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Teacher and Student Use of Gesture and Access to Secondary Mathematics for Students with Learning Disabilities: An Exploratory Study

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The researchers conducted an exploratory qualitative case study to describe the gesturing processes of tutors and students when engaging in secondary mathematics. The use of gestures ranged in complexity from simple gestures, such as pointing and moving the pointing finger in an arching motion to demonstrate mathematics relationships within equations (e.g., distributive property), to more elaborate gestures such as using arm movements to demonstrate spatial relationships in multi-step geometry problems. These gestures were used both in isolation and in combination with research-supported special education interventions such as helping students with learning disabilities to organize their cognitive process and diagram problems. Findings from this study are described with regards to the potential role gesturing may provide as a support for students with learning disabilities as they engage in challenging mathematics.

Keywords: **Gesturing, visual representations, diagrams, learning disabilities, mathematics.**

Introduction

The Need for Students with LD to Acquire Mathematical Concepts

Due to changes in educational policy in the United States (e.g., Every Student Succeeds, 2015; Individuals with Disabilities Education Act, 2004; No Child Left Behind, 2002), the expectations for students with learning disabilities (LD) continue to rise as they participate in high-stakes assessments of grade-level mathematical content. All students, including students with LD, are expected to master complex concepts as more states implement the Common Core Standards for Mathematics (Council of Chief State School Officers and National Governors Association [CCSSO & NGA], 2010). As students with LD advance to middle school mathematics, they need to develop a complex understanding of mathematics concepts to succeed at this level and build a foundation for success in high school courses (e.g., Algebra I, Geometry, and Algebra II) and post-secondary outcomes (Confrey et al., 2012; Hartwig & Sitlington, 2008; National Council of Teachers of Mathematics [NCTM], 2000).

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Therefore, it is imperative for special education researchers and practitioners to understand how students with LD access complex mathematics in secondary settings and the key supports they will need to provide (Foegen, 2008; Maccini, Mulcahy, & Wilson, 2007; Powell, Fuchs, & Fuchs, 2013).

Challenges and Mathematical Strategies for Students with LD

Students with LD are often identified for special education services by low academic performance compared to their IQ scores or by a lack of improvement in academics despite small group or individualized instruction (see Gresham & Vellutino, 2010). In mathematics, students with LD tend to have difficulties with basic skills as well as more contextualized tasks, such as word problems, especially when these students also have LD in reading (Fuchs & Fuchs, 2002). These difficulties are associated with the many difficulties students with LD have with memory and processing (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Students with LD often struggle with working memory (i.e., processing, storing, and integrating more than one set of information) as well as cognitive and metacognitive processes such as effectively processing, diagramming, and solving multi-step mathematics problems (Baddeley, 2003; Swanson & Siegel, 2001; van Garderen, Scheuermann, & Poch, 2014). These struggles may be exacerbated by deficiencies in foundational knowledge, such as place value, addition, and subtraction (Geary et al., 2007). Rather than easily recalling information from long-term memory, students with LD may have difficulty connecting basic, foundational math knowledge to more complex problem solving processes as demanded by challenging mathematical tasks (Ericsson & Kintsch, 1995; Geary et al., 2007). In response to these challenges, researchers from special education and mathematics education have stated that, although there should be high expectations for students with LD, these students should also receive strategic interventions and scaffolds designed to help them meet these expectations (NCTM, 2000; Woodward, 2006).

Despite the challenges facing students with LD, some of these students have demonstrated the ability to succeed with challenging mathematics when intervention support is provided (Gersten, Chard, Jayanthi, Baker, & Morphy, 2009). In recent studies, students with LD have succeeded in middle school mathematics with the use of visual representations (for review, see Foegen, 2008; Maccini et al., 2007). Visual representations, such as diagrams, may provide scaffolds for students with LD related to difficulties with working memory; students can store information from earlier steps in diagrams to relieve the burden on short-term memory to facilitate concentration on processing, storing, and integrating information from upcoming steps in problems (Keeler & Swanson, 2001). Also, diagrams are often utilized by researchers to help students understand the mathematical concepts within problems (e.g., Jitendra et al., 2009; Xin, Jitendra, & Deatline-Buchman, 2005) increasing their mathematical understanding while simultaneously supporting memory and processing of information. Therefore, the strategic use of visual representations can serve as both an instructional and scaffolding strategy, helping students organize multi-step problems and connect information to find a solution (Ives, 2008; van Garderen, 2007).

Special education researchers have also utilized visual representations when teaching with graduated instructional sequences (e.g., Scheuermann, Deshler, &

Schumaker, 2009). Graduated instructionally sequenced instruction (often referred to as concrete-semiconcrete-abstract instruction) is often designed to develop students' understanding of a concept at concrete and semiconcrete levels before advancing students to abstract contexts (e.g., Scheuermann et al., 2009). Students with LD have demonstrated success with word problems involving equations with multiple variables and multi-step geometry and measurement problems after being taught using this strategy (Cass, Cates, Smith, & Jackson, 2003; Scheuermann et al., 2009).

