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Mathematical thinking and creativity through mathematical problem posing and solving

Pensamiento matemático y creatividad a través de la invención y resolución de problemas matemáticos

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Summary

This work shows the relationship between the development of mathematical thinking and creativity with mathematical problem posing and solving. Creativity and mathematics are disciplines that do not usually appear together. Both concepts constitute complex processes sharing elements, such as fluency (number of ideas), flexibility (range of ideas), novelty (unique idea) and elaboration (idea development). These factors contribute, among others, to the fact that schoolchildren are competent in mathematics. The problem solving and posing are a very powerful evaluation tool that shows the mathematical reasoning and creative level of a person. Creativity is part of the mathematics education and is a necessary ingredient to perform mathematical assignments. This contribution presents some important research works about problem posing and solving related to the development of mathematical knowledge and creativity. To that end, it is based on various beliefs reflected in the literature with respect to notions of creativity, problem solving and posing.

Keywords: Problem solving, problem posing, mathematics education and creativity.

Resumen

En este trabajo se muestra la relación entre el desarrollo del pensamiento matemático y la creatividad con la invención y resolución de problemas matemáticos. No son frecuentes las ocasiones en las que aparecen juntas Creatividad y Matemáticas. Ambas constituyen procesos complejos que comparten elementos como la fluidez (número de ideas), la flexibilidad (variedad de ideas), la novedad (idea única) y la elaboración (desarrollar una idea). Estos factores contribuyen, entre otras cosas, a que los escolares sean competentes en matemáticas. La invención y la resolución de problemas resultan ser un instrumento evaluador muy potente con el que se pone de manifiesto el nivel de razonamiento matemático y creativo de una persona. La creatividad forma parte de la educación matemática y constituye un ingrediente necesario para realizar las tareas matemáticas. Esta aportación presenta algunas investigaciones relevantes, que se han llevado a cabo sobre invención y resolución de problemas, vinculadas al desarrollo del conocimiento matemático y a la creatividad. Para ello se parte de las distintas creencias que se recogen en la literatura referente a las nociones de creatividad, invención de problemas y resolución de problemas.

Palabras clave: Resolución de problemas, invención de problemas, Educación Matemática y creatividad.

Introduction

The main objective of teaching mathematics is to develop thinking. To that end, it is necessary to carry out problem posing and solving assignments. When an individual has to pose a problem, they have to think, analyze the formulation critically, examine the data of such formulation and to handle such solving strategies that allow obtaining the solution to such problem. All of this is also practiced by solving problems. The solving and posing of problems help to strengthen what is learning. Therefore, Noda (2000) says that both activities are important for the mathematical knowledge building, which is an essential basic cognitive action for the education theory and practice. For that reason, the concept of number and the problem solving and posing are considered the backbone of the school mathematical knowledge (Ayllón, 2012).

Problem solving is a basic component for learning, as well as for knowledge acquisition. García (1998) considers that problem posing is a priority to consolidate and improve knowledge. He also advises that the researchers can reach achievements through their scientific theories when formulating, discovering or facing new problematic fields.

When a person has to solve a problem, a priori, they think it is not a very easy assignment, which is a challenge to develop their creativity and mathematical skills.

This circumstance also occurs in the problem posing. Problem posing allows getting meaningful learning and investigates the mathematical skills of the person, when establishing relationships among the different mathematical concepts, as well as numerical structures. To pose problems, it is necessary to have a high abstraction level and it demands reflection, which allows reaching a reasoning phase that facilitates the mathematical knowledge building. The person that poses a math problem is based on

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their own ideas, following a creative process. Therefore, a problem posing constitutes the individual´s own posing and not a reformulation of a problem already posed.

Since birth, people look for explanations for everything around them, putting their creativity to work. Such creativity must be developed, stimulated and encouraged. For this reason, it is necessary to educate it and include it in schools. Barbarán and Huguet (2013) consider that a current education system does not encourage creativity. They also say that it hinders it in some cases.

Several researchers relate the problem posing to mathematical knowledge and creativity development. Authors like Krutetskii (1969) and Ellertoh (1986) indicate the existence of an implicit relationship among the ability required to pose a problem and the creativity level, as well as the mathematical competence. There is also an evident relationship between creativity and problem solving (Callejo, 2003) since when every person has to solve a problem, the creativity is activated when making that assignment. Ayllón and Gómez (2014) say that problem posing assignments develop creativity in schoolchildren and improve the acquisition of mathematical concepts.

