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The effect of visuals on non-routine problem solving success and kinds of errors made when using visuals

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This study aims at determining to what extent visuals contribute to success in non-routine problem solving process and what types of errors are made when solving with visuals. Comparative model was utilized for identifying the effect of visuals on student achievement, and clinical interview technique was used to determine the types of errors. In the study, 370 primary fourth-grade students were applied with the four non-routine problems; first in verbal form, then in visual form by changing only the numbers without changing the operational stages that should be used when solving the question. As a result, it was seen that the use of visuals decreased the number of unanswered questions by 11%, and increased the number of correctly answered questions by 12%. It was found at the end of the study that when students utilized the visuals they drew rather than the given visuals, this contributed to the problem solving achievement more. Another finding achieved in the study is that the students made errors originating from lack of knowledge, misinterpretation, incorrect structuring, incomplete structuring, and misplacement of what is given respectively.

Key words: Primary school, mathematics, non-routine problems, use of visuals, problem solving success.

INTRODUCTION

There is a significant emphasis on the need for building a link between mathematics education and real life in today's mathematics education programs (Yağcı and Arseven, 2010). Kaur and Yeap (2009) emphasize that the most important factor that has enabled Singapore to remain among the top in international mathematics evaluation studies is its mathematics education focusing on problem solving, and the attention paid while choosing problems. Kaur and Yeap (2009) state that when the subject has first started to be taught, exercise-type problems are preferred more whereas after the subject has been learnt, problems that have been prepared upon real-life situations are preferred more.

Altun (2005) defined exercise-type problems as routine while those requiring real-life knowledge as non-routine, and gave the sentence *"If Ali buys two pencils, paying 3 TL for one, how much does he pay total?"* as an example for routine while he gave the sentence *"If one pays 3 TL to divide an iron bar into two, how much does s/he pay to divide it into four?"* as an example for non-routine problem. When the answers for both questions were analyzed, it was seen that most of the students gave the

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answer “3x2=6” to both questions (Ulu, 2011), but in fact, it was expected that the students were going to reach the result “3x3=9” in the second question thinking that 3 cuttings would be required to divide the iron bar into 4.

In this context, while operational skills are required to solve routine problems, for non-routine problems, operational skills are also required but not sufficient. While solving non-routine problems, data should be organized, classified and adapted to real-life (Yazgan and Bintaş, 2005; Altun, 2005).

Research reveals that primary school students make a lot of errors because they solve non-routine problems like routine ones (Verschaffel et al., 1999; 2000; Xin et al., 2007; Ulu 2011; Çelik and Güler, 2013). This has caused students’ solving approaches to be examined. Hegarty et al. (1995) argued there are two general approaches to understanding mathematical word problems that have been introduced by previous researchers: "a short-cut approach" and a "meaningful approach" that is based on an elaborated problem model. In the short-cut approach, which refer to as direct translation, the problem solver attempts to select the numbers in the problem and key relational terms and develops a solution plan that involves combining the numbers in the problem applying the arithmetic operations that are primed by the keywords or cues (e.g., addition if the keyword is "more" and subtraction if it is "less").

Thus, directly the problem solver attempts to translate the key propositions in the problem statement to a set of computations that will produce the answer and does not construct a qualitative representation of the situation described in the problem. In the meaningful approach, which refers to as the problem model approach, the problem solver translates the problem statement into a mental model of the situation described in the problem.

Short-cut approach is defined as how students use readily available solutions in their memories when they are confronted with a problem similar to a number of problems they have solved before (Jitendra and Hoff, 1996; Steele and Johanning, 2004; Viennot and Moreau, 2007). This theory assumes that solving different kinds of questions will enhance the problem solving achievement as it will increase the number of solution methods stored in the memory. Yet, when the schema is changed a little bit or students confront with a new schema, this theory may fail to solve the problem (Viennot and Moreau, 2007). The readily available solutions mentioned in the short-cut approach are addressed within the scope of procedural knowledge (Anderson, 2010; Brynes and Wasik, 1991; Baroody et al., 2007).

The fact that readily available solutions or procedural knowledge do not work in every situation was revealed in the studies performed by Viennot and Moreau (2007) and Soylu and Soylu (2006). In their study, Viennot and Moreau (2007) first asked the problem “*Luke comes to the school with 15 marbles with him. He plays two games, loses 7 marbles in the first one. Luke, who is a good player, earns 34 marbles at the end of the second game. If Luke has lost the second game, how many marbles has he lost; if not, how many has he earned?”*, then asked it again after changing the phrase *“Luke, who is a good player,”* into *“Luke, who is a bad player,”*

At the end of the study, they found that some students who gave the answer *“He has earned 26 marbles”* to the question with the phrase *“Luke, who is a good player”* answered the question with the phrase *“Luke, who is a good player,”* with *“He has lost 26 marbles.”* These errors were also observed in the study performed by Soylu and Soylu (2006). In the study, the question *“Ali has 10 apples left after giving 5 of them to Ayşe. So, how many apples has Ali had in the first place?”* was answered “10 - 5 = 5” by most of the students. It was determined as a result of the interviews that over regularization of the phrase “apples left” caused the students to perform subtraction instead of addition. It can be said that the reason why the students made errors is the overreguralization of the schema-based theory (if you see the “more”, add; if you see the “less”, subtract); similar situations are observed in the studies of Panasuk and Beyranevand, (2010), Moreno and Mayer (1999) and Hegarty et al. (1995) and Marshall (1995).

In their study, Baki and Kartal (2004) concluded that the students with low levels of problem solving achievement first watched out if the problem resembled other problems they had previously solved while solving it, and they gave up solving it when they could not found any resemblance or achieve irrational results.

In the studies performed by Anderson (2010), Brynes and Wasik (1991) and Baroody et al. (2007), it was observed that the procedural knowledge may fall insufficient in the first-time situations and contextual knowledge is needed for such situations. Kieren (1993) and Baroody and Lia (2007) defined conceptual knowledge as associating recently-learned knowledge with previously-learned knowledge and real life, constructing the knowledge in accordance with individual traits and processing it through a rational sieve.

The studies conducted by Schoenfeld (1991), Verschaffel et al. (1999, 2000) and Xin et al. (2007) shows that students achieve results which are mathematically correct but do not agree with real life since they cannot associate the procedural result they find with real life. To give a reason for the case, it was explained that procedural thinking alone was not enough for sense-making and students made errors due to not thinking conceptually. Problem solving process is defined requiring multiple skills together. The constituents of this process are listed as

1. Problem comprehension
2. Choosing the required information among the data
3. Converting this information into mathematical symbols

and
4. Reaching a solution through required operations (Mayer, 1985; Artzt and Thomas, 1992; Hong, 1995; Morales, 1998).

Mayer (1985) explains the first three stages of this process under "problem representation" title and the fourth stage under "problem solution" title. According to Mayer (1985), problem representation is composed of two substages: problem translation, which relies on linguistic skills needed to comprehend what the problem is saying, and problem integration, which depends on the ability to mathematically interpret the relationships among the problem parts to form a structural representation. In representation stage, one is expected to convert the problem situation in verbal form into mathematical symbols by using problem solving strategies (Van Garderen and Montague, 2003).

Healy and Hoyles (1999) explain mathematical representation as converting a given expression into other expressions without damaging the original form and distinguishing it from other expressions that bear no resemblance to it. Moreno and Mayer (1990) explain representation as a process to convert problem sentence in verbal form into mathematical structures by using visuals and symbols separately or together.

Moreno and Mayer (1990) emphasized that it is a one-way representation when internal representation is externalized using only one technique and it is multi-way representation when it is externalized using multiple techniques, the interpretation increases when the problem solving process is diversified (symbolical, visual, verbal), and this, in parallel, increases the problem solving achievement.

