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SOME COMPONENTS OF GEOMETRIC KNOWLEDGE OF FUTURE ELEMENTARY SCHOOL TEACHERS

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Abstract: Geometric experience, spatial representation, spatial visualization, understanding the world around us, and developing the ability of spatial reasoning are fundamental aims in the teaching of mathematics. (Freudenthal, 1972) Learning is a process which involves advancing from level to level. In primary school the focus is on the first two levels of the Van Hiele model (level 1. recognizing figures; level 2. analysing figures); on laying the foundation of basic terms in geometry.

The aim of the research is to investigate the knowledge of teacher training students in the area of association and relationship building between geometric content and real life objects (e.g. buildings, sculptures, fountains); to provide means for revealing and studying some components of the students' geometric knowledge and point out any lacuna, as well as facilitate the completion of existing lacunas.

The participants involved in this research (N=115) had to identify two-dimensional and three-dimensional figures on the basis of 12 photos of real three-dimensional objects (selection criteria: a multitude of shapes and figures). We have focused on the accuracy, frequency and variation of terminology in students' answers, as well as on analysing mistakes. (Pjanic & Nesimovic, 2015)

Key words: *mathematics education, geometric shape, plane figure, solid, spatial visualization*.

1. Introduction

'Spatial sense can be described as the ability to understand the outside world'. (Freudenthal, 1972)

One of the tasks of teaching mathematics is developing students' spatial ability or spatial reasoning. The association of a geometrical content with real situations and objects is recommended for the realization of this task.

The curriculum should include developing spatial visualization, spatial reasoning, and the ability to recognize figures in different settings. Pupils should be able to accurately describe figures, shapes, and the properties of these, using appropriate geometric terms. (Szilágyiné, 2013).

According to the Van Hiele theory the development of geometric thinking and knowledge gaining is a process which involves several stages. A number of studies were carried out during the 80s to prove this theory. (Mayberry, 1983; Fuys – Geddes – Tischler, 1988; Burger – Shaughnessy, 1986; Usiskin, 1982)

Thus, competencies related to spatial thinking, spatial visualization abilities, the ability to recognize geometric shapes met in different environments, and the ability to describe these shapes using geometric terminology should be found among the competencies of the teachers. (Herendiné, 2003)

The aim of this paper was to analyze the responses of 115 students of Primary School and Kindergarten Teacher Training College students (at Partium Christian University, Oradea,

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Nyíregyháza University and Babes- Bolyai University, Satu-Mare) in an activity of identifying geometric shapes using 12 photos of real objects. We have focused on the accuracy, frequency and variation of terminology in students' answers, as well as on analysing mistakes. (Pjanic - Nesimovic, 2015)

2. Kindergarten and elementary school experience gaining in geometry

The Mathematics curriculum for primary education states that students should be able to distinguish geometrical solids from geometrical plane figures, to recognize and distinguish geometrical solids: sphere, cube, cuboid, cylinder, pyramid and cone, to recognize and distinguish geometrical plane figures: circle, square, rectangle and triangle, as well as to describe features of aforementioned geometric shapes. Students should be able to recognize and distinguish shapes in their environment.

Identifying geometric shapes among objects in the environment is the students' first step in the process of describing and organizing their own environment.

The development of geometric thinking and knowledge gaining is a process which involves several stages. According to the Van Hiele theory these are the following:

Level 1. recognising figures

Level 2. analysing figures

Level 3. understanding logical implications (informal deduction)

Level 4. constructing proofs (formal deduction)

Level 5. axiomatic construction

Level 1. and 2. are completed in elementary school, level 3. in middle school, while level 4. and 5. in high school and higher education.

At level 1. children perceive geometric figures as a whole, they cannot distinguish their components. They recognise figures based on their shape, memorise their names and use them confidently. They cannot compare figures, the square is not a rectangle for them, they cannot recognise the cuboid in a cube, etc.

At level 2. pupils distinguish and work with the components of figures. They recognise the relationship between the figure and its components. They recognise the faces, edges and vertices of solid figures. They can analyse figures and enumerate their properties. However, they cannot yet recognise the logical connections between properties. They also do not have a need for definitions. (Szilágyiné, 2013).

The curriculum should include developing spatial visualization, spatial reasoning, and the ability to recognize figures in different settings. Pupils should be able to accurately describe figures, shapes, and the properties of these, using appropriate geometric terms.

At elementary level the foundation for basic terms in geometry is laid. The development of geometric terms is a lengthy process, which is not completed within the first four years of study. The goal is to shape these terms, thus the activities that serve as a starting point should be the ones specific to this particular age group. The experiences gained from such activities will eventually lead to the recognition and formulation of more general relationships.