Special education researchers have found encouraging results for students with LD who struggle with cognition and metacognition by teaching these students strategic thinking processes for word problem solving (e.g., read, paraphrase, visualize, hypothesize, estimate, compute, and check) (Montague, Krawec, Enders, & Dietz, 2014). Special education researchers have also integrated cognitive/metacognitive and diagramming instruction to help students with LD learn to solve multi-step word problems, including problems that require algebraic problem solving, by creating diagrams with subsections for each step of a multi-step problem including a final step of critical thinking to evaluate the logic of the solution (van Garderen, 2007; Xin et al., 2005).

Gesturing During Mathematics Learning

Gesturing, which is any physical motion (e.g., hand waving to indicate motion, pointing and moving the pointer finger, etc.), can be helpful for students in the context of mathematics learning (Cook, Duffy, & Fenn, 2013; Goldin-Meadow, Nusbaum, Kelly, & Wagner, 2001). While gesturing is often an unintentional and natural method of communication, it can provide crucial, supplemental information that may not be easily conveyed by verbal language alone (Cook et al., 2013; Goldin-Meadow et al., 2001). Gesturing can often be used in combination with verbal language to explain ideas in a meaningful way by both teachers and students (Cook et al., 2013; Goldin-Meadow & Alibali, 2013; Rasmussen, Stephan, & Allen, 2004). For example, Goldin-Meadow and colleagues (2001) provided this example: "a speaker might say, 'I ran all the way upstairs' while moving her index finger upward in a spiral. It is through the speaker's gesture, and only her gestures, that the listener knows the staircase is spiral" (p. 516).

When engaging with challenging mathematics, students often benefit from their own use of gestures, and use of gestures by their teachers, when they are thinking through problems because gesture can convey meaning without requiring an overwhelming amount of cognitive resources (Cook et al., 2013; Goldin-Meadow et al., 2001). Gesturing is potentially, very beneficial regarding math interventions for students with LD because of the relationships between gesturing and a decrease in cognitive load as well as the predisposition of many students with LD to struggle with working memory (Goldin-Meadow et al., 2001; Swanson & Siegel, 2001). As students with LD struggle to process and integrate multiple pieces of information and cognitive resources are often diverted away from maintaining some of these pieces in short-term memory (Swanson & Beebe-Frankenberger, 2004). Gesturing can be used to represent concepts that students are trying to remember and help student integrate different pieces of information when processing through a multistep task (Alibali & Nathan, 2012; Ping & Goldin-Meadow, 2010).

Gesturing may play a particularly key role in the context of teachers and students with LD are communicating about a concept that is new to students. As students are learning a new concept, they are sometimes able to gesture to convey meaning of the concept before they can comfortably and easily use language to represent the concept (Goldin-Meadow & Alibali, 2013). In these intermediate stages, teachers can also observe students' gestures and strategically build on student knowledge using student gestures for formative assessment (Goldin-Meadow & Singer, 2003). Along with research-supported, special education interventions, such as diagrams and cognitive strategies, gesturing is potentially a key scaffold for students with LD as they are attempting to access challenging mathematics.

Rationale and Research Questions

While psychology research on gesturing has provided valuable, foundational information to the field of special education, these teaching techniques have not been thoroughly researched specifically in special education settings. Teacher and student gesturing strategies are of particular relevance to students with LD because these strategies can ease the demands on working memory while students are interpreting and explaining mathematics (see Goldin-Meadow et al., 2001); these techniques may be especially important when teachers and students are discussing the complex math concepts students with LD face as they transition from middle school to high school (Confrey et al., 2012; Foegen, 2008; Swanson & Beebe-Frankenberger, 2004). The purpose of this study is to describe the use of gesturing by teachers and students in situations where students with LD are likely to struggle (e.g., multi-step situations). The specific research questions of this study are: (1) How do tutors utilize gestures to support students with LD when solving secondary mathematics problems? (2) How do students with LD use their own gestures to access secondary mathematical concepts? (3) How do students with LD respond to the use of gesturing by tutors and benefit from their own use of gestures in mathematical problem solving situations?

The researchers conducted an exploratory, qualitative microanalysis of six sessions of tutors working with three different students with LD. We utilized a case study methodology to describe the use of gestures, by tutors and students, in this context in order to inform further study of this topic on a more macro-level and future studies involving the purposeful, strategic embedding of gestures into math interventions (Stake, 2010).

