. Therefore, classical mathematical knowledge is made more up-to-date in the course of the process. In other words, it becomes applicable to (usable in) real life. The modelling process comes to an end as the knowledge made relevant to reality is handled and used in real life.

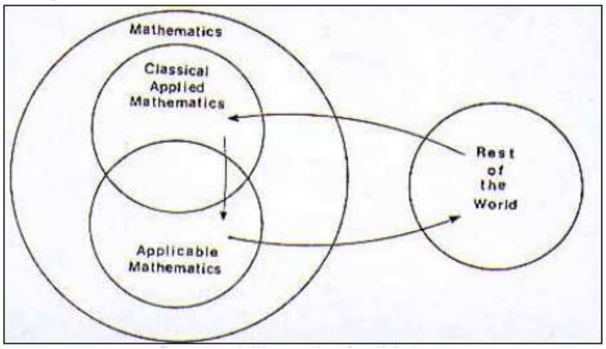


Figure 9. Modelling Process (Pollak, 1979, 203)

According to Pollak's (1979) model, it can be said that mathematical modelling processes embody mathematical applications. Although this model seems to conflict with the idea put forward by Niss et al. (2007), it suggests the same thing in essence: *Although every mathematical modelling can be deemed as a mathematical application, not every mathematical application can be regarded as a modelling as it does not involve all stages of the modelling circle. Differences between the processes vary depending on real life situation.* Accordingly, it can be stated in general that mathematical modelling processes are a special kind of mathematical applications.

Another concept that is closely associated with using mathematics in the daily life in the literature is mathematical literacy. To see the relationship between the concept of mathematical literacy and mathematical modelling, an attempt is made to explain the concept of literacy in this section.

3. Mathematical Literacy

Mathematical literacy is a concept introduced by the Organisation for Economic Co-operation and Development (OECD) via Programme for International Student Assessment (PISA) implemented by it as of 2000. PISA survey uses the concept of mathematical literacy while assessing to what extent individuals can use their mathematical knowledge and skills in the daily life. OECD defines mathematical literacy as follows:

"*Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety* *of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens (OECD, 2013).*

As defined in PISA, mathematical literacy highlights students' levels of using mathematics and associates their levels of using mathematics with the quality of education they receive (OECD, 2013). Three main mathematical processes are defined in the assessment dimension of mathematical literacy: *formulating situations mathematically; employing mathematical concepts, facts, procedures, and reasoning; and interpreting, applying and evaluating mathematical outcomes* (OECD, 2013). Accordingly, the basic components of the concept of mathematical literacy are accepted to be *formulizing, using,* and *interpreting*. Hence, it can be said that the processes of using mathematics are among the key components of the concept of mathematical literacy.

Basic mathematical skills that are used in PISA survey within the scope of mathematical literacy are *communication*, *mathematising*, *representation*, *reasoning and argument*, *devising strategies for solving problems*, *using symbolic, formal, and technical language and operations*, and *using mathematical tools*. The skills mentioned here overlap to a large extent. In other words, several skills need to be displayed at the same time during the use of mathematics. Thus, it is stated in PISA survey that taking and evaluating these skills separately will not yield accurate results; some structures must be used to efficiently explain and assess students' mathematical literacy capacities; and one way of this is to form skill clusters based on the kinds of cognitive demands required to solve different kinds of problems. These skill clusters are reproduction (standard definitions and representations, routine computations, routine procedures, routine problem-solving), connection (modelling, standard problem-solving, translation and interpretation, multiple well-defined methods), and reflection (complex problem-solving and posing, deflection and insight, original mathematical approach, multiple complex methods, generalization). Six competence levels (Table 2) were created for these skill clusters, and evaluations were made based on these levels.

The Table 2 implies that different levels of mathematical literacy involve different modelling skills. While the first two levels include no process concerning modelling, students at the third level can make sense of and use mathematical representations based on direct inferences. While students at the fourth level can effectively work with the open models related to complex situations, students at the fifth level can develop models for complex situations themselves and work with these models. Students at the sixth level can model complex problem situations and conceptualize, generalize, and use the knowledge they acquire. In this regard, it can be said that a student who can complete the modelling circle precisely in a complex problem situation is at the highest level of mathematical literacy. Similarly, Niss et al. (2007) and Stacey (2015) state that the process of mathematical literacy is expressed and evaluated based on the modelling circle, but different literacy questions involve different steps of the modelling circle.

