Finding quality geometry apps

Not as simple as $a^2 + b^2 = c^2$

Kevin Larkin
Griffith University, QLD
<k.larkin@griffith.edu.au>

Trying to find quality apps for use in mathematics classes can be time consuming and bewildering. This article outlines a process for evaluating apps and provides teachers with access to comprehensive qualitative evaluations of 53 geometrical apps based on pedagogical, mathematical and cognitive fidelities.

A recent review of 53 geometry apps by the author has unfortunately confirmed earlier research regarding number and algebra apps, published in *Australian Primary Mathematics Classroom* (Larkin, 2014). As was the case with the earlier research, finding an appropriate geometry app to use with primary school students is difficult—in terms of the time it requires and also due to the poor quality of apps available. This article outlines a process for evaluating apps and serves two purposes: it saves classroom teachers time as potentially useful apps to use have been located for them; and it provides an online resource where further app reviews can be documented as new geometry apps are released. In previous research, apps were reviewed using the Queensland Education Department's *Productive Pedagogies* (2002) and Gee's *Principles for Game Design* (2003). These measures were not considered as appropriate for reviewing geometry apps given the additional necessity for mathematical accuracy in external geometric representations. Thus, in this research, apps were critiqued using Dick's (2008) three measures of fidelity—cognitive, mathematical and pedagogical. In addition the author also evaluated the apps qualitatively against the geometry sub-strand components of the *Australian Curriculum: Mathematics* (ACARA, 2015).

Whilst not surprising, given the earlier research, it is still disappointing to report that most of the 53 apps reviewed (already pre-selected as having at least some potential and thus not representative of a broader sample of even worse apps) are inappropriate for teachers to use with primary school students. It is hoped that, after reading this article, teachers have both a starting point for using geometry apps and also a strategy to follow in evaluating new geometry apps as they become available. iTunes apps, rather than Android apps, were chosen for review as iPads remain the dominant mobile device in schools. It is generally accepted in the mathematics community that materials (manipulatives), both concrete and digital, if used thoughtfully, enhance mathematical learning. Given this, I will focus immediately on geometry apps as they are the manipulatives evaluated in this research and also the type of manipulative of most interest to many teachers challenged with using iPads in mathematics classrooms.

Dick (2008) suggests that pedagogical fidelity refers to the functionality of the tool to further learning and includes "the extent to which teachers (as well as students) believe that a tool allows students to act mathematically in ways that correspond to the nature of mathematical learning that underlies a teacher's practice" (Zbiek, Heid, Blume & Dick, 2007, p.1187). Thus, the effectiveness of a tool in terms of pedagogy (and this is taken to mean the action of the teacher and also the action of the students in using the apps) must support how students initially develop conceptual knowledge, and then later procedural and declarative knowledge. For example Coordinate Geometry (Ventura Educational Systems, 2013) scaffolds the learning throughout the app as students learn new concepts, have the opportunity to apply these concepts, and then are quizzed on what they have learned.

Mathematical fidelity is defined as the "faithfulness of the tool in reflecting the mathematical properties, conventions, and behaviors (as would be understood or expected by the mathematical community)" (Zbiek, et al., 2007, p.1173). Dick (2008) cautions that the drive for user friendliness can sometimes run contrary to faithfulness to accurate mathematical structures.

Table 1. Levels of fidelity in apps—adapted from (Bos, 2009) (See Larkin, 2015.)

Type of fidelity	Low level (1-3)	Medium level (4-7)	High level (8-10)
Pedagogical The degree to which the app can be used to further student learning.	App is difficult to work with. Accessing all aspects of the app is difficult. App is not appropriate for the mathematics concepts it uses. Transitions are inconsistent or illogical.	Using app is not initially intuitive; but with practice becomes so. Mathematical activities presented are appropriate but could be developed without app. Transitions evident but only made via trial and error.	Manipulation of app is intuitive and encourages user participation. Little or no training or instructions are required. Transitions are logical and aid sense making.
Mathematical The degree to which the app reflects mathematical properties, conventions and behaviours.	Mathematical concepts are underdeveloped or overly complex. Lack of patterns. Lack of connection to real world mathematics.	Application of mathematics concepts unclear. Patterning is evident but lacks predictability or is unclear. Some connection to real world mathematics.	Mathematics concepts developed are correct and age appropriate. Patterns are accurate and predictable. Clear connection with real world mathematics.
Cognitive The degree to which the app assists the learner's thought processes while engaged in mathematical activity.	No opportunities to explore or test conjectures. Static or inaccurate representations. Patterns do not connect with concept development.	Limited opportunities to explore or test conjectures. Minor errors with representations but still make sense. Patterns connect in a limited way with concept development.	App encourages exploration and testing of conjectures. Representations are accurate and easily manipulated. Patterns clearly aid concept development.