Method

Participants and Setting

The study was conducted in one-on-one and small group (e.g., one tutor and two students) settings in a classroom in an urban secondary school (grades 7 through 12) in the Midwestern United States. All four of the participants, one female and three males, were African-American, eighth-grade, school-identified students with LD who were all enrolled in Algebra I in a school with 75% of its students from a low socioeconomic status, 95% of its students from ethnic and racial minority groups, and 26% of its students identified as students with disabilities. Each of the participants' special education files indicated a history of difficulties in math, reading, and writing, but all four of these participants were currently earning passing grades in Algebra I while receiving extra help in smaller settings and participating in the general education classroom. All students were receiving a significant amount of support from a special education teacher and volunteer tutors. Each participant received tutoring for at least one class period per week throughout the school year.

The research team consisted of three pre-service teachers from special education, two special education graduate students, and a special education professor. There were three tutors in this study. They were all Caucasian and from middle-class backgrounds. Sophia and Matt were both undergraduates. Sophia was majoring in special education and Matt was studying middle childhood education at the time of the study. Jane was graduate student with decades of experience in schools as an occupational therapist who was currently pursuing her doctorate in educational studies. All of the tutors were involved in discussions with first author of this study on how to meet the needs of students with LD by drawing from special education (see Gersten et al., 2009), math education (e.g., Steffe, 1990), and educational psychology research (e.g., Swanson & Beebe-Frankenberger, 2004, von Glasersfeld, 1995). The research team and the tutors had multiple discussions about how to help students build on their existing knowledge inside and outside of school (see von Glasersfeld, 1995) by making key teacher adaptations based on student needs (Steffe, 1990; Woodward, 2006) with special considerations for potential difficulties with working memory the students with LD might face when engaging in challenging mathematics (Barrouillet, Bernardin, Portrat, Vergauwe, & Camos, 2007; Swanson & Beebe-Frankenberger, 2004).

Data Collection and Analysis

This collection and analysis of data included multiple phases. To begin data collection, a member of the research team observed and collected field notes with a particular focus on the use of gestures by the tutors and the students with LD. During the *first phase* of analysis, using the field notes as a guide for listening to audio-recorded sessions, the first author of this study conducted a preliminary analysis of these sessions to determine the key instances where gesturing was utilized by the tutors or students while working on math curriculum that could be directly linked to Common Core Standards for Algebra I or II and high school Geometry or prerequisite skills for these courses.

During the *second phase* of analysis, the key situations were transcribed verbatim and the first author coded the use of gestures regarding the nature of the physical movements (e.g., moving of fingers, hands, arms, etc.) and the perceived purpose of the gestures (e.g., to demonstrate a mathematical process or to demonstrate a mathematical relationship). The researchers had team meetings to discuss the placement of key instances into categories to further refine the coding of data; we refined our analysis to move from subjective, reflective comments, to emerging themes, to dividing themes into sub-themes (Brantlinger, Jimenez, Klinger, Pugach, & Richardson, 2005) which led to a focus on gesturing by students and gesturing by the tutors. We also focused on the purposes of gestures such as to represent problem solving processes (e.g., using the distributive property to solve an equation) and the use of gestures to demonstrate spatial relationships (e.g., relationships between angles).

During the *third phase* of analysis, the research team members, who did not participate in the initial data analysis, audited the findings regarding themes and sub-themes that emerged and our inferences about those themes monitor interpretive validity and determine inter-rater reliability (Brantlinger et al., 2005; Maxwell, 1992). After we came to a mutual agreement on all of our findings, we consulted with an external auditor, a local special education teacher, who evaluated the logic of the researchers' inferences during coding of specific instances as well as during the overall analysis of themes that emerged (Brantlinger et al., 2005). The teacher agreed with these interpretations and commented that she had noticed similar problem solving tendencies among many of her students with LD.

Findings

We report descriptive data from six teaching sessions of teaching and learning rooted in the context of tutoring eighth grade students with LD enrolled in Algebra I. The included sessions focused on the mathematics topics of reciprocals, distributive property, and the Pythagorean Theorem. Our findings indicated that student use of gesturing was likely quite important for some participants when they were communicating about novel concepts. Teacher use of gesturing occurred in situations when students seemed to be struggling with interpreting lengthy equations and when they had to interpret complex spatial relationships. Each key example will be presented with background information on the participant, the tutor, and the context of the mathematical situation.

Students' Use of Gestures to Communicate about Mathematics

Luke had a history of difficulty with mathematics and reading. He often struggled with academic anxiety and was especially self-conscious about appearing incompetent in front of his peers. Luke had a positive attitude toward math and often caught on quickly when concepts were demonstrated. Luke and his peer Henry, who was also school-identified as a student with LD in math and reading, were working with their tutor on negative exponent problems. Luke and Henry enjoyed working together with a tutor, often competing to solve problems faster than each other so they could help the other. Their tutor, Matt, was a middle childhood education pre-service teacher. Luke asked Matt for help solving a problem on which he had worked independently, but could not figure out how to solve.