In their study, they revealed that students using the multi-way representation (symbolical, visual, verbal) were more successful than students using the one-way representation technique (verbal). It can be said that when no transition is done between multi-representations, mathematics cannot be comprehended in a conceptual way (Ainsworth, 1999; Meij et al., 2006). Panasuk and Beyranevand (2010) designed the problem situation in three different ways (symbolical, visual, and verbal), and examined students' reactions to the different structures of the same problem in their study. As a result, the successful students stated that all three situations represented the same thing differently while the unsuccessful ones stated that each situation was different and each situation could be represented in one way.

Polya (1990) emphasized visual representation because it facilitates comprehending problem situation and choosing decisive procedures. Halmos (1980) advocated the opinion "to be a scholar of mathematics you must be born with the ability to visualize" and thus spread this emphasis to all fields of mathematics. Lean and Clements (1981) defined visual imagery as "imagery which occurs as a picture in 'the mind's eye'". Drawing pictures on paper while comprehending the word problem might help less successful problem solvers in the process of building effective problem representations (Bryant and Tversky, 1999). Numerous psychological studies prove that the representation of mathematical objects and pictorial representations play an important role in the learning process. Illustrations facilitate a better understanding of concepts or problem situations and they help develop mathematical reasoning. Therefore, there is a need for examining students' skills to do multi-representation and difficulties arising in the process.

Diezman (2005), Duval (2002) and Gagatsis and Demetriou (2007) emphasized the importance of visuals in the development conceptual thinking. According to them, use of visuals in problem solving process has advantages such as integrating the information given in the problem text by associating them, facilitating the realization of similarities between the problems solved before and the newly confronted problems, and contributing the mental development due to ensuring organization in the mind. Ainsworth (1999) stated that visuals used in mathematics classrooms should be used to understand and solve the problem text, and Gyamfi (1993) stated that individuals make two types of representations (internal and external) in the problem solving process and the internal representation can become tangible and observable only by using the external representation techniques.

Duval (2002) emphasized that for students to orientate towards one-way representation when understanding a problem and describing each problem in the same way should not be appropriate in terms of the structure of mathematics. In the study performed by Elia et al. (2007) with primary first-, second- and third-grade students, an achievement test composed of routine problems was asked to the students first in written and then by using visuals, and the effect of visuals on the problem solving achievement differed by each problem in the end. Pape (2004) found that the rate of benefiting from visuals in the solutions found by students with higher problem solving achievement was high.

In the studies performed by Ulu (2008) and Olkun et al. (2010), it was determined that the students tend to solve non-routine problems without using visuals by transforming the verbal situation into symbolical expressions in general. In the study conducted by Uesaka et al. (2007), it was seen that for individuals to create their own visuals rather than using the readily available ones contributed to their problem solving achievements more. Yet, it was found in the study by Altun and Arslan (2006) that majority of the students did not prefer using visuals in their solutions in spite of the education given.

Non-routine problems that needed to be solved with conceptual skills were emphasized in this study rather than procedural skills. It is seen that the results will not coincide with the real-life conditions when the questions used in the study are solved with procedural knowledge, thus it is required that the students need to resort to their conceptual knowledge for a solution. In this context, visuals were utilized in the study to activate students' conceptual knowledge.

Elia et al. (2007) stated that problems can be solved without using visuals, but visuals may help students in the problem solving process. The studies conducted by Viennot and Moreau (2007) and Soylu and Soylu () revealed that short cut approach may fall insufficient in the problems requiring conceptual knowledge and students achieved faulty solutions by using their procedural knowledge. Diezman (2005), Duval (2002) and Elia et al. (2007) stated that visuals may help the conceptualization when solving problems, but many errors may arise while converting verbal form into visuals. In this context, this study aims to determine the effect of visuals on problem solving success and what types of errors are made when solving with visuals; therefore, answers were sought to the following questions:

1. Are there any differences between the numbers of correct answers given to the verbal form and visual form of the problem solving achievement test for primary fourth-grade students?
2. What types of errors do the primary fourth-grade students make while converting problem sentence in verbal form into visuals?

METHODOLOGY

The research model

The first research question, “Is there any differences between the numbers of correct answers given to the verbal form and visual form of the problem solving achievement test for primary fourth-grade students?” was answered with comparative survey model. The survey model aims to reveal a current situation as it is, and the researcher cannot have any manipulative influence in this model (Karasar, 2002).

The second research question, “What types of errors do the primary fourth-grade students make while converting problem sentence in verbal form into visuals?” was answered with the clinical interview method. Piaget argues that errors made by children provide important information on their nature of thinking and it is necessary to use the clinical interview method, which is a method of asking flexible questions, to explore the richness of students' thoughts and evaluate the cognitive skill (Karataş and Güven, 2004).

According to Frederiksen et al. (1990), standard tests can only determine to what extent the students solve the problem correctly or incorrectly but do not question why they do it correctly and what to do so that they achieve the right result. Karataş and Güven (2004) regard the clinical interview method as one of the measurement methods that can be used for evaluating the problem solving skill and think that the reason for the errors made by students when solving problems can be revealed as they are solving it. Hunting (1997) stated that the clinical interview method is dynamic and the errors made by the student are identified rather by him/her. Goldin (1998) indicates that one of the purpose of using clinical interviews is to have information about individual’s mathematical behaviours through problem solving.

Study group

The study group of the research was gathered in two different ways. The cluster sampling method was used for the survey model. Karasar (2002) proposes to select the whole group by using cluster sampling method when the members in the population do not have the chance to be selected one by one. In cluster sampling method, equal selection chance is not for a member but for the whole group with its members.

In this context, while deciding the study group, not individuals but classes are selected. First, the schools were divided into three groups (high, moderate, and low) based on the scores of TEOG (Transition from Primary to Secondary Education) exams, and a school of each group was chosen with the unbiased selection method. 370 fourth-grade students attending the 18 classes of the primary schools chosen were applied with the problem solving achievement test. The sample is composed of 196 female and 174 male students. The study group of clinical interview was selected using maximum variation sampling method of purposeful sampling.

According to Patton (2002), purposeful sampling enables examining deeply the cases thought to have rich information about the research question, while maximum variation sampling is used to increase variety of responses individuals give to the research question. In the study, purposeful sampling was preferred because the study group to whom clinical interview was conducted focused only on the students who made errors and maximum variation sampling was preferred because as different as possible incorrect solutions were examined among students who made errors.

When determining the study group which would be subjected to the clinical interview, first of all, the errors made in each question were encoded and then 5 students were interviewed for each code. As a result of the encoding, since 9, 7, 8, and 10 error codes were determined for the first, second, third and fourth question respectively, the interviews were performed for 170 solutions (5x(9+7+8+10)) in total. Among the students subjected to the clinical interview, interviews were conducted for only 1 question with some students, for 2 questions with some students, for 3 questions with some students and for all questions with some students. In this context, the study group for clinical interviews was composed of 58 students (36 female, 22 male) in two primary schools at Kütahya city center at moderate level according to scores in TEOG exams.

Data collection tool

First of all, a problem solving achievement test was developed to classify the errors made by the students in the research. The problem solving achievement test is composed of 4 word problems used in the studies performed by Ulu, (2011), Altun (2005) and Yazgan and Bintaş (2005). The scope validity of the questions in the scale was achieved with an expert opinion.

The validity and reliability study of the problem solving achievement test used in the study was performed on 117 fourth-grade students attending the school having the closest score to the Kütahya average based on 2014/2015 YEP (Placement Scores). Primarily, the item difficulty and item distinctiveness of each question were calculated for determining the validity.

