**Introduction**

Every year the School of Education at the University of Waikato attempts to meet the needs and aspirations of Māori parents and schools by recruiting and accepting applications from people who wish to become teachers for mainstream, bilingual and immersion classrooms. Those who wish to focus especially on bilingual and/or, Māori immersion teaching as well as mainstream, are grouped together. Over the years, the great majority of this group has been Māori. A significant number of them have arrived as mathematically anxious people with real concerns about their own mathematics content knowledge (Hawera, 2004; Herewini, 1998). Past mathematical experiences have involved very little mathematics learning using an investigative approach (Hawera, 2006). These people are aware that in three years time they will be expected to teach mathematics competently in any New Zealand primary school and are therefore keen to learn more mathematics as well as how to teach mathematics.

This study reports on some of the responses of Māori preservice primary teachers to using an investigative approach for their own mathematics learning as an integral part of their mathematics education programme at university.

**Why Use Mathematics Investigations?**

According to MINZC (Ministry of Education, 1992), engaging in a mathematical investigation places a greater emphasis on the process of problem solving. The open-endedness of a problem or investigation is an important feature. As well as dealing with the mathematics content, learners often have to pose questions and decide which avenue to follow to move towards a solution or solutions. This active involvement and mathematical thinking is usually over an extended period of time, thereby offering learners an opportunity to learn to persist and sustain their engagement in a mathematics task (Bastow, Hughes, Kissane & Mortlock, 1984; Maxwell, 2001; Ministry of Education, 1992). Maxwell (2001) explains that the benefits of being engaged in this approach can encourage persistence, risk-taking and open-mindedness. This means learners can develop a positive disposition towards mathematics and enjoy their mathematics learning.

Past experiences can have a very negative impact on a person’s disposition towards learning mathematics (Ambrose, 2004; Biddulph, 1999, Cornell, 1999; Tooke & Lindstrom, 1998). Such feelings can affect the learning and teaching of mathematics and need to be addressed (Biddulph, 1999; Grootenboer, 2003; Hawera, 2004; Schuck & Grootenboer, 2004). Humanistic learning theory indicates that notions of relevance, control, temporary discomfort, success and genuine interaction, are integral to learning mathematics and can be influential in promoting positive engagement in mathematics tasks (Biddulph, 1997).

People’s feelings about engaging with investigations may well be part of the subtle emotional balance that exists between them and their teachers in mathematics lessons. Hackenberg (2005) suggests that reciprocal mathematical ‘caring’ for the teacher and the learner during the mathematical learning process is important. For learners to participate in mathematical tasks, there needs to be some stimulation to aid their engagement. They will also at times experience a ‘depletion’ stage when they will need support to persevere with their investigating. Teachers need to anticipate or be aware of different ways learners may progress in their mathematics task or thinking, thereby helping if necessary, learner engagement to continue (Maxwell, 2001; Pendlington, 2006). In return, teachers are cared for by the learners’ active involvement and engagement.

**How Might This Link To Māori Learners?**

In traditional Māori society one method used for educating learners was through exposure (Hemara, 2000). Learners would be put into or presented with a situation and asked how they would respond. Teachers would expect learners to ask questions which could contribute to the thinking and learning process (Hemara, 2000). Education was oral, thematic and holistic (Barton & Fairhall, 1995; Riini & Riini, 1993) and learning mathematics was integrated into community practices.
Studies show that face-to-face relationships and interactions between teachers and Māori learners are major influences on Māori educational achievement today (Bishop, Berryman, Tiakiwai & Richardson, 2003). If relationships and interactions are of poor quality, teachers may employ deficit thinking and have low expectations of Māori learners. Such thinking can support the long-term pathologizing of Māori where they are blamed for their lack of educational achievement (Bishop, 2005).

There has been concern in New Zealand for some time about the underachievement of Māori in mathematics. (Barton & Fairhall, 1995; Forbes, 2002; Garden, 1997; Garden, 1996; Knight, 1994; Ohia, 1995; Young-Loveridge, 2004). Despite recent government initiatives to address the situation (Ministry of Education, 1992; Ministry of Education, 2001), the gap regarding mathematics achievement between Māori and non-Māori is still widening (Young-Loveridge, 2004).

To enhance the learning and teaching of Māori, Macfarlane (2004) suggests that there are five key principles that should be considered. These are ideas about valuing cooperative learning, the safety of the classroom, the bonding as a class, the building of relationships with members in and out of the school and teacher effectiveness. These notions reflect a social constructivist approach to learning mathematics where ‘knowledge’ is not seen as an external body of truths but is socially constructed by the learner (Ernest, 1994). This encourages “problem-solving, communication, active participation and social interaction” (Holt, 2001, p.24) that will benefit Māori as well as other learners.

**What Was This Study About?**

This study focused on the responses of 18 Māori preservice primary teachers when carrying out mathematics investigations during their Year 1 compulsory course of mathematics education. Learners were expected to attend a two hour class each week and then use the rest of the course time to complete other tasks and associated work. The tasks described here were the major component of work during the first five of twelve weeks of the course. Learners were involved in tasks that required them to engage in four mathematical investigations, reflect on that process, and consider links to how people learned. Homework tasks were different from those undertaken in class.