> **Matt:** If you wanted to get rid of this [POINTING to the negative exponent in the denominator] and make it one number you have to multiply by the reciprocal. Do you remember what that is?
>
> **Luke:** When you switch right? (in an unsure, questioning way) [TWISTING his hand back and forth in a circular motion with his palm facing toward Matt; see Table 1, row 1]
>
> **Matt:** Yeah. So, what would it be if you want to multiply by the reciprocal?
>
> **Luke:** Uh…The same thing, but twist, right? [moving his hand in the same TWISTING motion]

Luke then easily solved the rest of the problem. The hand-twisting motion seemed to help Luke make some sense of the concept of reciprocals so he could focus on solving the rest of the problem. Even though he supplied the gesture, continued use of the gesture seemed to reinforce his thinking.

Then, Matt noticed Henry was also struggling with the same problem and directed Luke to explain his method to Henry.

Luke: Okay, so first you've got to find out what does this (pointing to 10^{-2}) equals.
Henry: Uh… 1 over 100.
Luke: Write that down.
Henry: (writes silently)
Luke: Okay, now what is the reciprocal of that?
Henry: Uh…
Luke: Do you know what a reciprocal is?
Henry: Can you explain?
Luke: It is when you flip the numbers [while using the same HAND-TWISTING gesture he used while learning the problem; see Table 1, row 1].
Henry: A hundred over one.
Luke: Yeah.

Henry quietly solved the rest of the problem. Gesturing seemed to help Luke and Henry process the concept of reciprocals while engaging in a multi-step problem about a concept they had not yet mastered. Luke used gesturing to demonstrate his knowledge of the term reciprocal, which allowed Matt to guide him through the steps necessary for solving the problem. Luke also used the same gesture to describe the concept to Henry (i.e., to help him visually and more easily process the concept) and how to work through the problem. In this case, gesturing by the student served two purposes: the students were able to use gesture as a means to build their understanding and ability to communicate about a new topic and as a valuable assessment tool for the tutor which Matt utilized to facilitate Luke's progress (Goldin-Meadow & Alibali, 2013; Goldin-Meadow & Singer, 2003)

Teacher Use of Gestures to Communicate about Equations

Greg was a quiet student who would often work through multiple problems in a steady rhythm. He was quite proficient with solving equations needing minimal, occasional assistance. However, during one of the sessions, he was working independently on his classwork when Jane, an occupational therapist and doctoral student, noticed he was struggling with this problem: $2(2x + 9) = 50$. Jane noticed that the multi-step nature of the problem and the abundance of mathematical symbols to process (according to Greg's current skill level in mathematics) seemed to be potentially causing Greg some difficulties with cognitive overload (see Barrouillet et al., 2007; Swanson & Beebe-Frankenberger, 2004). Jane also found that Greg had the misconception that he needed to solve what was inside the parentheses first, when he actually needed to utilize the distributive property because the variable inside the parenthesis prevented him from solving. Regardless of whether Greg's difficulties stemmed from interpreting the mathematical symbol (possibly blocking out temporarily irrelevant information) or Greg's struggles were due to missing prerequisite understanding, Jane decided that Greg would benefit from gesturing along with spoken word to help him access concepts that were not readily apparent for him (Ping & Goldin-Meadow, 2010; Swanson & Siegel, 2001).

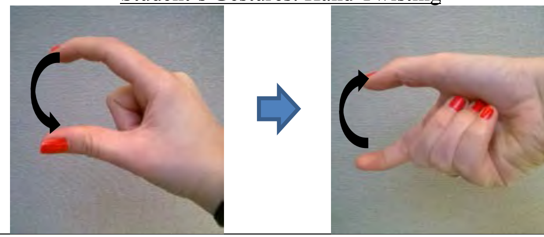
Table 1. *Gesturing Examples with Corresponding Problems*

Gesturing to Describe a Mathematical Concept

Student's Gestures: Hand Twisting

Use the properties of exponents to evaluate the following expressions.