Problem Solving.

Throughout the scientific literature, different studies address the problem posing field and they indicate that it is an important task to acquire mathematical knowledge (Stoyanova & Ellerton, 1996; Nicolaou & Pilippou 2007; DeHaan, 2009; Kesan, Kaya & Guvercin, 2010; Ayllón, 2012; Fernández & Barbarán, 2012; Pintér, 2012; Espinoza & Segovia, 2013; Fernández, 2013; Ayllón & Goméz, 2014). It is true that research works on problem posing are not very common and not older than the research works on problem solving. However, since the last decades, this field has been increasingly addressed by authors.

Researchers refer to the term problem posing in several ways, such as problems (Kilpatrick, 1987), problem posing (Brown & Walter, 1993), formulation and generation of problems (Silver, 1994). These terms allow the formulation of new problems or the reformulation of particular situations (Silver & Cai, 1996). That is, a problem can be posed during its solving (Silver, Mamona-Down, Leung & Kenny, 1996) through changes in it, reformulating and/or focusing on a particular case in order to better understand the problem, and thus, solve it. A problem can be also posed once it has been solved, modifying objectives, conditions and/or questions about it (Silver, 1994). Therefore, Brown and Walter (1993) explain how to pose a new problem from a given problem following the strategy “What if not?”, which consists in modifying the conditions of a given problem.

Shukkwan (1993) proposes a problem formulation from a given solution, and Mamona-Downs (1993), views problem posing as the activity which is precipitated when the problem invites the generation of other problems.

Espinoza, Lupiañez and Segovia (2013) consider several ways to formulate math problems: free situation, semi-structured situations and structured situations (Stoyanova, 1998). Free situation does not require any condition to pose a problem; semi-structured situation requires a specific starting situation, and the structured situation refers to situation in which a condition of a given problem is modified.

Problem posing requires to make a personal, own and creative contribution, as well as to use mathematical knowledge already acquired and to relate several concepts. The problem posing is based on specific situations and personal interpretations (Koichu & Kontorovich, 2012).

In 2005, Christou, Mousoulides, Pittalis and Pitta-Pantazi carried out a study that showed that the problem posing stimulates mathematical

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learning. It shows that when a person poses a mathematical problem, they are obliged to establish relationships among several mathematical concepts, which involves several cognitive processes, like edit, select, understand, organize and translate the information from a representation form to another.

Barbarán and Huguet (2013) carried out a study with first-grade secondary students, in which it was evidenced that mathematical problem posing and reconstruction assignments encourage the creativity development. They say that they actively stimulate the creativity of students since they oblige them to project their ideas generating original problems. They also say that the problem posing allows the teachers to use a methodology in which the main character of their own learning is the student, thus encouraging the development of their creativity.

Ayllón and Gómez (2014) consider the benefits of the problem posing, making a collection of the contributions of several researchers and categorizing such aspects in 6 positive factors:

1. Increase of mathematical and linguistic knowledge. Posing a problem requires a clear writing of the formulation, in an organized and accurate way. It requires data analysis and critical reasoning, discussion and questioning of ideas and solutions. Therefore, the creator must write in a clear, accurate and organized way (Davidson & Pearce, 1988; Burçin, 2005; Whitin, 2006).
2. Motivation increase. Problem posing stimulates the curiosity and motivation of students (Silver, 1994; Akay & Boz, 2010).
3. The decrease of anxiety that some students suffer in some opportunities causes their relationship with mathematics. Authors like Burçin (2005) and Song, Yim, Shin and Lee (2007) say that, when a student makes problem posing assignments, their fear of mathematics decreases.

4. Overcoming mathematical mistakes often made by students. Brown and Walter (1993) say that the problem posing reduces resolving errors and mistakes made by students since they choose the information and the data in an appropriate way to solve the problem.
5. Creativity increase. Authors like Krutetskii (1969), Ellertoh (1986) and Silver (1994) evidence the relationship among the skill to pose problems and the creativity level and mathematical competence.
6. As an evaluation tool. Problem posing is an evaluation tool for teachers. Teachers can better know the concepts learnt and their students by means of this tool (Lin, 2004; Ayllón, 2005; Sheikhzade, 2008). They can also value their type of mathematical reasoning.