According to Tekin (1997), items with an item difficulty index varying between 0 and 1 and difficulty indexes varying between

Table 1. Verbal form of the problem solving achievement test.

1. 2 liras are paid to cut an iron bar into 2 pieces. How many liras are paid to cut it into 4 pieces?
2. A ball dropped from high bounces the distance half of the height it has been dropped from. Now that the ball bounces 10 meters on the fourth bounce, from what height has it been dropped?
3. Trees will be planted at intervals of 10 meters on both sides of a 50-meter-long road. How many trees will it take to plant along the road?
4. A snail is put in a jar with its lid open. The height of the jar is 9 cm. If the snail which climbs up 3 cm every day slides back down 1 cm, in how many days can it get out of the jar?

0.30 and 0.70 are at the moderate difficulty level. The item difficulty indexes of items in the scale vary between 0.38 and 0.54; this indicates that all of the problems in the test have moderate difficulty. The distinctiveness index varies between -1 and +1 and this value being 0.40 and higher shows the distinctiveness of items (Tekin, 1997)**.** The distinctiveness indexes of items in the test vary between 0.41 and 0.67; this indicates that all of the items are distinctive. Secondly, KR 20 coefficient was calculated for determining the reliability and found 0.76. Kr 20 value being 0.70 and higher shows high reliability for the test, thus showing reliability of the test sufficient (Büyüköztürk, 2002). The verbal form of the scale is given in Table 1.

The visual form of the scale was obtained by changing the numbers in its valid and reliable verbal form, and drawing the illustrations right under the questions. The visual form of the scale is given in Table 2.

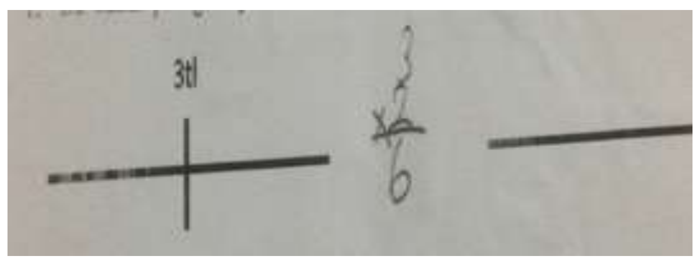
Collection and analysis of the data

Since 15 min were enough for solution during the pilot application of the problem solving scale, this duration was taken as reference in the real application. Firstly, the verbal form of the problem solving achievement test was applied, the questions answered correctly, incorrectly and left blank were identified. After a one month interval, the visual form of the achievement test was applied and the questions answered correctly, incorrectly and left blank were identified for this form. Each incorrect answer was encoded and clinical interviews were made with five students who had made each error. The clinical interviews were examined and categorized by three classroom teaching mathematics experts who completed PhD in problem solving skills in classroom teaching mathematics education. The types of errors are given in Table 3. The clinical interviews were classified by the error analysis inventory while the kappa coefficients were looked up to determine the correlation between the experts. The data obtained from the kappa coefficient are interpreted as "Weak Correlation=< 0.20; Acceptable Correlation=0.20-0.40; Moderate Correlation=0.40-.60; Good Correlation=0.60-0.80; Very Good Correlation=0.80-1.00" (Şencan, 2005). The kappa coefficients were found to be 0.93 for incorrect rating/proportioning, 0.91 for lack of knowledge, 0.97 for misinterpretation, 0.94 for incomplete structuring, 0.91 for incorrect structuring, and 0.92 for misplacement of what is given. These values enabled us to see that the experts exhibit very good correlation in the classification of errors. The errors were categorized in accordance with the expert opinions before the frequency and percentage analyses.

FINDINGS

Here, answers were sought to the research problems for each question and the whole test. In this context, the descriptive analyzes of the answers given to the verbal and visual forms of the problem solving achievement test are given in Table 4.

According to Table 4, blanks for question 1 decreased with the use of visuals and the number of correct answers increased 17%. This can be said to arise from the fact that reaching a solution by using visuals increased solution rate and correct solution rate. In the next phase of the study, the solutions of 175 students who gave the incorrect answer when solving the visual form of the first question are examined. The first error type determined upon clinical interview in first question arose from solution without using the illustration and can be seen in Interview 1.



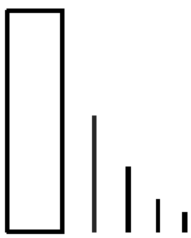
T: What is asked of us in the question?
S: 3 liras are paid to cut an iron bar into 2 pieces; it asks us to find how many liras will be paid to cut it into 4 pieces.
T: Can you explain your operation?
S: I multiplied 3 by 2 to get 6.
T: Why did you do that?
S: Because, if 3 liras are paid to cut it into 2 pieces, 2 times more money is paid to cut it into 4 pieces. That's why.
T: There is an illustration above. Why didn't you use it?
S: I didn't feel the need because I had the answer in my mind.
T: You used the illustration in some of the other questions?
S: The illustration help when I don't understand the question but I understood this question right.

As seen in Interview 1, the students who made this error rated/proportioned directly without touching the given

Table 2. Visual form of the problem solving achievement test.

2 liras are paid to cut an iron bar into 3 pieces. How many liras are paid to cut it into 4 pieces?

A ball dropped from high bounces the distance half of the height it has been dropped from. Now that the ball bounces 5 meters on the fourth bounce, from what height has it been dropped?



Trees will be planted at intervals of 5 meters on both sides of a 40-meter-long road. How many trees will it take to plant along the road?

A snail is put in a jar with its lid open. The height of the jar is 16 cm. If the snail which climbs up 4 cm every day slides back down 1 cm, in how many days can it get out of the jar?

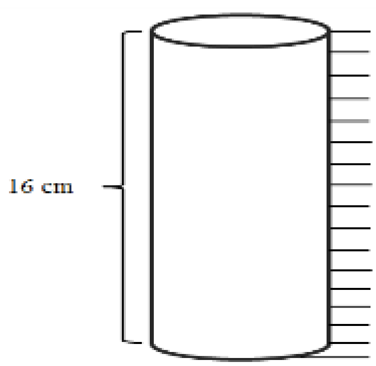


illustration but they also could not see that the rate/ proportion might contrast with the real life. The student justified not using the illustration with the fact that he/she had found the answer well in advance. There are 45 students who gave the incorrect answer to the question without using the illustration even though one was given in the visual form; the ratio of this error type to the errors made in the first question in general is 25.71% (45/175). There are 130 students who solved the visual form of the question incorrectly using the illustration. The second error type determined upon clinical interview in first question arose from lack of knowledge despite using the illustration and can be seen in Interview 2.

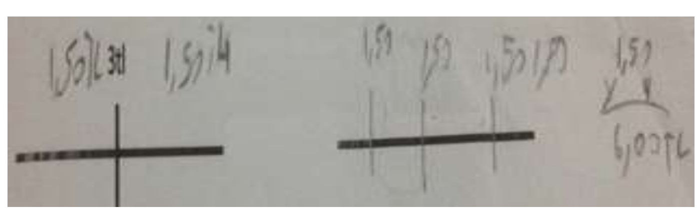


Table 3. Types of errors.

Types of error	Sample behaviors
Incorrect rating/proportioning	The students who made this error rated/proportioned directly without using the visual given in the problem
Lack of knowledge	The students who made this error drew their visuals incorrectly due to the lack of real-life or mathematical knowledge
Misinterpretation	The students who made this error structured their visuals correctly but misinterpreted them
Incomplete structuring	The students who made this error used their visuals correctly but drew them incompletely
Incorrect structuring	The students who made this error made incorrect illustrations on the given visual
Misplacement of what is given	The students who made this error used the visual correctly but could not communicate what was given in the problem question to their illustrations.