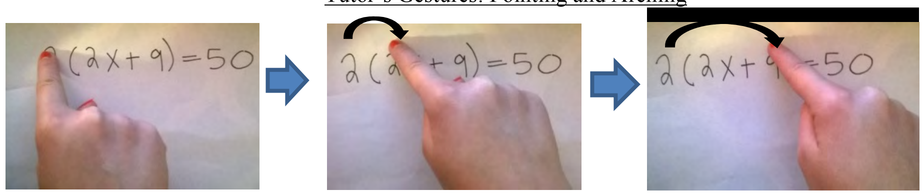
$$\frac{10^2}{10^{-2}}$$



Gesturing to Teach Mathematical Processes

Tutor's Gestures: Pointing and Arching

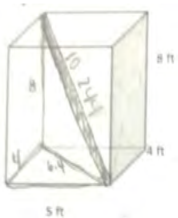
$2(2x + 9) = 50$

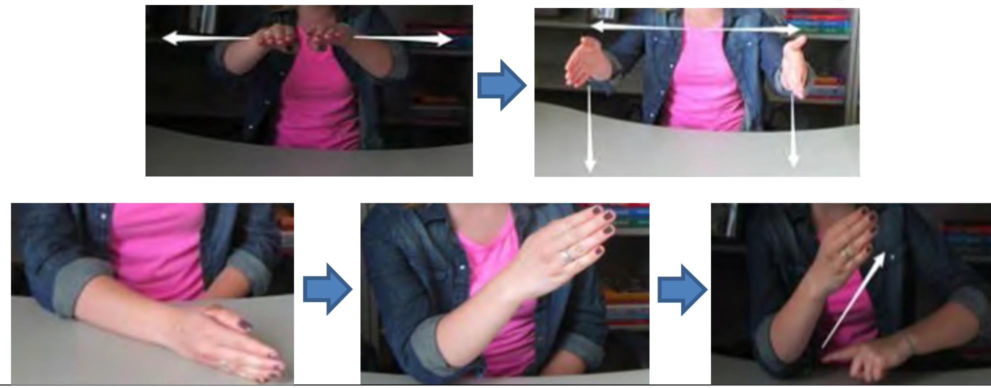


Gesturing to Demonstrate Spatial Relationships

Tutor's Gestures: Motions to demonstrate lines and shapes

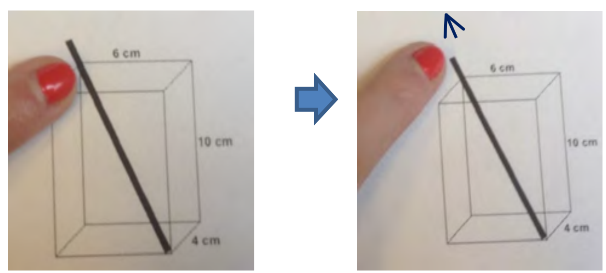
You are trying to fit a 10 foot pole in the back of a truck. The truck is 5 feet long, 4 feet wide, and 8 feet tall. Will the pole fit in the back?





Tutor's Gestures: Pointing and Sliding

You are trying to determine if a straw is long enough to use for a carton of juice. The carton is 10 cm high and the top and base are 4 cm by 6 cm. In order to drink from the straw, it will need to stick out at least 1.5 cm from the carton. If the straw fits in diagonally, as shown in the picture, what is the shortest length the straw could be and still be useable?



Note. Gestures were recreated and photographed to show the gestures used during instruction.

Jane: Now, what do we do with that?
Greg: Um. The parentheses…?
Jane: We would do what is inside the parentheses first, but since we can't… What about this number out in front? What does the 2 out in front tell us?
Greg: Times?
Jane: Times what? (pauses for a while) And, how will get that [POINTING to the 2 outside the parentheses and SWEEPING her pointer finger in an arching motion over the left parenthesis to the 2x; see Table 1, row 2] across these?
Greg: (silence)
Jane: So… What they call the distributive property... We are going to take this 2 and make it go to this one [POINTING at the 2 outside the parentheses and ARCHING toward the 2x inside the parentheses]. We are going to take it across each of these [repeating the ARCHING gesture]. It will be 2 times 2x and 2 times 9. (after some wait time) 2 times 2x and I would do this to this [POINTING to the 2 outside the parentheses and to the 9 inside the parentheses using the same ARCHING gesture; see Table 1, row 2]. I would keep this plus in here… Plus 2 x 9…So, I gave that 2 to each of the numbers inside the parentheses and I made it *times* because it says times there [POINTING to the left parenthesis]. So, then I figure this [POINTING and ARCHING across the parenthesis from 2 to 2x] out. Two times two is what?
Greg: 4x. (right away)
Jane: 4x! Good job of remembering that x. Then, I just bring this down [POINTING below the current step].
Greg: (works quietly and gets 4x + 18 = 50 without further prompting)

Jane: Very good!

Jane's gesture to illustrate the process of the distributive property seemed to help Greg understand how to solve the problem. He then easily solved the next step equation (4x + 18 = 50) for x by subtracting 18 from 50 and dividing 32 by 4 to get 8. In this situation, Greg seemed comfortable solving for x with 4x + 18 = 50, but the need for distribution in the problem was something unfamiliar and challenging for Greg.

On the next problem, Greg was required to solve for x in the equation, 3(3x – 5) = 12. Greg, without assistance, multiplied 3 and 3x to get 9x. Then, Greg was stuck for a moment and Jane went back to the problem before using the same gesturing process for distribution.