Children can be given concrete tasks in order to analyse figures, such as:

- observations, grouping, measuring, drawing, folding, cutting, sticking

- modelling, parquetry, mirroring

During these tasks the pupils will observe the following:

- the number of faces, edges, sides and vertices
- the shape of faces
- the length of edges and sides
- the parallelism and perpendicularity of faces and sides
- symmetries

Detailed observations, noticing and formulating properties through guided attention is more than enough. Pupils do not need definitions at this stage. They are not yet aware of the hierarchical relationships between figures.

The teachers in primary education are those who systematically have to foster spatial thinking in children, there is a need to recognize this ability within primary education as well.

In order to impart the knowledge and observations discussed above and guide the activities related to these kindergarten and elementary school teachers need the following skills and abilities:

- spatial visualisation skills
- spatial reasoning
- recognizing geometric figures and shapes in different environments
- the ability to describe these figures and shapes and their properties proficiently and accurately, with the use of appropriate terminology

Future elementary school teachers also need to have at least these skills in order to be able to implement the Van Hiele level 1. and 2.

A good teacher should be able to recognize geometric shapes in various everyday settings or in illustrations and to properly and accurately describe those shapes using geometric terminology. (Pjanic - Nesimovic, 2015)

3. Research

The aim of the research is to investigate the ability of teacher training students to associate and relate geometric figures to real objects (e.g. buildings, sculptures, fountains); to create an opportunity for investigating and revealing students' knowledge in geometry; and to highlight shortcomings and facilitate their rectification. We hypothesise that students are not familiar with accurate geometric terminology, they do not know the names of figures and shapes, they cannot recognize them. (They are expected to cope with the Van Hiele level 1. and 2. at least)

4. Participants

Research participants are 41 (17 second year and 24 third year) *Teacher Training* majors at Partium Christian University (hereafter referred to as group 1.), 37 *Kindergarten- and School Pedagogy* majors at Satu-Mare Department of Babes-Bolyai University (hereafter referred to as group 2.), 20 *Teacher Training* majors at Nyíregyháza College (hereafter referred to as group 3.), as well as 17 *Bank and Finances* majors at Partium Christian University (hereafter referred to as group 4.). This latter group was included in the research based on the assumption that they have more experience in mathematics, and because we wanted to draw a contrast between students majoring in teacher training and students majoring in other fields. A total of N=41+37+20+17=115 students.

5. Methodology

Research participants, 98 teacher training students, as well as, 17 students in group 4. were shown photos of real three-dimensional objects and were asked to identify different two-dimensional and three-dimensional figures (see Figure 1). Associating and connecting geometric contents to real objects (e.g. buildings, sculptures, fountains, etc.) provides means for investigating and revealing students' knowledge in geometry, as well as, highlighting shortcomings and facilitating the rectification of these. Research participants represent different institutions, which provides an opportunity to compare and contrast their knowledge in geometry.

The selection criteria for the 12 photos in the test were the following:

- photos should be characterized by a multitude of two-, and three-dimensional figures
- the presence of ”unusual” geometric figures in the photos

We focused on the frequency, variation and accurate use of terminology in students' response and analysed mistakes.

6. The test

1. (A) Sebilj fountain, Sarajevo

2. (B) Lodging

3. (C) Atomium, Brussel

4. (D) Street light, Osijek

5. (E) Lodging

6. (F) Casket, Museum, Berlin

7. (G) Tokyo Big Sight, Tokyo

8. (H) Fuji TV building, Tokyo

9. (I) Lodging

10. (J) Well

11. (K) Building, Tokyo

12. (L) Fountain

Figure 1. *Photos of real objects*

7. Research results

Students are not familiar with the correct geometric terms for figures and shapes; they cannot identify these. They are more able to identify two-dimensional shapes (60%) than three-dimensional ones (40%). The figures students find the easiest to identify are the *circle*, the *square*, the *rectangle*, and the *triangle*, as well as the *sphere*, the *cube* and the *cuboid*. Identifying truncated shapes posed the most problems for students. They found it difficult to identify shapes and figures in unusual positions.