Table 4. Comparison of the numbers of correct and incorrect answers given to the verbal and visual forms of the first question.

Solution		Verbal form			Visual form		
		Correct	Wrong	Total	Correct	Wrong	Total
Blank	Frequency (f)	-	-	57	-	-	23
	Percentage (%)	-	-	15.40	-	-	6.21
Without using illustration	Frequency (f)	48	177	225	18	45	63
	Percentage (%)	21.33	78.67	60.81	28.57	71.42	17.03
By using illustration	Frequency (f)	62	26	88	154	130	284
	Percentage (%)	70.45	29.55	23.79	54.22	45.78	76.76
Total	Frequency (f)	110	203	370	172	175	370
	Percentage (%)	29.73	54.87	100.00	46.49	47.30	100

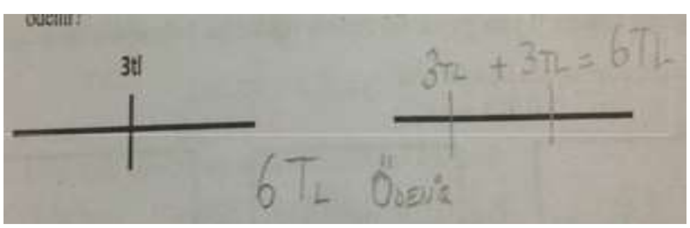
Not: There are 4 students who solved the verbal form of the question correctly without using illustration but solved its visual form incorrectly by using illustration.

T: Can you explain your solution on the illustration?
S: If 3 liras are paid to cut it into 2 pieces, 1.5 liras are paid for each piece. Then I drew 3 lines on the other illustration because it asks us to cut it into 4 pieces.
T: Has it been cut into 4 pieces now?
S: Yes. (The student counted the pieces in the second illustration.
T: What did you do next?
S: Since 4 pieces will make 6 if each piece is 1.5, I multiplied 1.5 with 4.
T: Why do you think the blacksmith will take money from us? For each piece he cut or for each cutting?
S: He will take 1.5 liras for each piece.
T: How much would we pay if he took it for each cutting?
S: We would pay 9 liras because we would cut it for 3 times (the student made the operation 3+3+3 verbally).

As seen in Interview 2, this type of error was originated not from the cutting operation applied to the iron bar but the requirement of a payment for each cut piece of the iron bar. It was seen that the first one of the incorrect solutions using the illustration originated from lack of knowledge and the ratio of this error type to the incorrect solutions using the illustration was 17.71% (31/130). The third error type determined upon clinical interview in first question arose from the misinterpretation of the illustration and can be seen in Interview 3.

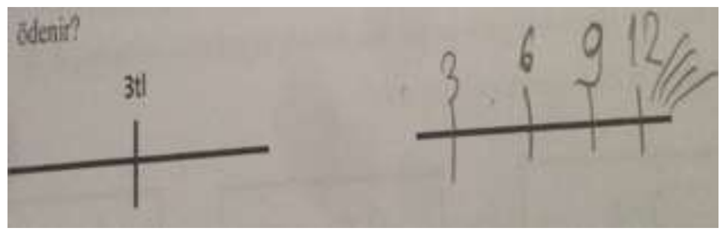
T: Can you explain your solution on the illustration?
S: It asks us to find how many liras we will pay to cut it into 4 pieces. And I cut the iron bar into 4 pieces.
T: Why did you write "3 liras" above each cutting?
S: Because we cut it once to cut into 2 pieces. It requires 3 liras.
T: How many times did the blacksmith cut it?
S: Three times.
T: How much will we pay then?
S:6 liras.
T: But it is 3+3+3=9 according to the illustration.
S: I didn't understand that because it should be 6.
T: Why?
S: If 3 liras are paid to cut it into 2 pieces, 6 liras should be paid to cut it into 4 pieces.

As seen in Interview 3, the students who made this error drew the illustration correctly but saw that the solution they structured in their mind did not coincide with the solution they found on the illustration. Consequently, they decided that the formulary answer they obtained was true despite the illustration they drew. The ratio of this error type to the incorrect solutions is 14.86% (26/130). The fourth error type determined upon clinical interview arose in first question from the drawing of incomplete illustration, and this can be seen in Interview 4.



T: Can you explain your solution on the illustration?
S: 3 liras were paid in the first illustration to cut it into 2 pieces; I cut the other illustration into 2 pieces once again. And I found that 6 liras were paid.
T: Can you look at your illustration, how many pieces was the iron bar cut into?
S: 3 (The student counted it verbally).
T: The question asks us to cut it into how many pieces?
S: Cut it into 4 pieces?
T: What do we need to cut it into 4 pieces?
S: To draw another line?
T: How much would we pay, then?
S: 9 liras but the answer should be 6 liras.
T: Why?
S: Because if 3 liras are paid to cut it into 2 pieces, 6 liras should be paid to cut it into 4 pieces.

As seen in Interview 4, the student tried to transfer the solution he/she structured in his/her mind but cut the iron bar into 3 pieces rather than 4 pieces. The student remained under the influence of his/her rate/proportion, too. The ratio of this error type to the incorrect solutions is 21.14% (37/130). The fifth error type determined upon clinical interview arose in first question from the drawing of incorrect illustration, and this can be seen in Interview 5.



T: Can you explain your solution on the illustration?
S: 3 liras were paid to cut it into 2 pieces in the first illustration; it asks how much we will pay if we cut it into 4 pieces. I cut it into 4 pieces and find 12 liras.
T: Is your illustration cut into 4 pieces now?
S: Yes.
T: Can you count? How many pieces are there?
S: 4.
T: Can you look at the first illustration. How many pieces are there after a cut?
S: 2.
T: How many pieces are there in the second illustration? How about count again?
S: 5, now but I should've cut it into 4 pieces.
(The student counted each piece verbally)
T: How many liras would we pay, then?
S: 9. I drew an extra line.
T: Why?
S: 4...

As seen in Interview 5, the student cut the iron bar into 5 pieces rather than 4 pieces. The ratio of this error type to the incorrect solutions is 14.29% (25/130). The sixth error type determined in the first question upon clinical interview arose from the drawing of incorrect illustration, and this can be seen in Interview 6.

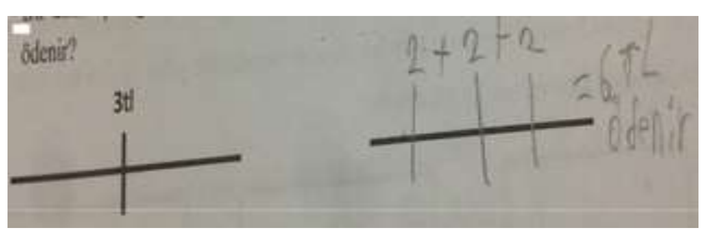


Table 5. Comparison of the numbers of correct and incorrect answers given to the verbal and visual forms of the second question.