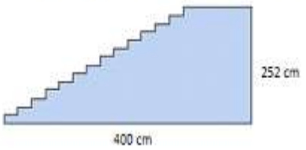
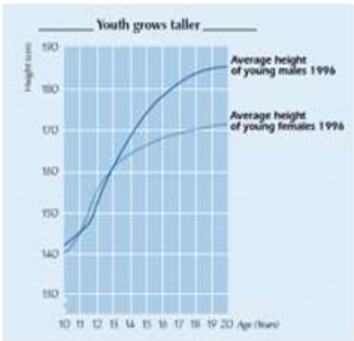
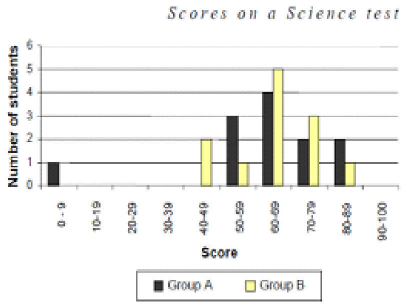
Level	Problem	Description
1	Mei-Ling from Singapore was preparing to go to South Africa for 3 months as an exchange student. She needed to change some Singapore dollars (SGD) into South African rand (ZAR). Mei-Ling found out that the exchange rate between Singapore dollars and South African rand was: 1 SGD = 4.2 ZAR Mei-Ling changed 3000 Singapore dollars into South African rand at this exchange rate. How much money in South African rand did Mei-Ling get?	• Students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. • They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. • They can perform actions that are obvious and follow immediately from the given stimuli.
2	The diagram below illustrates a staircase with 14 steps and a total height of 252 cm:  What is the height of each of the 14 steps?	• Students can interpret and recognize situations in contexts that require no more than direct inference. • They can extract relevant information from a single source and make use of a single representational mode. • Students at this level can employ basic algorithms, formulae, procedures, or conventions. • They are capable of direct reasoning and making literal interpretations of the results.
3	In 1998 the average height of both young males and young females in the Netherlands is represented in this graph.  According to this graph, on average, during which period in their life are females taller than males of the same age?	• Students can execute clearly described procedures, including those that require sequential decisions. • They can select and apply simple problem-solving strategies. • Students at this level can interpret and use representations based on different information sources and reason directly from them. • They can develop short communications reporting their interpretations, results and reasoning.
4	During these 3 months the exchange rate had changed from 4.2 to 4.0 ZAR per SGD. Was it in Mei-Ling's favour that the exchange rate now was 4.0 ZAR instead of 4.2 ZAR, when she changed her South African rand back to Singapore dollars? Give an explanation to support your answer	• Students can work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions. • They can select and integrate different representations, including symbolic ones, linking them directly to aspects of real-world situations. • Students at this level can utilise well-developed skills and reason flexibly, with some insight, in these contexts. • They can construct and communicate explanations and arguments based on their interpretations, arguments and actions.

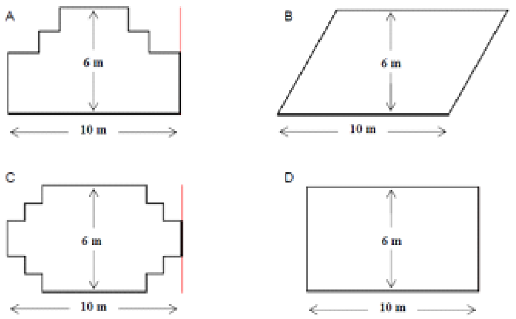
Figure 10. PISA 2012 Mathematical Literacy Competence Levels

Figure 10. cont.

<table>
<tr>
<td>5</td>
<td>The diagram below shows the results on a Science test for two groups, labelled as Group A and Group B. The mean score for Group A is 62.0 and the mean for Group B is 64.5. Students pass this test when their score is 50 or above.


Looking at the diagram, the teacher claims that Group B did better than Group A in this test. The students in Group A don't agree with their teacher. They try to convince the teacher that Group B may not necessarily have done better. Give one mathematical argument, using the graph, that the students in Group A could use.</td>
<td>

Students can develop and work with models for complex situations, identifying constraints and specifying assumptions.
They can select, compare, and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models.
Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriately linked representations, symbolic and formal characterisations, and insight pertaining to these situations.
They can reflect on their actions and formulate and communicate their interpretations and reasoning.

</td>
</tr>
<tr>
<td>6</td>
<td>A carpenter has 32 metres of timber and wants to make a border around a garden bed. He is considering the following designs for the garden bed.