Problem Solving.

Scientific literature is the problem posing with different meanings. Researchers identify the problem solving with different considerations. Koestler (1964) relates the problem solving to creativity. He says that when a person solves a problem, it is supposed to be a contribution to the creativity spark. Agre (1982) considers that the problem solving corresponds to a process in which knowledge learnt through a new but not familiar situation is used. Puig (1993) says that the problem solving is a mental activity produced from the moment the problem is proposed to the solver, who is conscious that he has a problem and feels the need to solve it and finishes the activity once it is solved. Contreras (1998) states that problem solving corresponds to a situation where a person who is facing a problem, they are capable to understand it, since they have knowledge previously learnt, but they do not know how to solve it in that moment, although they try to find a solution by facing such situation. Contreras also states that the problem solving corresponds to a task merely perceptive and conceptual.

García (1998) considers 5 lines of research in the problem solving field: problem solving as a strategy to generate conceptual, methodological

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and attitudinal changes; cognitive organization of knowledge and the problem-solving capability; comparison between expert individuals and beginners; design of heuristics for problem solving, and the creativity as a problem solving. The last line indicates that the problem solving involves processes where it is necessary to modify a situation by generating creative ideas. For that reason, it considers that creativity is a way to solve problems and vice versa, that is, problem solving is an efficient way to develop creativity.

Schroeder and Lester (1989) differentiate the problem solving work in the classroom in three categories to teach *about* problem solving, *to* solve problems or *via* problem solving. They consider that teaching problem solving is about working solving strategies, that is, the objective is that the student know how to solve problems. Teaching to solve problems means how to use the mathematical knowledge already acquired in the problem solving. The last category, teaching via problem solving, consists in building on problems to learn new mathematical problems. These threes objectives are different: the aim of the first one is that the students know how to solve problems; the aim of the second one is that the students strengthen their knowledge, and the aim of the third one is to use a problem solving methodology.

One of the ideal frameworks for the meaningful learning construction is problem solving, since it contributes to increasing the liking for mathematics and encourages the development of a critical and open attitude (Carrillo, 1996). As mentioned above, a person that faces a problem is learning and using mathematical concepts already learnt.

Authors, such as Polya (1965) states that problem solving is a barometer of mathematical knowledge. Along with this author, Gagné (1965) and Brown (1978) state that problem solving does not only use knowledge learnt, but generates new knowledge.

However, to know how to solve problems, people must previously learn it. Brown (1978) states that problem solving, as well as the simple memorization, algorithmic learning and conceptual learning are a way of learning. He states that the problem solving is a tool for construction and learning indicated.

Creativity.

The development of a country depends on its scientific development. To that end, it is necessary creative people. Ramos (2006, p. 9) states that "educating to be creative is an essential requirement at the beginning of the 21^{st} Century".

The Royal Spanish Academy (22^{nd} Ed.) defines creativity as "*the right to create, creation capacity*". Throughout the literature there is no agreement on creativity. Stein (1956) refers to creativity as a process which results in a useful novel work; De Bono (1974) states that creativity is a mental attitude and a thinking instrument. Therefore, it is a way to use the mind and information. De la Torre (1984) opines that creativity is an ability, while Goleman, Kaufman and Ray (1992) say that creativity is an attitude. However, Sorin (1992) says that every person can be creative, provided that they are unique in their field and produce innovations.

Other definitions relate creativity to a thinking style. There are those based creative processes, while others are based on creative products. Nadjafikhah and Yaftian (2013) state that creative thinking is a dynamic mental process that includes divergent and convergent thinking. The divergent thinking comprises 4 necessary elements to consider creative a production: fluency (number of ideas), flexibility (variety of ideas), novelty (unique idea) and elaboration (develop an idea).

Therefore, creativity can be understand and defined in different ways, although it can be summed up as a personal or group activity aimed

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at producing something new. Nevertheless, the definition of the National Advisory Committee on Creative and Cultural Education (NACCCE, 1999, p. 30) is more recognized and it states that creativity is “any imaginative creativity aimed at producing original and highly valuable results”. Bolden, Harries and Newton (2010) state that this reference of “originality” and “value” has an implicit social aspect, since ultimately society validates the product.