An example of one investigation was:  

**Popcorn.**

A supermarket has a competition where groups of customers make containers to hold popcorn. The winner is the customer who makes the container with the largest volume (by cutting corners from the given piece of paper – no flaps). Pretend you are among this group of customers – your challenge is to create the largest container possible out of this piece of paper. How do you know your suggestion has the largest volume?

The preservice teachers were asked to keep journals of their investigative work, including reflections about their mathematical learning. Observations recorded by their teacher captured some of the spontaneous sharing of ideas and questions that were generated during class time. On two occasions tape recorders were used to record conversations between learners as they worked on class investigations. Learners were encouraged to work together in class time but to write about their mathematical thinking and reflecting individually. They were able to work together on homework investigations too if they wished, but again they needed to record their thinking independently.
What Did Learners Have To Say?
There was a range of responses to the tasks.

1. Multiple Solutions:
Ten of the eighteen learners commented on their surprise at the idea of mathematics having more than one solution to a problem or investigation. This seemed at odds with their perspective of mathematics and past mathematics experiences.

“I thought there would only be one answer like there was at school” (S).

2. Different Strategies:
Eleven learners expressed surprise at learning that there might be more than one way of gaining a solution.

“I like finding out different ways of answering a question and solving a problem” (Anon).

“ I didn’t realize there would be other ways of doing this” (T).

3. The Value of Equipment:
Using equipment and discussing their workings also seemed to help in the making sense of mathematical ideas for seven of the learners.

“Length, width and height now makes sense to me…..what volume is. Using a 1cm square grid sheet also helped me to understand how many cubes fit into each container” (M).

4. Links with Prior Knowledge:
Six of the learners attempted to make links with past learning regarding formulae.

Learner 2: First we all estimated it and we just looked and worked out this was a bit from here and a bit from there.
Learner 4: So it’s base times height?
Learner 1 and 2: Divided by two.
Learner 1: Because base times height gives the rectangle or square and you divide that by two and it gives you a triangle.

Although not always mathematically “correct”, past learning provided a place to start so as to move further along in the investigation.

“…trying to remember what I learned from school, the formula for finding the volume of a circle is pi r squared” (H).

5. The Value of a Mathematical Challenge:
Eleven learners commented on finding the mathematics challenging.

“I have really been enjoying the maths we have been doing as it is stimulating my brain and I do love challenges, especially those I overcome” (J).

“I really enjoyed this task because it made me think” (W).

“These tasks are very challenging. Got us out of our comfort zone” (H).

6. Accessibility of Tasks:
Three learners commented on the ease of moving back into doing mathematics.

“Don’t worry if you don’t know much maths. You can explore and enjoy it” (M).

“This is a good introduction, so we could get back into maths, as a lot of people left school years ago” (E).

7. Links with Real-Life Experiences:
At times it was evident that learners were attempting to make links between their mathematics and real life experiences. Four learners made such links.

“This is what I did when I was thinking about the plumbing pipes/ spouting that are on houses and applied that to my maths investigation” (H).

8. Collaboration:
Seven learners also commented on how helpful they found it to be able to work collaboratively with others as opposed to having to work individually on their tasks.

“It’s good to be able to work with someone else on this maths because by the time we share our ideas we actually manage to do quite a bit. I don’t know if I could do this much by myself. We seem to get something out of what we both put in. I’m surprised” (M).

9. Teacher-learner interactions:
Three learners commented on their interaction with their teacher and seemed to appreciate the opportunities given to pursue their own solutions.

“It’s good she doesn’t really give us the answer, ay? She just gives us a bit so we can move on… like ask us a question when we’re stuck” (H).

10. The Openness of Tasks:
Two learners found the idea of multiple solutions “daunting” at times. For one student,

“There are too many possible answers! Maths should just have one answer and be right or wrong!……. This is what I don’t like about maths investigations, and when do you know when to stop?” (T).

One learner wondered how multiple solutions might affect her as a teacher. Would she have to know every possible solution to a problem or investigation?

Two learners commented that understanding more than one way of finding a solution was challenging.

“I find it difficult when a task is solved that differs from the way I would’ve solved it” (M).

For four of the learners, following an idea right through appeared to be too difficult for them. It seemed that posing questions and attempting some exploration without reaching any conclusions caused them concern. Their investigations were just left to “hang”.

Three learners expressed dissatisfaction with the challenging nature of the tasks.

“No I don’t like challenges. Just tell me what you’re doing” (T).

11. Writing and reflecting:
Writing and reflecting on their engagement with the mathematics learning...
process was required of the learners. The timing of this was not always appropriate for them, especially when asked to stop their engagement in their mathematics investigations and write a reflection on their mathematics working at a specific time. Two learners commented on this.

Learner 2: “Isn’t it funny how it’s like school? You just get into something and then get stopped.”

Learner 1: “I could’ve done this for a while.”