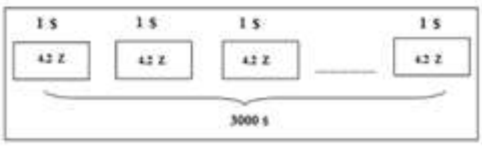
Circle either "Yes" or "No" for each design to indicate whether the garden bed can be made with 32 metres of timber.
<table>
<tr><th>Garden bed design</th><th>Using this design, can the garden bed be made with 32 metres of timber?</th></tr>
<tr><td>Design A</td><td>Yes / No</td></tr>
<tr><td>Design B</td><td>Yes / No</td></tr>
<tr><td>Design C</td><td>Yes / No</td></tr>
<tr><td>Design D</td><td>Yes / No</td></tr>
</table>
</td>
<td>

Students can conceptualise, generalise, and utilise information based on their investigations and modelling of complex problem situations.
They can link different information sources and representations and flexibly translate among them.
Students at this level are capable of advanced mathematical thinking and reasoning.
These students can apply insight and understanding along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for dealing with novel situations.
Students at this level can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments and the appropriateness of these to the original situations.

</td>
</tr>
</table>

Let's compare the first level and sixth level questions in the Table 2 and their solution processes in order to observe the modelling processes in different dimensions of mathematical literacy more clearly. The first question is a clearly expressed question requiring the use of mathematics in the daily life. The data to be used for reaching the solution are given clearly. One can easily reach the solution by using the routine procedures he knows. To solve this problem, he just has to complete the first two stages of the modelling circle. The solution is reached at the end of the second stage.

1. *To understand the problem and to establish a model based on reality*

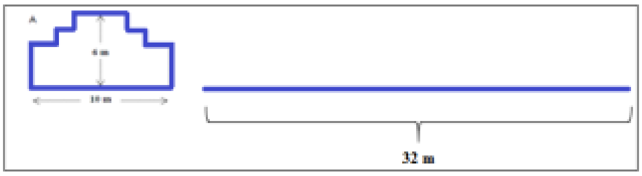


2. *To establish a mathematical model by using the real model*

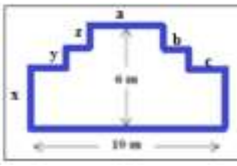
$$\boxed{3000\ \$ = 3000 \times 4.2 = 12600\ Z}$$

On the other hand, the sixth level question involves all stages of the modelling circle. The solution process of the problem corresponding to the sixth level in the Table 2 will be as indicated below.

1. *To understand the problem and to establish a model based on reality*



2. *To establish a mathematical model by using the real model*



$$x + y + z = 6$$
$$a + 2b + 2c = 10$$
$$x + c + y + b + z + a + z + b + y + c + x + 10 = 23$$

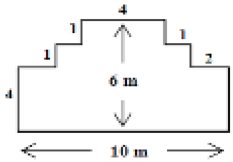
3. *To answer the mathematical question by using the formed mathematical model*

$$2x + 2y + 2z + a + 2b + 2c + 10 = 32$$
$$2x + 2y + 2z + a + 2b + 2c = 22$$
$$2\,\underbrace{(x + y + z)}_{6} + \underbrace{a + 2b + 2c}_{10} = 22$$

4. *To interpret the obtained mathematical results in real world*

As 2.6 + 10 = 22, the lengths x, y, z, a, b, and c can be accepted to design a garden like the one in the figure.

5. *To validate the solution*



The solution of the problem is correct for all the x, y, z, a, b, and c values that yield the equations of x + y + z = 6 and a + 2b + 2c = 10. For example, the correctness of the solution can be proven by selecting the following values: x=4, y=1, z=1 and a=4, b=1, c=2.

The solution processes provided above show that the solution processes of mathematical literacy questions involve different stages of the modelling circle. Based on all this, it can be said that an individual who is at the highest level of mathematical literacy can use mathematics as an effective tool in his life and will also be successful in mathematical modelling processes.

4. Conclusion and Discussion

From past to present, mathematics has always been a tool human beings use for dealing with the situations, problems, and issues emerging in fields other than mathematics (Niss, 2012). In this regard, scientific studies carried out in the field of mathematics education seek ways of teaching the functional aspect of mathematics and come up with a great variety of concepts. The present study has attempted to explain some theoretical concepts concerning the use of mathematics in the daily life (i.e. using mathematics, mathematical applications, mathematical modelling, and mathematical literacy) in connection with one another.