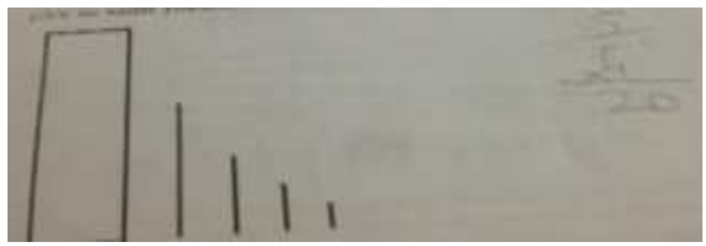
Solutions		Verbal form			Visual form		
		Correct	Wrong	Total	Correct	Wrong	Total
Blank	Frequency (f)	-	-	62	-	-	26
	Percentage (%)	-	-	16.76	-	-	7.03
Without using illustration	Frequency (f)	32	246	278	12	75	87
	Percentage (%)	11.51	88.49	75.13	13.79	86.21	23.51
By using illustration	Frequency (f)	16	14	30	87	170	257
	Percentage (%)	53.33	46.67	8.11	33.85	66.15	69.46
Total	Frequency (f)	48	260	370	99	245	370
	Percentage (%)	12.97	70.27	100	26.75	66.22	100

Note: There are 8 students who solved the verbal form of the question correctly without using illustration but solved its visual form incorrectly by using illustration.

T: Can you explain your solution on the illustration?
S: 3 liras are paid to cut it into 2 pieces; it asks how many liras we will pay if we cut it into 4 pieces.
T: What did you do, then?
S: I cut it for three times to cut it into 4 pieces. And found that we should pay 6 liras.
T: How many liras does it take to cut the iron bar once?
S: 3.
T: You wrote 2 liras down in your illustration. Did you do it by accident?
S: I thought the answer should've been 6. That was why I wrote 2 liras.
T: So, you knew it was 3 liras when you were solving the problem?
S: Yes, but the answer was 9 liras, then

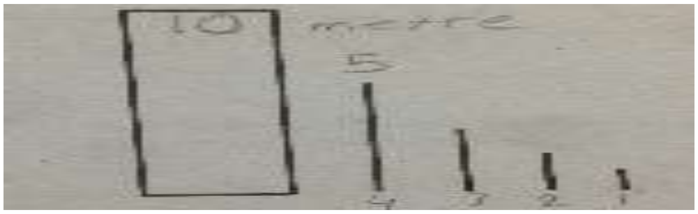
As seen in Interview 6, it was seen that the students who made this error structured the illustration correctly but manipulated what was given in the problem so that they could achieve the answer 6 in their minds. The ratio of this error type to the incorrect solutions is 6.29% (11/130). In the next phase of the study, the answers given to the verbal form of the second question were compared to the answers given to its visual form by the numbers of correct and incorrect answer, and the findings obtained are presented in Table 5.

According to Table 5, blanks for question 2 decreased with the use of visuals and the number of correct answers increased to 14%. This can be said to arise from the fact that reaching a solution by using visuals increased solution rate and correct solution rate. In the next phase of the study, the solutions of 245 students who gave the incorrect answer when solving the visual form of the second question and information on the clinical interviews performed with them are presented. The first error type determined upon clinical interview in second question arose from solution without using the illustration, and this can be seen in Interview 7.



T: What is asked of you in the second problem?
S: To find the height from which the ball was dropped?
T: Can you explain the operation you did with your reasons?
S: I multiplied 4 with 5 because the ball bounces for 4 times from the height from which it's been dropped.
T: Can you explain your operation a bit more?
S: If it bounces 5 meters in the fourth bounce, it bounces four times of the height in the first bounce.
T: Why didn't you use the illustration?
S: Because it's easier to solve it in this way.
T: You didn't find it necessary, then?
S: No.
T: You didn't use it in other questions, either? Why?
S: Because those questions were very easy, I didn't want to waste time.

As seen in Interview 7, it was revealed that the students who made this error used direct rate/proportion without benefiting from the illustration. There are 75 students who gave the incorrect answer to the question without using the illustration even though one was given in the visual form; the ratio of this error type to the errors made in the second question in general is 30.61% (75/245). There are 170 students who solved the visual form of the question incorrectly with using the illustration. The second error type determined in second question upon clinical interview arose from lack of knowledge which is presented in Interview 8.

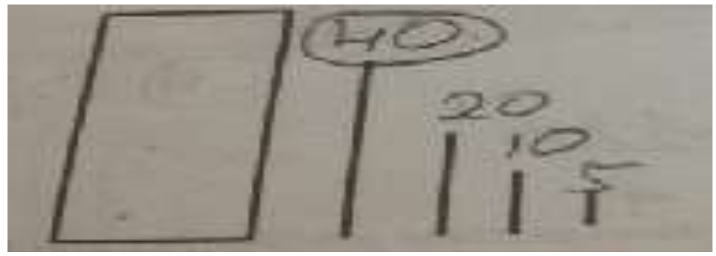


T: Can you explain your solution in the second question on the illustration?
S: It bounced 5 meters in the fourth bounce, I thought it was dropped from a height of 10 meters.
T: Where is the fourth bounce, you think?
S: There, one under which it writes four (The student pointed at the line before the rectangular shape).
T: How did you understand that?
S: I counted.
T: Can you count again and show me the fourth bounce?
S: The one before the rectangular (The student counted starting from the shortest line).

As seen in Interview 8, the students who made this type of error structured the illustration incorrectly as a result of the misperception of the concept of the fourth bounce. Actually, the fourth bounce is the shortest line, the students counted them from the end and perceived the first bounce as the fourth bounce. The ratio of this error type to the incorrect solutions is 26.12% (64/170). The third error type determined in second question upon clinical interview arose from misinterpreting illustration as seen in Interview 9.

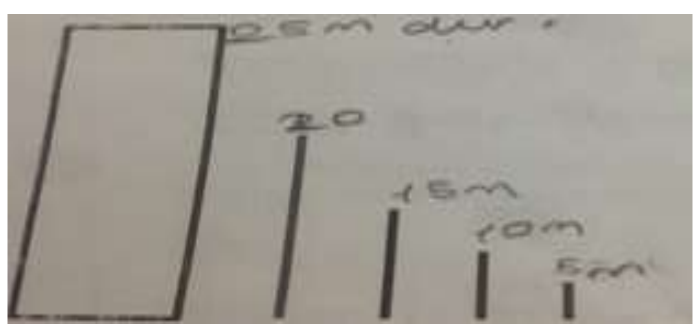
T: Can you explain your solution in the second question on the illustration?
S: I doubled it all the way starting from the fourth bounce. Next, I added them all and found 155.
T: Why add?
S: To find from which height the ball was dropped.
T: What do you think?
S: 155 meters.
T: Can you show me the height from which it was dropped on the illustration.
S: Here (showing the rectangular shape).
T: How high is that?
S: 80 meters.
T: Do you think your result is correct?
S: Wrong. I didn't have to perform the addition.

As seen in Interview 9, the students who made this error transformed what was given into the illustration in a correct format but gave the incorrect answer because they misperceived what was asked of them as the total height to which the ball bounces rather than the height from which it is dropped. The ratio of this error type to the incorrect solutions is 11.02% (27/170). The fourth error type determined in second question upon clinical interview arose from structuring the illustration incompletely and can be seen in Interview 10.



T: Can you explain your solution in the second question on the illustration?
S: The ball bounces two times less each time. It bounced 5 meters in the fourth bounce. I doubled all the way and found 40 meters.
T: What is asked of us in the question?
S: The height from which the ball was dropped.
T: Does your result shows the height from which the ball was dropped?
S: Yes.
T: From which height do you think the ball was dropped? Can you show me on the illustration?
S: From the height on which it writes 40.

As seen in Interview 10, the students who made this error doubled the height starting from the fourth bounce back to the first one but forgot calculating the height to which the ball first bounced. The ratio of this error type to the incorrect solutions is 15.51% (38/170). The fifth error type determined in second question upon clinical interview arose from misplacing what is given this can be seen in Interview 11.



T: Can you explain your solution in the second question on the illustration?
S: The ball bounced 5 meters in its fourth bounce; I doubled all the time to find 25.
T: Do you think you doubled all the time?
S: Yes.