This means, for example, that in the app Shape Rotate (Sums Online Ltd., 2012), instead of students having to determine how to draw specific angles, the app allows them to enter a numerical value and then does the drawing for them. Given that non-educators design many apps, perhaps looking for a quick profit, the poor mathematical structuring of future apps is likely to continue. Cognitive fidelity, critical in geometry apps that require multiple external representations, refers to "the faithfulness of the tool in reflecting the learner's thought processes or strategic choices while engaged in mathematical activity" (Zbiek, et al., 2007, p.1173). For example, in Shapes 3D Geometry Learning (Setapp, 2015), students can pull apart and put back together the 3D objects to see the connections between vertices in a 3D object and corners in its 2D net representation. A disappointing aspect of this research is that, as seen in the Shapes 3D app, the technological capability of the iPads is sufficient for high levels of fidelity. Unfortunately, perhaps for the financial reasons noted above, few apps utilise the full functionality of the iPads.

Methodology

In this article I have used Bos' (2009) framework for evaluating educational software but have adapted it for reviewing iPad apps as they serve a different purpose to more comprehensive computer based software.

Findings and discussion

A collection of comprehensive, qualitative evaluations for 53 apps is available for download at http://tinyurl.com/Geometry-Apps. Each of the reviews contains the geometry content, matched to the relevant *Australian Curriculum* year level, with the often inaccurate description of the app from the iTunes store and a thorough review by the author. A sample is provided in Table 2.

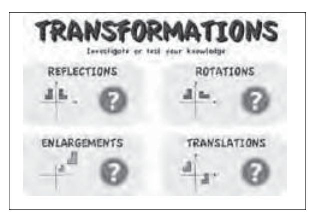

Figure 1. Transformations app.

A number of generic observations come to light as a consequence of this review. Firstly, as any teacher who has attempted to locate useful mathematics apps already knows, the sheer number of apps available makes locating useful ones a Herculean task. As of February 2016, there were approximately 201 000 education apps available for download (Pocketgamer.biz, 2016). If this isn't a big enough problem it gets worse: apps are misnamed (i.e. the name at the iTunes store doesn't match the name when it is opened on an iPad); similarly named apps (a dozen apps had variations an iPad);

Table 2. Example qualitative geometry app review.

App name	Content	Year level	Description of app at iTunes store
Transformations	Flips Slides Turns Dilations	Years 5-7	Transformations is designed to provide interactive teaching and learning tools for the four types of geometric transformations—reflections, rotations, enlargements and translations. Interactive diagrams and built-in interactive tests make this an essential app.
Reviewer comments regarding fidelity This app is an example of where the design of the app requires additional adult assistance to use (particularly the quiz) but where mathematics learning is encouraged. Much of the following therefore relates to the very worthwhile investigation element for each of the transformations. The app is very good in the investigation section, but way too complex in the quiz. App is intuitive and very clearly develops concepts more effectively than could be done with pencil and paper. Transitions between the different transformations are in evidence. Mathematics concepts are correct and age-appropriate and are accurate and predictably represented. No real- world connections. Exploration is encouraged and the representations are easily manipulable in such a way as to add to conceptual development. As indicated, quiz section is correct mathematically but beyond the level of understanding of most primary school students; however, the investigations sections alone make this a very useful app.			

similarly named apps (a dozen apps had variations on the word geometry); the rapid turnover of apps at the store; and finally a very poor search engine (apps not sorted according to category or alphabetically). As reported in the full qualitative reviews, the descriptions of the apps at the iTunes store essentially promise "solutions for all your teaching woes" but are little more than infomercials. Readers familiar with the ABC program The Checkout are aware of the "packet vs. product" comparison—here what is described at the iTunes store regarding each app regularly bears little resemblance to what is seen when the app is opened. Thus the independant qualitative reviews of the apps are very helpful for teachers to become aware of what is actually "in the app box" rather than "on the cover of the box" at the iTunes store.

The quantitative data tells much the same story. In Table 3 readers can see that only a small number of apps scored 6 or higher. Just under half of all the apps reviewed (26 out of 53) failed to score a six on any of the three measures—and thus do not support pedagogy, are mathematical inaccurate, and cognitively inert. These can be immediately excluded from any consideration for use in the classroom. The mean score for the 53 apps failed to reach a passing grade (scoring 12.9/30) and none of the individual categories reached a passing grade. In short, each of the three fidelities, whilst evident in some apps, were poorly represented overall. This data confirms the qualitative review findings and is a clear indication that the hard sell of the apps at the iTunes store does not match the hard reality of classroom use. As a general trend, the apps that scored at least one 6 or more tended to score well on the pedagogical fidelity dimension, less well in terms of mathematics fidelity, and generally poorly in cognitive fidelity.