García (1998) explains that creativity is characterized by the following elements or capacities:

a. Sensitivity to problems: This capacity allows the person to complicate situations and find solutions.
b. Flexibility: This capacity provides the possibility to change the approaches of a problem, making people seek different strategies to solve it.
c. Thinking fluency: It allows generating ideas at any given time.
d. Originality: Based on knowledge learnt, new knowledge are generated.
e. Capacity to perceive non-obvious connections among facts; not previously established relationships among different experiences are discovered.
f. Representation capacity: new models are established and different relationships among their elements are discovered.

Creativity and Mathematics Education.

As a tradition, creativity was only attributed to the world of art and literature. Nowadays, it is related to the scientific world. To be creative, a scientific idea must be new and useful. Mathematical thinking encourages the development of creativity since it requires to make conjectures and distinguish opinions to solve a situation set out.

In mathematics education, creativity is based on knowledge. It consists in building something new, previously getting rid of the ways of thinking established, considering new possibilities and applying a wide range of mathematical knowledge (Bolden, Harries & Newton, 2010). The justification to carry out it is the cognitive flexibility, one of the three main mental functions involved in the creative problem solving (Ausubel, 1963, 2000). On the other hand, DeHaan (2009, 2011) says that it is possible to apply ideas to new contexts, to which he refers as "transfer" of knowledge capacity. This allows the active development of the representations of students and the conversion of the information received into something more useful, practical and constructive.

Two ways of research referred to creativity and mathematics education are considered. They use the word creativity as a subject: *creativity in mathematics education*, or as an adjective: *recreational mathematics education* (Sequera, 2007). Research works on creativity in mathematics education consider creativity as a methodological element that helps to acquire the mathematical learning and makes sure that when applying the problem solving, not only reasoning skills are developed, but also creative ones. In this discussion, creativity prevails, from which mathematics is learnt. The studies referring to the creative mathematics education state that based on mathematics learning, creativity arises. Therefore, during a problem solving process, a creative process arises.

Problem Creativity and Posing.

There are different methods to find out when a person is creative in research works. One of them is used by Getzels and Jackson (1962), in which problem posing tasks are carried out to measure creativity. The same technique was used by Balka in 1974. He studied the productions of students based on the originality of formulations posed.

Silver (1997) says that when teaching mathematics based on problem posing tasks, the teacher helps the students to develop their creativity increasing their capacity of fluency, flexibility and novelty. Contributions from other authors are considered like Van den Brink (1987), Streefland (1987), Healy (1993) and Skinner (1991), who state that problem posing encourages the development of fluency, being this the main characteristic of creativity.

Singer, Perczer and Voica (2011) analyzed the creativity of 11 to 13 aged students who pose problems. Their research work evidence that those students who pose coherent and original problems through changes made in their formulations have creative skills.

Van Harpen and Sriraman (2013) carried out a study in China and The United States of America with secondary students to analyze their creativity. To that end, they performed activities based on problem posing with geometrical concepts. The results showed that when a student poses a mathematical problem, they develop fluency skills and it also showed that students formulate problems with high quality elaboration and originality. These authors propose to include problem posing in the school curriculum, in the mathematics course due to its benefits for the students.

Dickman (2014) carried out a study in which he investigated the existing connection between the problem posing and creativity. To that end, he asked a group of primary teachers, psychologists who work in mathematics education and mathematicians to evaluate the problems posed to know how creative they are. This study showed that there is no an agreement between them since they do not share the same definition of creativity, although a relationship between creativity and problem posing is admitted.

Creativity and Problem Solving as Processes.

Muñoz, in 1994, referred to the research works performed until that time about creativity, in aspects like the person, the process, the product and the environment.