Use of mathematics can be divided into two: the use of mathematics in real world and the use of mathematics in the mathematical world. The use of mathematics in the mathematical world refers to well-known pure mathematical applications. The use of mathematics in real life can be named as “the application of mathematics to real life” or “using mathematics in real life”. Both of them mean the same thing. There are a variety of usage areas of mathematics in real life. The first one is the use of mathematics by professionals in other disciplines or areas of life (e.g. engineering, medicine, architecture, art). For example, at the present time, experts use mathematics, advanced mathematics, and mathematical models for constructing modern buildings, diagnosing and treating illnesses, and producing excellent works of art. Use of mathematics in the daily life, however, involves mathematical applications that do not require expertise. In these kinds of situations, individuals try to understand the situations they encounter in their lives, analyze and interpret them, and reach certain results or make certain decisions about them. To engage in these kinds of applications, one does not need to have advanced knowledge of mathematics. However, such use of mathematics embodies certain levels as well. The situations we encounter in our lives may require different levels of mathematical skills as they may involve a variety of mathematical knowledge and solution processes. While some individuals can effectively use their mathematical knowledge in all complex situations, some are eager to use only a specific kind of mathematics, but are unwilling to produce opinions or think on real life situations that are difficult or unfamiliar to them.

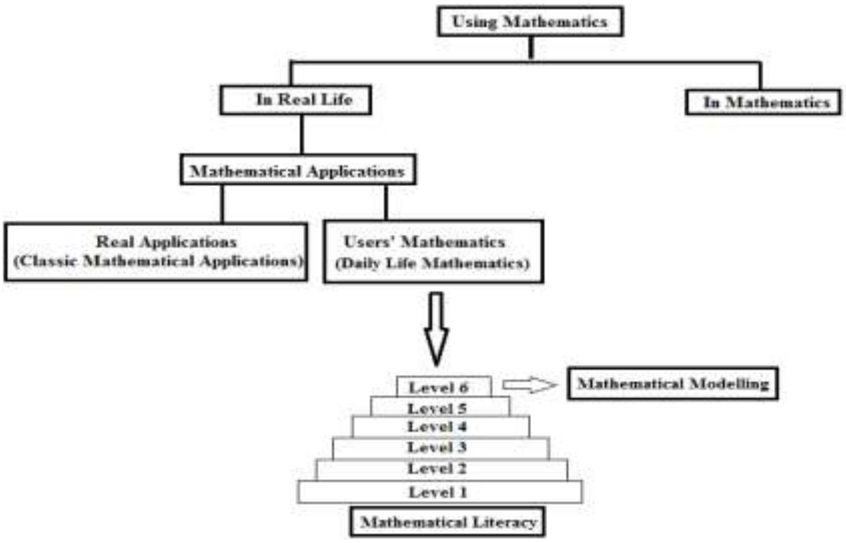


Figure 11. The Theoretical Structure of the Concept of Using Mathematics

It can be said that the individuals that can effectively use the mathematics they learn in any situation they encounter in their lives are at the highest level of mathematical literacy. These individuals have the mathematical modelling skill as well. This is because the uppermost step of the literacy ladder represents modelling. The individuals who cannot use mathematics as an effective tool in every case are those individuals who are not successful in mathematical modelling processes in every case. All in all, the following inference can be made for all these concepts (Figure 7). All the theoretical concepts discussed in this study involve the use of mathematics. An individual's success in these processes depends on the extent to which he can use mathematics effectively. The basic requirement for this is to achieve a complete learning of mathematics and to have mathematical skills that can be improved through application and experience. Hence, the area of use of mathematics, which gains more importance every passing day due to the demands of the developing and changing world, should be highlighted in educational environments more. Complete learning and understanding of mathematics by students requires them to have an idea and knowledge of the daily-life usage areas of the mathematical concepts they learn. According to Skovsmore & Valero (2008), not only the students of a specific segment but all the students must be equipped with mathematical literacy, and every student must have the personal, technological, and thinking skills that are required for applying mathematics to the daily life and use mathematics effectively.

The literature contains quite a limited number of studies about the use of mathematics in the daily life. Therefore, new research is needed in this matter. In this regard, activities and academic studies and projects can be planned and carried out to show the ways of reaching the information sources that teachers can use in their lessons to give their students more detailed information about where mathematics is used. Moreover, there is a need for research that takes and interprets the concept of using mathematics in its different theoretical dimensions.

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