Given that pedagogical fidelity can include elements of procedural and declarative knowledge, which are perhaps more easily mimicked by non-mathematical designers of apps, it is not surprising that this type of fidelity is most commonly found in the apps. However, many of the apps met only one of the pedagogical criteria (Table 1); namely, they were easy to use without instruction. Other apps partially met the criteria of developing concepts in an appropriate manner, without necessarily doing anything more than could be easily replicated with an IWB or physical manipulatives. Although some of the apps scored highly in mathematical fidelity, (Attribute Blocks [Brainingcamp, 2015] and Simitri [Grindall, 2016]) overall the apps were weak in the areas of real world connections to geometry as children may experience it; they tended to fall into the trap of many mathematics textbooks/workbooks/web based manipulatives in presenting prototypical shapes utilising standard orientations; and finally were inconsistent with nomenclature (e.g. squares classified as non-rectangles or triangles not included as polygons). Whilst these mathematical issues are important they could largely be easily corrected by teachers using the apps. More problematic is the very low-level of cognition supported by the apps. As already indicated, the iPads are technically capable of supporting high-level dynamic representations (e.g. Montessori Geometry [Edoki Academy, 2016]); however, perhaps for reasons of cost or low mathematical knowledge of the designers, very few apps demonstrate cognitive fidelity and thus are immediately reduced to the role of revision apps, at best. Thus many apps failed

Table 3. Number of apps scoring 6 or more in respective fidelities.

Type of Fidelity	Number of Apps (n=53)	Percentage (to nearest 0.1)	Average Score /10
Pedagogical	21	39.6%	4.9
Mathematical	13	24.5%	4.3
Cognitive	8	15.1%	3.7
Overall average score for apps on the three measures/30			12.9

to mimic the physical activity of transformations such as translations, rotations ans reflections and instead had the users input numbers to perform the transformations. This is a signature failure of cognitive fidelity as there is no link between the action of the user and the resultant mathematics outcome as would be the case if students were physically manipulating 'real' objects.

So, having painted a rather gloomy picture of the pedagogical, mathematical and cognitive fidelity of the apps, teachers might be thinking that iPads are best stored in a cupboard during mathematics; however, there are a few apps which are well worth keeping the iPads stored handily in the student's tidy trays. Table 4 presents in summary form the geometric equivalent of the Magnificent Seven. Each of the Magnificent Seven scored a six or more for each of the three fidelities and thus are definitely ones that teachers should consider using in their geometry teaching. Even though these apps are well worth a look, only the top three consistently scored very highly across the dimensions and thus, even some of the top seven only just achieved the minimum standard of six in one or two of the fidelities.

By way of example Pattern Shapes (The Math Learning Center, 2014.) (Figure 2) made the list as it scored at least a six in each category, but the app is only really useful in terms of supporting pedagogy (either that of a teacher using the app in a lesson or an individual student closely following the structure inherent in the app). Even with this app though, it is not particularly accurate in terms of mathematics

Figure 2. Pattern shapes icon.

(with errors in language and classification) and it is not particularly supportive of connecting representations with mathematical concept knowledge. This pattern of strength in one area, and weakness in one or both remaining areas, is apparent in a number of other apps indicating that significant, prior planning is required by teachers if the app is to be useful rather than potentially detrimental to some forms of mathematics knowledge. For example, and respectively for pedagogical, mathematical and cognitive fidelity, Geoeng (Geoeng Systems, 2013)—8, 6 and 5; Geometry 4 Kids (Nth Fusion LLC, 2014)—8, 6 and 3; and Montessori Geometry (Edoki Academy, 2016)—8, 6 and 5 each scored a high mark in supporting pedagogy but barely average marks in mathematical fidelity and often poor marks in supporting cognition. What this means is that, even with the apps recommended for teachers and even more so with apps just outside of the Magnificent Seven, teachers still have a critical role to play in deciding the how and when of app use. In other words, are these apps going to be useful when introducing a concept (cognitive fidelity), or when making connections between mathematics and external representations (mathematical fidelity) or when

Table 4. Apps that scored 6 or more out of ten on each of the three fidelities.

App name	Pedagogical	Mathematical	Cognitive	Total
Co-ordinate Geometry	9	8	9	26
Transformations	9	8	9	26
Attribute Blocks	8	8	8	24
Shapes – 3D Geometry	9	6	8	23
Shapes and Colors	7	6	7	20
Pattern Shapes	8	6	6	20
Isometry Manipulative	7	6	6	19

practising and reinforcing a concept, skill or strategy (pedagogical fidelity)? As is always the case with quality mathematics teaching, the materials, in this case iPad apps, cannot do the teaching themselves and teachers remain the fundamental determinant of the mathematics experience of their students.

Conclusion

As was the case with the review of number and algebra apps published in *Australian Primary Mathematics Classroom* (Larkin, 2014), this review discovered a wide discrepancy in the quality of the geometry apps. Utilising the qualitative reviews found at http://tinyurl.com/Geometry-Apps minimises the chance that teachers will be led astray by the iTunes infomercials into selecting inappropriate apps. In addition, the full summaries provide important information as to how the apps might be used in the classroom. The list of 53 apps is editable, and it is hoped that teachers will contribute reviews of their own to keep the list 'alive'. Unlike matter, which is neither created nor destroyed only transformed, apps come into and out of existence very rapidly and thus reviews such as these can easily become the digital equivalent of 'yesterday's news is today's fish and chips paper'. Please contact the author for further information as to how to contribute to the reviews to keep the list current.

Acknowledgement

This project was initially funded through a Griffith University, School of Education grant scheme. Aspects of this article were presented at the 2015 MERGA conference.

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