Table 6. Comparison of the numbers of correct and incorrect answers given to the verbal and visual forms of the third question.

Solutions		Verbal form			Visual form		
		Correct	Wrong	Total	Correct	Wrong	Total
Blank	Frequency (f)	-	-	68	-	-	13
	Percentage (%)	-	-	18.38	-	-	3.51
Without using illustration	Frequency (f)	64	184	248	18	53	71
	Percentage (%)	25.81	74.19	67.03	25.35	74.65	19.19
By using illustration	Frequency (f)	22	32	54	130	156	286
	Percentage (%)	40.74	59.26	14.60	45.45	54.55	77.30
Total	Frequency (f)	86	216	370	148	209	370
	Percentage (%)	23.24	58.38	100	40.00	56.49	100

Note: Note: There are 8 students who solved the verbal form of the question correctly without using illustration but solved its visual form incorrectly by using illustration.

T: Shall we take a look? For example, did you double it from the fourth bounce back to the third one?
S: Yes. I increased it from 5 meters to 10 meters.
T: So, did you double it from the third bounce back to the second one?
S: No, 15 is not the doubled amount of 10.
T: How many meters should it be?
S: 20 meters.
T: Next?
S: Here should it be 40 (showing the first bounce). It should be 80 meters in the end.

As seen in Interview 11, the student who made this error were able to use the illustration correctly but misplaced what was given because s/he structured the concept of two times incorrectly. The ratio of this error type to the incorrect solutions is 16.74% (41/170). In the next phase of the study, the answers given to the verbal form of the third question were compared to the answers given to its visual form by the numbers of correct and incorrect answer, and the findings obtained are presented in Table 6.

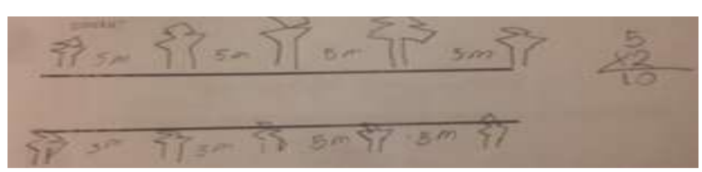
According to Table 5, blanks for question 3 decreased with the use of visuals and the number of correct answers increased to 17%.

This can be said to arise from the fact that reaching a solution by using visuals increased solution rate and correct solution rate. In the next phase of the study, the solutions of 209 students who gave the incorrect answer when solving the visual form of the third question and information on the clinical interviews performed with them are presented.

The first error type determined upon clinical interview in third question arose from solution without using the illustration as shown in Interview 12.

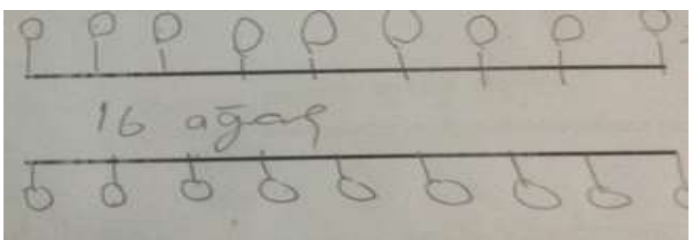
T: What is asked of us in the third problem?
S: To find the total number of trees to be planted?
T: Can you explain the operation you did with your reasons?
S: I found the number of trees to be planted by dividing 40 by 5.
T: Why didn't you use the illustration?
S: Because the question was so easy, I didn't find it necessary; I found the answer even without doing any operations.
T: Why did you use it only in the fourth question?
S: That question was a bit difficult.

As seen in Interview 12, the students who made this error used direct rate/proportion without benefiting from the illustration. There are 53 students who gave the incorrect answer to the question without using the illustration even though one was given in the visual form; the ratio of this error type to the errors made in the second question in general is 25.36% (53/209). There are 156 students who solved the visual form of the question incorrectly without using the illustration. The second error type determined in third question upon clinical interview arose from lack of knowledge as shown in Interview 13.



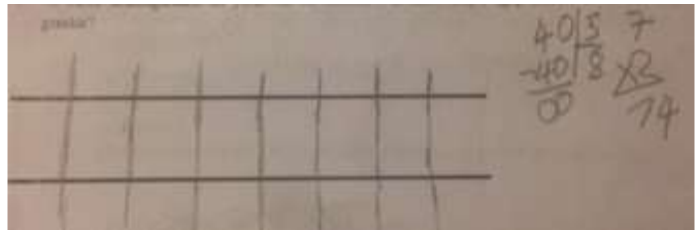
T: Can you explain your solution in the second question on the illustration?
S: I planted trees at intervals of 5 meters, then I counted, it is 10.
T: How long is the road?
S: 40 meters.
T: Is it 40 meters in your illustration? Can you show me?
S: 5-10-15-20 up the road, 5-10-15-20 down the road; it is 40 meters in total.
T: Where is the road, lines or between the lines?
S: Between the lines.
T: How long is it there?
S: I see. Both sides should be 80 meters.
T: What would it be, then?
S: The number of trees would increase.
How many would there be?
S:18 (The student solved the problem all over again).

As seen in Interview 13, the students, who made the error originating from lack of real-life knowledge, did wrong was dividing 40 meters, which is the length of the road into two lines which are 20 meters each. The ratio of this error type to the incorrect solutions is 11.00% (23/ 156). The fourth error type determined in third question upon clinical interview arose from misinterpretation of the illustration as shown in Interview 14.



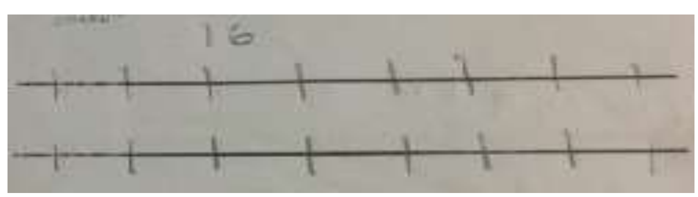
T: Can you explain your solution in the second question on the illustration?
S: There is a 40-meter-long road. Trees will be planted on both sides of it at intervals of 5 meters. I placed the trees at intervals of 5 meters. And I found the answer as 16.
T: How many trees are there in your illustration?
S: 1, 2, 3……18 trees.
T: Which one do you think is right? The number of trees in the illustration or the one you found?
S: If we divide 40 by 5, there will be 8 trees; it will be 16 trees because there are two sides. I think my illustration is incorrect because it should've been like this.

As seen in Interview 14, the students who made this error drew the illustration correctly but saw that the solution they structured in their mind did not coincide with the solution they found on the illustration. Consequently, they decided that the formulary answer they obtained was true despite the illustration they drew. The ratio of this error type to the incorrect solutions is 20.57% (43/156). The fifth error type determined in third question upon clinical interview arose from structuring the illustration incompletely and can be seen in Interview 15.



T: Can you explain your solution in the third question on the illustration?
S: I placed the trees at intervals of 5 meters along the 40-meter-long road, then I counted and found 14 trees.
T: How long is the road?
S: 40 meters.
T: T: Is it 40 meters in your illustration?
S: Yes.
T: So, were the trees planted at intervals of 5 meters?
S: Yes.
T: How many meters are there between the starting point of the road and the first tree?
S: 5 meters.
T: Shouldn't we plant a tree at the starting point, then?
S: Okay.
T: How many meters are there between the end of the road and the last tree?
S: 5 meters.
T: How about planting a tree at the end of the road?
S: Yes, we can. I already did but erased it later.
T: What is the result, then?
S: 9 tress on one side of the road, 9 on the other side; it is 18 trees.