12. **INTERPRETATION OF INVESTIGATION:**

Unless learners asked, they were left to interpret the tasks themselves. At times their interpretation differed from that of the teacher. For example, Learner (P) thought that with a task involving a plumbing problem, she had to be concerned with the amount of water running into the spouting and therefore became concerned with trying to calculate the volume of water. Such learners thought they knew what to do and therefore did not see a need to seek clarification. This led to mathematical misunderstandings and frustration for them and the teacher.

“**These tasks are set up to trick you and make you fail**” (D).

**WHAT MIGHT THIS MEAN?**

**BENEFITS:**

The investigative approach to learning mathematics that these learners experienced supported the notion of education through exposure to problematical situations (Hemara, 2000). Most of these learners responded positively to the various mathematics investigations. They were able to construct possible solutions and at times, the best outcome. This approach encouraged the construction of ideas in ways that made sense. They used problem-solving strategies, critical thinking and collaboration with others to discern which ideas were more feasible than others. These processes were very familiar to Māori in earlier times (Hemara, 2000) and are key facets of social constructivism. However, in terms of learning mathematics today, it seemed that this approach was new to most of these learners.

The attempt by some of these learners to link the investigations and thinking to their real-life experiences is consistent with humanistic learning theory. The contexts in which the investigations were embedded enabled them to access and grapple with the mathematics ideas. Being able to work collaboratively with others also seemed to support learning. These factors appeared to make the investigations and consequently the mathematics, more accessible. They encouraged different ways of beginning, provoked discussion and supported the use of equipment for modelling thereby optimising opportunities for learners to persevere and reach mathematical conclusions.

The challenging nature of the tasks seemed also to be appreciated and enjoyed by most of these learners. Offering conjectures, taking risks and making decisions seemed to help increase the positive disposition of learners to mathematics as proposed by Maxwell (2001). The emphasis on learners displaying their investigative process and thinking skills seemed to relieve the pressure of finding quickly the “right answer” and justified the exploration process. This is especially important with learners who are mathematically anxious and may have finished any formal study of mathematics some time ago (Biddulph, 1999; Grootenboer, 2003; Hawera, 2004; Herewini, 1998).

Teacher awareness and practice about mathematical caring needs to be reflected so that intellectual development may be nurtured (Hackenberg, 2005). This is important to note in view of deficit thinking and hence low expectations of some mathematics teachers of Māori (Bishop, 2005; Bishop et al., 2003).

A greater emphasis on an investigative approach to mathematics teaching and learning within the framework of social constructivism and humanistic learning may be worthwhile pursuing to help improve the mathematics achievement of Māori learners. Exposing more Māori preservice and in-service teachers to this approach may bode well for Māori children learning mathematics in schools.

**CONCERNS:**

A concern that arose from this study was the need to ensure that investigations are understood by the learners. Communication skills of the teacher are paramount. It is worth spending time clarifying tasks to ensure that Māori learners are able to maximize their opportunities to explore mathematics successfully.

Carrying out an investigation often requires learners recording or tracking information so that patterns may be exposed and noticed. Being aware of how one thinks or conducts an investigation can aid further development. Any intervention by the teacher to ensure that such recording and reflecting occurs needs to be carefully considered so that learners’ train of thought or work is not compromised.

With this approach to mathematics there is a need to ensure that there is sufficient support and flexibility for learners to engage in the investigations. How to support those who struggle with maintaining their engagement in mathematics investigations is an issue and requires consideration of the mathematical caring that Hackenberg (2005) suggests.

**LIMITATIONS**

An obvious limitation of this study is the small number of learners who were involved. While this approach seems to have been beneficial for many of these learners in some way, it would be useful to gather more information with a larger sample size.

It may also be useful to promote mathematics investigations that relate specifically to Māori cultural contexts too for those learners who wish to start from that arena.

**HOW MIGHT THIS AFFECT ME AND MY CLASSROOM PRACTICE?**

The results from this study would indicate that an investigative approach to mathematics learning was beneficial for many of these Māori learners. They gained new perspectives regarding mathematical processes and strategies. It seemed helpful for them to focus on their own development as learners of mathematics. These aspects, combined with support, helped enhance their disposition towards engaging in mathematics. Perhaps ensuring that Māori children enjoy similar opportunities to focus on themselves as learners of mathematics could be worthwhile.

The issue of time needs consideration. This may mean prioritising key mathematics ideas that can be explored by learners, some of whom may be anxious about their mathematics abilities. Meaningful mathematics exploration can be time consuming, yet the benefits for the learner may outweigh such concerns.

Māori achievement in mathematics is a very complex issue. Despite our best intentions the gap between Māori and non-Māori continues (Ministry of
Education, 2006). On the whole, while some very small successes may be claimed here, the results indicate that implementing the investigative approach is worthy of time and effort. It would be useful to research whether long-term benefits were gained for these people as classroom teachers. Additional research with Māori children using this approach could also be worthwhile to ascertain if there are benefits for them as well. Such research would supplement the paucity of literature regarding possible approaches for supporting and improving Māori achievement in mathematics.

**IS THIS A WAY?**

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