Jane: You had the right thing here [POINTING to 9x]. You had 3 × 3x. That was correct. But, we have more to put down. Keep going with this part [POINTING to the 3 and the 3x] what will that be?
Greg: 9x (quickly)
Jane: What's this? [with an ARCHING gesture from the 3 to the 5] 3 × 5…
Greg: 15…

Then, Jane and Greg worked together to solve the rest of the equation. On the next problem, $3(x + 5) = 33$, Greg worked quietly for about a minute. Greg had appropriately distributed the 3 to the x. But, he seemed stuck at that point.

Greg: So, 3 plus…

Jane: Times… It's going to be 3 times 5. So, you gave this 3 to the x [ARCHING gesture again]. Give this 3 to the 5. We are going to do the same process.

Greg then worked quietly to set up the equation as $3x + 15 = 33$. They worked together to solve the equation. Greg seemed to do better with distributing terms that were right next to each other, such as the 3 and x with $3(x + 5)$, but struggled to connect the 3 and the 5. He was certainly at a beginner's level with these problems and still needed to improve, but the tutor seemed to be able to use gesturing and some other interventions to help Greg begin to build his understanding of distribution. Interestingly, seeing the numbers right next to each other seemed to cue Greg as he began to develop some understanding as opposed to the in the beginning when he needed that part explained to him. However, he seemed to have trouble processing across the equation to the far term, the 5 in the $3(x + 5)$, but gestures, along with verbal reminders, seemed to help him make the connection.

While it is impossible to measure the impact of gestures in comparison to the other prompts given by Jane, it does seem that gesturing was a valuable tool for Jane as she explained distribution to Greg as he began to develop some preliminary understanding of the concept (Goldin-Meadow & Alibali, 2013). It is also important to consider the role that gesture may have played in Greg's struggles to process the information in the equation; working memory-related challenges were likely to have impacted Greg in this multi-step, novel problem context and gesturing was potentially a key facilitator as he engaged in mathematics that was challenging for him (Barrouillet et al., 2007; Cook et al., 2013; Swanson & Beebe-Frankenberger, 2004)

Teacher Use of Gestures to Communicate about Spatial Relationships

Lisa was a student with LD who often worked independently with the use of scratch paper and self-created visuals to support her working memory as she worked through problems. She often was able to work through problems with little support. Sophia, a special education undergraduate was tutoring her in the context of the Pythagorean Theorem. During the first teaching session, Lisa easily solved for unknown sides of right triangles using the Pythagorean Theorem, used the Pythagorean Theorem to prove an obtuse triangle was not a right triangle, and solved a Pythagorean Theorem word problem about a ladder leaning against a building. Eventually, a multistep, word problem that required three-dimensional use of the Pythagorean Theorem caused some difficulty for Lisa and Sophia intervened. The problem (see Table 1, row 3) was about fitting a long pole in the back of a loading truck, but the provided diagram consisted only of a rectangular prism with some labeled dimensions; so, Sophia drew some diagonals on the diagram, labeled more of the dimensions, and provided explicit explanations of the mathematical relationships within the problem.

Sophia: (Reads problem and draws in diagonal for the pole.) This will be your pole (as she draws a diagonal within the rectangular prism). And they want to know if there is enough space to fit it diagonally like that. This is not like the other ones. It's harder.

Lisa: Is it still Pythagorean Theorem?

Sophia: It is still Pythagorean Theorem. Do you have an idea of how you might solve it?

Lisa: Yeah, by drawing a picture of it like out of the truck. (She drew a triangle using dimensions given, but not the right dimensions.)

Sophia: [POINTS out where the diagonal she would be solving for using her chosen dimensions would be.]

Lisa: (Starts to guess again using a different combination of dimensions.)

Even with the semiconcrete visual and verbal supports, Lisa continued to struggle. Accordingly, Sophia offered more intensive support as she moved Lisa forward through the process utilizing the diagram as a visual representation and enhancing this representation by drawing diagonals and labeling dimensions.

Sophia: Let me give you a hint. You want to use the 5 and the 4. Because, do you see this dotted line here [POINTING to the line]?

Lisa: Yeah, is that supposed to be the way it is farthest in the truck?

To help Lisa gain a better understanding of the space of the truck, Sophia used her arms to block out the space of the prism, creating a visual of what the truck would look like in three-dimensions and showing where the pole (i.e., the triangle side they were ultimately solving for) would be in relation to the prism (see Table 1, row 3).

Sophia: Yeah, that line is going to be, if you are in the truck, it's going to be the distance. If this is our truck [MOTIONING ALONG EDGES of a rectangular prism with her hands], it's going to be here to here on the floor if you laid the pole straight down. Because what we're going to do is make that triangle in there (draws in triangle) 5 × 4 and this will be our hypotenuse. And, then what we're going to do when we know what this is…We're going to be able to make this triangle (draws in second triangle) right here. So, this will then be a leg. We can fill in 8 for this side, and then this will be the hypotenuse for the pole. So, it's like a double Pythagorean Theorem problem.