As seen in Interview 15, the students who made this type of error did not plant a tree in the front and/or back end. The ratio of this error type to the incorrect solutions using the illustration is 13.40% (28/156). The sixth error type determined in the third question upon clinical interview arose from structuring the illustration incorrectly and can be seen in Interview 16.

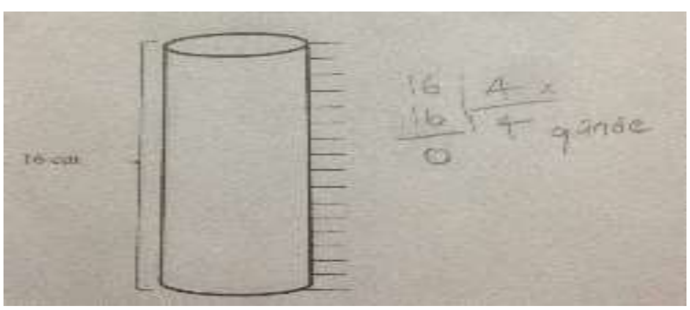


T: Can you explain your solution in the second question on the illustration?
S: I planted trees at intervals of 5 meters, then I counted, it is 16 trees.
T: How long is the road?
S: 40 meters.
T: Is it 40 meters in your illustration? Can you show me?
S: I'll calculate the length between the trees, first. It is 35

meters (the student counted in groups of five), then it is 2.5 meters here (showing the gap between the front end of the road and the first tree), it is 2.5 meters here again (showing the gap between the last tree and the back end of the road). It is 40 meters in total.
T: Why did you start drawing 2.5 meters away from the front end instead of drawing the trees in the first place?
S: Because 8 trees are required on one side.
T: How did you get that?
S: If we divide 40 by 5, there will be 8 trees on one side?
T: Will it be equal if you plant them like this?
S: The front and back ends won't be equal.
T: How about starting all over again?
S: It is 9 trees, then.
T: Which of the results is correct, 8 or 9?
S: 9 is correct because it will be equal, then.

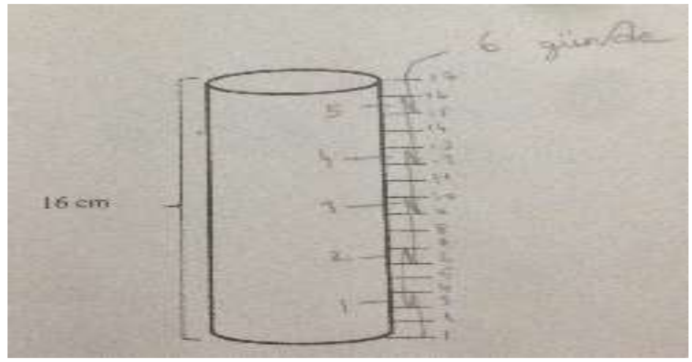
As seen in Interview 16, the students who made this error did wrong because they thought that the trees were to be planted 2.5 meters away from the starting point rather than beginning with the front end. The ratio of this error type to the incorrect solutions is 29.67% (62/156). In the next phase of the study, the answers given to the verbal form of the fourth question were compared to the answers given to its visual form by the numbers of correct and incorrect answer, and the findings obtained are presented in Table 7.

According to Table 5, blanks for question 4 illustrations made the least increase in student achievement in the fourth question. In the next phase of the study, the solutions of 243 students who gave the incorrect answer when solving the visual form of the fourth question and information on the clinical interviews performed with them are presented. The first error type determined upon clinical interview in fourth question arose from solution without using the illustration, and this can be seen in Interview 17.



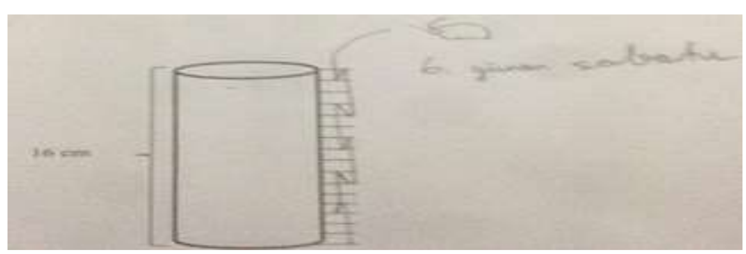
T: Can you explain your solution in the fourth question?
S: I divided 16 by 4 because if it climbs up 4 cm each day, it gets out of the jar of which height is 16 cm.
T: Why didn't you ever use the illustration?
S: Because the question was so easy.

As seen in Interview 17, the students who made this error used direct rate/proportion without benefiting from the illustration. The ratio of this error type to the errors made in the fourth question in general is 36.63% (89/243). There are 154 students who solved the visual form of the question incorrectly without using the illustration. The second error type determined in fourth question upon clinical interview arose from lack of knowledge, and this can be seen in Interview 18.



T: Can you explain your solution in the second question on the illustration?
S: The snail climbs up 4 cm on the first day, it comes to 3 at night, climbs up to 7 on the second day and comes back to 6. It climbs up to 10 on the third day, slides down to 9, climbs up to 13 on the fourth day and slides back down to 12. It climbs up to 16 on the fifth day, slides back down to 15. It gets out of the jar on the sixth day.
T: Is the height of the jar is 17 cm?
S: It says 16 but there are 17 lines.
T: Do you have a ruler with you?
S: Yes.
T: Can you look at it with your ruler to see from which number it starts?
S: It starts from 0; I started from 1.
T: Will the result change if you solve it again?
S: It will be 16 cm, then. It gets out of the jar because it won't be able to slide again on the fifth day.

As seen in Interview 18, the students who made this error lack knowledge on how to determined the starting point as 1 cm rather than 0 when measuring the length. The ratio of this error type to the incorrect solutions is 19.34% (47/154). The third error type determined in fourth question upon clinical interview arose from mis-interpretation of the illustration as presented in Interview 19.



T: Can you explain your solution in the second question on the illustration?

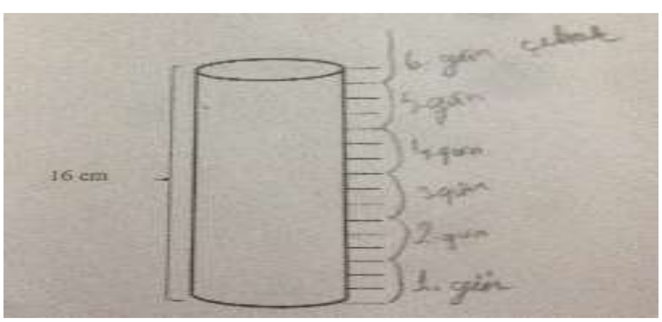
Table 7. Comparison of the numbers of correct and incorrect answers given to the verbal and visual forms of the fourth question.

Solutions		Verbal form			Visual form		
		Correct	Wrong	Total	Correct	Wrong	Total
Blank	Frequency (f)	-	-	65	-	-	23
	Percentage (%)	-	-	17.57	-	-	6.22
Without using illustration	Frequency (f)	86	179	265	34	89	123
	Percentage (%)	32.45	67.55	71.62	27.64	72.36	33.24
By using illustration	Frequency (f)	16	24	40	70	154	224
	Percentage (%)	40.00	60.00	10.81	31.25	68.75	60.54
Total	Frequency (f)	102	203	370	104	243	370
	Percentage (%)	27.57	54.86	100	28.11	65.67	100

Note: There are 28 students who solved the verbal form of the question correctly without using illustration but solved its visual form incorrectly by using illustration.