Table 1. *The rate of correct recognition of two-dimensional figures (percentages)*

Figure	*group 1. (N=41)*	*group 2. (N=37)*	*group 3. (N=20)*	*group 4. (N=17)*	*Averages*
Rectangle	59.81	64.09	56.81	76.36	64.27
Square	36.54	61.18	38.63	54.45	47.7
Circle	26.5	36.83	38.33	50	37.92
Semicircle	27	59	40	35	40.25
Trapezoid	56.75	53.5	55	69	58.56
Triangle	42.75	62	40	56	50.19
Hexagon	63	66.5	80	73.5	70.75
Octagon	22.66	10	50	47.33	32.5
Polygon	21.25	15	6.25	12	13.63
Average	39.58	47.56	45.00	52.62	46.2
Dispersion	15.71	20.48	18.63	19.08	18.48

The two-dimensional figures students find the easiest to identify are the *hexagon*, the *rectangle*, the *trapezoid*, the *triangle*, the *square*, the *semicircle* and the *circle*. The octagon and the polygon were the least often recognized. Group 4. achieved better results, except in the case of the semicircle. None of the students used the term 'regular octagon' or 'regular hexagon' (they only used 'hexagon'). (see Table 1).

We wished to compare and contrast the achievement of groups in terms of each figure, in order to investigate the differences between the results. Since there were 9 samples to compare (there were 9 different two-dimensional figures in the photos), which is more than two, we applied the single-factor analysis of variance (ANOVA test) to the values in each row in Table 1. Hypothesis H0 was that the expected value of the random variable will be the same for each sample. The results of the ANOVA test are shown in Table 2:

Table 2. *The single-factor analysis of variance for the correct recognition of two-dimensional figures (ANOVA test for each figure)*

ANOVA

Source of Variation	*SS*	*Df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	9812.205	8	1226.52565	9.77221367	2.905636	2.30531317
Within Groups	3388.8117	27	125.511544			
Total	13201.017	35				

Since the calculated value of sample F (F=9.77) is higher than the critical value of F (F=2.30), hypothesis H0 is dropped and hypothesis H1 is retained: the averages of the random variables show a significant difference when samples are compared. Thus, as regards recognising two-dimensional figures F reveals significant differences: certain figures (*hexagon, rectangular, trapezoid, triangle, square*) are far more easily recognised than other figures (e.g. *circle, semicircle, octagon* and *polygon*).

The results of the different groups were also compared and contrasted to investigate whether there are any differences between the expected values. Since there were 4 samples to compare (there were 4 groups), which is more than two, we applied the single-factor analysis of variance (ANOVA test) to the values in each column in Table 1. Hypothesis H0 was that the expected value for the random variable would be the same for each group. The results of the ANOVA test are shown in Table 3:

Table 3. *The single-factor analysis of variance for the correct recognition of two-dimensional figures (ANOVA test for each group)*

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	795.329722	3	265.1094	0.683842	0.568482129	2.901114
Within Groups	12405.6868	32	387.6771			
Total	13201.0167	35				

Since the calculated value of sample F (F=0.68) is lower than the critical value of F (F=2.90), hypothesis H0 is retained: sample F does not reveal significant differences between the averages of the analysed random variables. Thus, with regard to recognising two-dimensional figures F does not reveal significant differences between the research groups. The achievement of the 98 teacher training students and the 17 *Bank and Finances* majors is almost identical.

Table 4. *The rate of recognising three-dimensional figures (percentages)*

Figure	*group 1. (N=41)*	*group 2. (N=37)*	*group 3. (N=20)*	*group 4. (N=17)*	*Averages*
Sphere	83	64	66.25	69	70.56
Hemisphere	16	25.85	27.5	56	31.34
Cone	50	65	60	67.5	60.63
Cylinder	53.2	58	69	68.2	62.1
Truncated pyramid	29.33	0	8.33	20	14.42
Cube	39.6	41	27	49.6	39.3
Cuboid	55.2	55.2	56	56.6	55.75
Prism	14.08	1.35	14	10.5	9.98
Pyramid	0	14	0	0	3.5
Polyhedron (figure with polygonal faces)	0	0	0	0	0
Average	34.04	32.44	32.80	39.74	34.76
Dispersion	25.61	25.99	26.21	27.36	26.3

The most often recognised three-dimensional figures were the *sphere*, the *cone*, the *cylinder*, the *cuboid* and the *cube*. The least often recognised were the polyhedron, the pyramid, the prism, the truncated pyramid, and the hemisphere. The control group achieved better results, except in the case of the truncated pyramid (see Tabel 4).

None of the students used the term ‘regular hexagonal prism’, ‘regular octagonal prism’ or ‘polyhedron’ (figure with polygonal faces).