Lisa: Oh man.

Sophia: So, can you kind of see what I'm saying?

Lisa: Yeah.

Sophia: Okay, we will talk through it. So, first we will have to find the floor diagonal. So, our triangle is going to have 5 and 4 as the legs and we will find the hypotenuse.

Lisa: It's 5 and 4 as the legs?

Sophia: Okay, perfect… So, now we know that the floor diagonal is going to be 6.4. So I'm going to fill it in here. Can you draw a picture of our next triangle for the second Pythagorean Theorem?

Lisa: Is the next one going to be in here?

Sophia: It's going to be here [POINTS to the line in the picture]. So, we're going to have the height of the truck, the floor diagonal, and then this, where the pole will be [POINTS to picture].
Lisa: Oh! Okay. (Then, she works quietly to check her work).
Sophia: (After some wait time…) Okay, so C… C is this diagonal. The bottom-to-top opposite corner… And it's going to be 10.24 feet. So, is a 10 foot pole going to fit back there?
Lisa: I'm not sure yet, because we don't know the whole truck.
Sophia: So, what we have here is going to be… If this was our truck, and you'll have to visualize that it's taller. From this corner to this top corner, that's what we have. [MOTIONING WITH HANDS along edges to represent edges of the truck using the desk… MOTIONING WITH HANDS to show that the space of the desk was used as the flat base of the truck with imaginary sides coming up around the desk and POINTING to her ARM to show the diagonal from the front right corner of the base to the back left top corner.] That's what the 10.24 is going to be.
Lisa: Oh, so that's going to be the whole answer.
Sophia: So is a 10 foot pole going to fit back there?
Lisa: Yeah, I think so.
Sophia: Yeah, why? You're right. I just want to know why…
Lisa: (Silence for few moments)
Sophia: Okay, so you know that this diagonal here is 10.2 ft. And your pole is 10 ft.
Lisa: Oh… It will fit.

Infusing gesturing into the conversation, supported by semiconcrete representation, was effective for helping Lisa make some sense of the problem (i.e., not a thoroughly developed understanding, but an emerging understanding of the concept). The following day, Lisa was given a similar three-dimensional Pythagorean Theorem problem; however, it also required an additional step at the end (see Table 1, row 4).

After Lisa solved for an irrelevant piece of the diagram (i.e., the hypotenuse of a right triangle that was not needed for finding the answer), Sophia intervened by utilizing the representation and gesturing:

Sophia: So, we found from this front corner to here [using the diagram on paper and POINTING to the back, right, top corner and then to the back, left, top corner of the prism] and we need to find to this back corner and further.
Lisa: Oh… Ok, we have to use 4 and, oh, yes.

Sophia gave a hint to start by labeling the picture to get Lisa started. Then, Lisa proceeded to set up the problem and solve for the unknown hypotenuse (i.e., where the straw would need to sit in the carton). Lisa thought she had found the answer to the problem, not realizing the straw extended further than the box.

Sophia: You found this, but the straw comes up more so what do we have to do next?
Lisa: Is it *times* 1.5?

Sophia: Would you times it if it was just [POINTS to additional piece of straw SLIDING her finger out from the corner of the box; see Table 1, row 4]?
Lisa: Oh… Add (nodding her head forward with a knowing look on her face)
Sophia: Yes

Along with some significant prompting, gesturing seemed to help Lisa process the information in the problem and visualize what was necessary to complete the problem. In this problem, Lisa needed to be able to visualize a hypotenuse that was not readily visible on the paper; gesturing is often quite useful for teachers and students in these situations (Alibali & Nathan, 2012). Lisa needed to visualize sides and lines and distinguish between parts of lines in these problems; the tracing of lines and pointing to key elements in tandem with strategic use of language was, as in other studies, useful for Lisa (Alibali et al., 2013). She was able to apply what she had learned about Pythagorean Theorem problems in three-dimensional space, but she seemed to need the gesture to help her understand the extension of the hypotenuse outside of the prism. In this case, gesture provided some access to the concepts of spatial relationships that were difficult for Lisa based on her current skill level in mathematics.

Discussion

Our findings indicated that the participants seemed to benefit from their own use of gestures while thinking and communicating about mathematics as well as from observing the tutors' gestures when the tutors were explaining a concept (see Cook et al., 2013). The gestures appeared to be most useful for students when they were developing an emerging understanding of a concept or in situations when working memory was likely to be taxed due to multi-step demands of tasks or when several pieces of information needed to be processed, stored, and integrated (Baddeley, 2003; Barrouillet et al., 2007; Goldin-Meadow & Alibali, 2013; Swanson & Beebe-Frankenberger, 2004). Yet, many factors contributed to the students' ability to move through the different problems. The tutors relied heavily on diagrams and equations displayed on paper as visual supports for memory and processing. There were also many instances when the tutors provided support for cognitive and metacognitive processes while solving and discussing the problems. However, in addition to these research-supported, teaching strategies, gesturing seemed to provide extra support for the learners that were not readily available through pencil and paper and verbal prompts (Gersten et al., 2009; Rasmussen et al., 2004).