Creativity as a process refers to the necessary phases whereby individuals go through to achieve a creative product. In 1908, Poincaré defined these phases and called them preparation, incubation and illumination. The first phase corresponds to the initial time of conscious and intense work; the second phase refers to a break time, and the third one corresponds to the time in which ideas arises from the subconscious. In 1992, Goleman, Kaufman and Ray considered these stages and added one more called "translation". It is about the time in which the idea stops being a thought and it turns into reality. Later, in 1998, García recalled and completed these stages in: a) encounter with the problem, stage in which the subject uses their critical thinking, they feel the need to pose, solve the problem or even, to reveal ideas that they are worried about; b) generation of ideas, stage in which the person seeks possible solutions based on inspiration and, thus, generates a new idea; c) idea development, when the creation or project is materialized, and d) creative transfer, which is the last stage of the posing process and where the new idea is related to other ones already known.

These creative process stages correspond to the ones specified by Hadamard (1945) and Polya (1965) with respect to the problem solving process. Hadamard defines them in four stages: a) conscious work of familiarization with the problem, b) semiconscious or unconscious work of ideas incubation, c) inspiration and illumination on the way to solve problems, and d) verification that the inspiration really leads to the solution. On the other hand, Polya defines them in: a) problem understanding, b) plan conception, stage in which a solving strategy is prepared, c) plan execution, and d) verification.

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The necessary stages to develop a creative product coincide with the stages required to solve a problem. There is a relationship between preparation or encounter with the problem (creativity) and problem understanding and familiarization with it, since the two stages refer to the preliminary moment to be faced. The second and third stages, incubation and illumination or generation and elaboration of ideas, are found in the creative process and in the problem solving, even in the identical names. The last stage, translation or creative transfer (creativity) and verification (problems), corresponds to the time in which the objective set has been achieved.

Conclusions

Nowadays, the priority mission of teachers is to identify and develop creativity, and they need adequate environments for creative learning. Such learning requires to be flexible, to associate materials and ideas, have indirect, fostering and imaginative methodologies and to favor the relationship between student and teacher (De la Torre, 1995). One of the greatest concerns of teachers is to achieve that the teaching-learning of mathematics develops logical thinking in students by means of constructivist and ludic methodologies, to make mathematics a tool to be applied to daily life. This learning, thanks to the mathematical knowledge acquired, is connected to the student´s creativity and its development.

In view of the foregoing, it is evident that there is a relationship between creativity and mathematics education. Such relationship works both ways. Mathematics is an instrument that allows solving problematic situations arising in our daily life. This tool is based on the creation of new concepts, solving strategies and/or different ways of thinking. Creativity is an instrument of a mathematical activity. Vianney and Navarro (2011) state that creativity requires to build relationships among several concepts in order to create something new. To develop a mathematical thinking in an appropriate manner, the problem posing and solving become important tasks, being the backbone of it, in which people have to relate ideas, associate concepts, use the memory and use the critical thinking. Therefore, it starts from the innovation and creativity in problem posing and solving tasks.

Creativity is part of the mathematical activity (Sequera, 2007), so it is implicit in the problem posing and solving activities, which identify the mathematical abilities of students, turning them into a powerful evaluation instrument for teachers.

Whitin (2006) and Sequera (2007) encourages that problem posing tasks are included in the classroom in order to provide the students with a creative mathematics education, that is, a live and rich mathematical learning in which the problems posed and their solutions are questioned, problem solving strategies encouraging imagination are forested and a flexible thinking is developed. Ayllón (2005) states that when an individual poses a problem, they internalize it, creating real and close contexts to them, increasing their desires to learn, daring to make mistakes and learning from them, and reducing fear felt at some point by some students when learning mathematics. The student must use their creativity to pose a mathematical problem.

Currently, teaching mathematics, in several opportunities, stops the development of creativity when establishing a very strict action guides in problem solving. It is necessary to foster creativity of students to improve mathematical teaching. Students must learn how to think, do or undone freely to let their imagination loose and generate new things, in the same way as when they build a meaning for symbols, signs and operations. Pichel (2001) proposes tasks that help children to think about the number, play cryptography, make up operations, strategies and analyze games, etc. This avoids repeating the same processes consolidated that hinter the development of creativity.

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Solving mathematical problems and identifying their meaning favor the search and development of reasoning that will lead to creativity (Haylock, 1987). This leads to an improved behavior in social skills, motivation, self-esteem and performance in all aspects. For that reason, it is important to develop mathematical strategies and instruments favoring the creative learning in our classrooms.

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