We set out to compare and contrast the results of different groups in terms of each figure to investigate whether there are differences between the expected values. Since there were 10 samples to compare (there were 10 different three-dimensional figures in the photos), which is more than two, we applied the single-factor analysis of variance (ANOVA test) to the values in each row in Table 4. Hypothesis H0 was that the expected value of the random variable will be the same for each sample. The results of the ANOVA test are shown in Table 5:

Table 5. *The single-factor analysis of variance for the correct recognition of three-dimensional figures (ANOVA test for each figure)*

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	25537.97	9	2837.541	34.2895555	2.189623	2.210696983
Within Groups	2482.575	30	82.75257			
Total	28020.56	39				

Since the calculated value of sample F (F=34.28) is higher than the critical value of F (F=2.21), hypothesis H0 is dropped and hypothesis H1 is retained: the averages of the random variables show a significant difference when samples are compared. Thus, as regards recognising three-dimensional figures F reveals significant differences: certain figures (the *sphere,* the *cube,* the *cylinder,* the *cuboid,* and the *cube*) are far more easily recognised than other figures (e.g. the *polyhedron*, the *pyramid,* the *prism,* the *truncated pyramid* and the *hemisphere*).

The results of different groups were also compared and contrasted to investigate whether there are any differences between the expected values. Since there were 4 samples to compare (there were 4 different groups), which is more than two, we applied the single-factor analysis of variance (ANOVA test) to the values in each column in Table 4. Hypothesis H0 was that the expected value for the random variable would be the same for each group. The results of the ANOVA test are shown in Table 6:

Table 6. *The single-factor analysis of variance for the correct recognition of three-dimensional figures (ANOVA test for each group)*

ANOVA						
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	345.10075	3	115.0332	0.1496343	0.929249958	2.8662651
Within Groups	27675.425	36	768.7617			
Total	28020.5246	39				

Since the calculated value of sample F (F=0.14) is lower than the critical value of F (F=2.86), hypothesis H0 is retained: sample F does not reveal significant differences between the averages of the analysed random variables. Thus, with regard to recognising three-dimensional figures F does not reveal significant differences between the research groups. The achievement of the 98 teacher training students and the 17 *Bank and Finances* majors is almost identical.

8. Results

The two-dimensional figures students find the easiest to identify are the *hexagon*, the *rectangle*, the *trapezoid*, the *triangle*, the *square*, the *semicircle* and the *circle*. The octagon and the polygon were the least often recognized. Group 4. achieved better results, except in the case of the semicircle. None of the students used the term' regular octagon' or 'regular hexagon' (they only used 'hexagon').

As regards recognising two-dimensional figures F reveals significant differences; certain figures (*hexagon, rectangular, trapezoid, triangle, square*) are far more easily recognised than other figures (e.g. *circle, semicircle, octagon* and *polygon*).

With respect to recognising two-dimensional figures F does not reveal significant differences between the achievement of the research groups. The achievement of the 98 teacher training students and the 17 *Bank and Finances* majors is almost identical.

The most often recognised three-dimensional figures were the *sphere*, the *cone*, the *cylinder*, the *cuboid* and the *cube*. The least often recognised were the polyhedron, the pyramid, the prism, the truncated pyramid, and the hemisphere. Group 4. achieved better results, except in the case of the truncated pyramid.

None of the students used the term 'regular hexagonal prism', 'regular octagonal prism' or 'polyhedron' (figure with polygonal faces).

As regards recognising three-dimensional figures F reveals significant differences: certain figures (the *sphere,* the *cube,* the *cylinder,* the *cuboid,* and the *cube*) are far more easily recognised than other figures (e.g. the *polyhedron,* the *pyramid*, the *prism,* the *truncated pyramid* and the *hemisphere*).

With respect to recognising three-dimensional figures F does not reveal significant differences between the research groups. The achievement of the 98 teacher training students and the 17 *Bank and Finances* majors is almost identical.

9. Conclusion

The hypothesis that students will not remember geometric terminology and the names of figures after a 4-5 year break in geometric studies proved to be accurate. Students recognized more two-dimensional figures than three-dimensional ones. The two-dimensional figures students found the easiest to identify are the *circle*, the *square*, the *rectangle*, and the *triangle*, while the most easily recognized three-dimensional figures were the *sphere*, the *cube* and the *cuboid.*

Students encountered difficulties in naming truncated figures, they tried to build different analogies when it came to naming these figures.

Students found it difficult to name and identify figures in unusual positions (the cube standing on its vertex in photo C, as well as, the truncated pyramid facing downwards in photo D. and G.). The insufficiencies related to analysing figures need to be rectified within mathematics courses by introducing a large number of concrete tasks. Future elementary school teachers also need to accomplish at least these levels in order to be able to implement the Van Hiele level 1. and 2.

The lacuna in students' knowledge of identifying shapes and figures needs to be filled in within the course that focuses on methods for teaching geometry, with the help of as many concrete tasks as possible.

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