Gesturing by Students and Teachers during the Explanation Process

Students utilized gesture to explain concepts to each other. As Luke made sense of the concept of reciprocals and when he explained it to Henry, gesturing seemed like a natural way for Luke to supplement his thinking processes and support Henry's learning. As described by Rasmussen and colleagues (2004), gesturing seemed to help Luke with "expressing, communicating, and reorganizing" his thinking process (p. 319). Gesturing was not something that occurred separately from other means of communication, but rather it was a potentially crucial supplement to the mathematical conversations that were taking place (Goldin-Meadow & Alibali, 2013). The concept was made more accessible, and therefore less likely to create an unnecessary strain on working memory (see Goldin-Meadow et al., 2001) as new concepts were being learned and applied within challenging math problems.

The tutors also relied on gesture to communicate mathematical concepts. In other situations, when paired with other means of visual representation, such as equations and diagrams, gestures seemed to facilitate the students' thinking processes more easily. For example, when Lisa seemed to be struggling with processing the information in multi-step, Pythagorean Theorem problems, gesturing seemed to provide spatial support when diagrams and equations on paper Lisa were not quite enough support for her; the multimodal nature of communication seems to provide the support Lisa needed in this challenging situation (Goldin-Meadow & Alibali, 2013). Greg had previously done quite well with solving equations until the extra, novel idea of distribution added new difficulty to the problems for him. When his tutor began gesturing, Greg began to make progress in developing his understanding of distribution. At the end of the sessions described, both Lisa and Greg needed more work to become proficient with those concepts, but gesturing seemed to be a key component that allowed them to persist with the problem rather than becoming overwhelmed with frustration.

Greg and Lisa were engaging in problems that were quite difficult for them. In previous cases, these students had not relied on gestures and the tutors had not needed to gesture extensively to support them. Yet, when the students were engaging in multi-step problems that were novel and difficult (see Barrouillet et al., 2007; Swanson & Beebe-Frankenberger, 2004), these students appeared to struggle with memory and processing. As supported by psychology research, gesturing seemed to ease the demand on working memory as students attempted to coordinate the multiple pieces of information (Cook et al., 2013; Swanson & Beebe-Frankenberger, 2004)

In general, a combination of effective techniques seemed to be helpful for the students as in other special education studies (e.g., Scheuermann et al., 2009). Purposeful conversations guided by visual representations have a history of working for students with LD in research studies. Conceptual diagrams to support students' thinking processes have been effective for word-problem solving (Jitendra et al., 2009; Xin et al., 2005). Strategic organization of students' thinking processes, as well as diagrams, have also helped students with LD engage in challenging mathematics (Montague et al., 2014; van Garderen, 2007). The role of gesturing in tandem with verbal description as a support for communication may be a key support for students with LD as they engage in challenging mathematics in gatekeeper courses. According to Goldin-Meadow and Alibali (2013), "A teacher's inclination to support difficult material with gesture may be precisely what their students need to grasp challenging material" (p. 275). Also, as Rasmussen and colleagues (2004) recommended for students in general education settings and otherwise, the potential benefits of exploring gesturing as a tool for both teachers and students when explaining math concepts may yield findings that could be applied to interventions for students with LD.

Limitations and Future Research

The descriptive findings of this study provide some insight into how a combination of visual scaffolds (i.e., gesturing, diagrams, and equations) supports the problem solving and justification process of students with LD while engaging in secondary mathematics. However, the limitations of this study warrant more studies that expand upon these findings. For instance, in this study, the instruction was delivered in a one-on-one or small group setting. Future studies should examine the experiences of students similar to the participants in large-group settings such as inclusion classrooms. Additionally, while conducting a microanalysis of six sessions of three students' work provided some noteworthy and potentially beneficial information, future studies should be focused on the patterns exhibited by students with LD over longer periods of time and with more students with LD who might exhibit different tendencies in different situations.

The findings of this qualitative, microanalysis are not generalizable. However, these findings may provide foundational knowledge for future research studies regarding the use of gesturing in a variety of situations for students with LD or other students who are struggling with mathematics. In general, gaining a more thorough understanding of how students with LD problem solve in more complex mathematical contexts will be essential for helping students meet the demands of high school mathematics (Foegen, 2008; Maccini et al., 2007; Powell, Fuchs, & Fuchs, 2013).

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