T: Where is the exit point?
S: Top of the jar.
T: In which cm?
S: 16.
T: Has the snail ever been there before?
S: Yes. On the fifth day.
T: Why did it slide back down?
S: It shouldn't have because it gets out on the fifth day.

As seen in Interview 19, the students who made this error were not able to see that the snail got out of the jar on the fifth day. The ratio of this error type to the incorrect solutions is 26.33% (64/154). The fourth error type determined in fourth question upon clinical interview arose from structuring the illustration incorrectly as presented in Interview 20.



T: Can you explain your solution in the second question on the illustration?
S: If it climbs up 4 cm and slides back down 1 cm, it climbs up 3 cm a day. I advanced it 3 cm all the time and found 6.
T: Why didn't you state that it slides back?
S: Because it climbs up 3 cm, that's why.

As seen in Interview 20, the students who made this error thought, the snail climbs up 3 cm each day, and drew their illustrations accordingly to find the answer 6. The ratio of this error type to the incorrect solutions is 17.70% (43/154). After each question had been examined, the numbers of correct and incorrect answers in the test generally were reviewed, and the findings obtained are given in Table 8.

According to Table 8, 17.03% (252) of the answers were left blank in the verbal form of the test, the percentage went back to 5.75% (85) when the visual form was applied. It is seen that 68.65% (1016) of the students solved the test without using illustration while the percentage decreased down to 23.24% (344) when the visual form was asked. It was revealed that 22.64% (230/1016) of the answers given to the verbal form of the test without using illustration were correct while 74.19% (184/248) were incorrect. 14.32% (212) of the solutions in the verbal form of the test were found by using the illustrations the students drew, 54.72% (116/212) of the students gave the correct answer while 45.28% (96/212) gave the incorrect answer.

It is seen that the percentage of the solutions using the illustration in the visual form increased up to 71.01% (1051), the percentage of the solutions without using the illustration decreased down to 23.24% (344). 41.96% (441/1051) of the solutions using the illustration in the visual form of the test were correct, 58.04% (610/1051) were incorrect. It was revealed that 23.84% (82/344) of the answers given to the visual form of the test without using illustration were correct while 76.16% (262/344) were incorrect. Whereas, the verbal form of the test was answered correctly by 23.38% (346/1480), it is 35.33%

Table 8. Comparison of the numbers of correct and incorrect answers given to the verbal and visual forms of the questions in the test

Solutions		Verbal Form			Visual Form		
		Correct	Wrong	Total	Correct	Wrong	Total
Blank	Frequency (f)	-	-	252	-	-	85
	Percentage (%)	-	-	17.03	-	-	5.75
Without using illustration	Frequency (f)	230	786	1016	82	262	344
	Percentage (%)	22.64	77.36	68.65	23.84	76.16	23.24
By using illustration	Frequency (f)	116	96	212	441	610	1051
	Percentage (%)	54.72	45.28	14.32	41.96	58.04	71.01
Total	Frequency (f)	346	882	1480	523	872	1480
	Percentage (%)	23.38	59.59	100	35.33	58.92	100

Note: There are 48 students who solved the verbal form of the test correctly without using illustration but solved its visual form incorrectly by using illustration.

Table 9. Descriptive statistics regarding the errors in the visual form of the problem solving achievement test.

Types of error		Question 1	Question 2	Question 3	Question 4	Total
Without using Illustration 1	Frequency (f)	45	75	53	89	262
Incorrect rating/proportioning	Percentage (%)	25.71	30.61	25.36	36.63	30.05
Errors using Illustration 2						
Lack of knowledge	Frequency (f)	31	64	23	47	165
	Percentage (%)	17.71	26.12	11.00	19.34	18.92
Misinterpretation	Frequency (f)	26	27	43	64	160
	Percentage (%)	14.86	11.02	20.57	26.33	18.35
Incomplete structuring	Frequency (f)	37	38	28	-	103
	Percentage (%)	21.14	15.51	13.40	-	11.81
Incorrect structuring	Frequency (f)	25	-	62	43	130
	Percentage (%)	14.29	-	29.67	17.70	14.91
Misplacement of what is given	Frequency (f)	11	41	-	-	62
	Percentage (%)	6.29	16.74	-	-	7.11
Total	Frequency (f)	175	245	209	243	872
	Percentage (%)	100	100	100	100	100

(523/1480) for its visual form.

In the next phase of the study, the descriptive statistics regarding the errors in the test applied with illustrations in general were obtained, and the findings are presented in Table 9.

According to Table 9, 872 errors were observed in the visual form of the test, 30.05% (262) of these errors were made without using the illustration. The most common error using the illustration was due to lack of knowledge by 18.92% (165) which was followed by misinterpretation by 18.35% (160), incorrect structuring by 14.91% (130), incomplete structuring 11.811% (103) and misplacement of what is given 7.11% (62), respectively.

DISCUSSION, CONCLUSION AND RECOMMENDATIONS

According to the result of the study, 17.03% of the answers were left blank in the verbal form of the test, the percentage went back to 5.75% when the visual form was applied. Decreasing number of the questions left blank in the visual form of the test shows that images encouraged some of the teachers to make an effort to solve the questions. Whereas, the verbal form of the test was answered correctly by 23.38%, it is 35.33% (523/35.33) with an increase by 12% for its visual form. Although images had a positive impact on the student achievement in the first three questions, the contribution was limited in the fourth one. The finding that the images obtained as a result of the research contributed to the problem solving achievement coincide with the finding obtained in the study performed by Elia et al. (2007).

While 22.64% of the students gave the correct answer to the questions in the verbal form of the test without using the illustration, the general percentage of correct answers in the visual form of the test increased to 23.84% on a limited level. Yet, the percentage of correct answers given by the students who drew their own illustrations in the verbal form of the test was 54.72% whereas, the percentage decreased to 41.96% in the visual form which was solved with readily available illustrations. This shows that using one's own images rather than readily available ones contributes to the problem solving achievement more; moreover, the finding is in parallel with the findings obtained in the study performed by Uesaka et al. (2007).

The percentage of the answers given without using the illustration in the verbal form was about 69% whereas it decreased down to about 23%, the percentage of the answers using the illustration increased from about 14% to about 71%. An increase by about 57% in the percentage of using images only by changing the item format indicates that the students adapted to the use of images quickly; and this finding obtained in the research coincides with the findings obtained in the study performed by Altun et al. (2004). It was also found that the percentage of using images in the verbal form was very low without any guidance given to the students, revealing that this finding coincides with the findings obtained in the study by Ulu (2008). Ulu (2008) revealed that the teachers benefited from images too much when solving a problem but the finding obtained as a result of the study shows that the students did not take their teachers as a model. In the study performed by Altun and Memnun (2006), it was found that the primary school students did not use the images during the exam although they had utilized them during the courses and the use of images decreased as the grade level increased. This indicates that the habit of utilizing images should be taught to the students at early ages.

882 incorrect solution observed in the visual form of the test was examined along with their reasons. The ratio of the incorrect solutions without using the illustration in the visual form to the errors in the test in general is 30.05% (262/882); it was seen that the students did not use the images and used direct rates/proportions because they thought the questions were so easy. Use of non-routine problems in the study caused solutions by direct rate/proportion to contradict with real-life conditions; this issue encountered as a result of the research has also been observed in the studies performed by Olkun (2010), Schoenfeld (1991), Verschaffel et al. (2000), Verschaffel et al. (1999) and Xin et al. (2007).

Whereas, the number of errors due to the use of images in the verbal form of the test was 96, it increased to 610 in the visual form. The increase in the number of errors due to use of images in the second test can be basically explained by the increase of the use of images; however, it is also a fact that images made the students fall into error on a remarkable level even though they increased the problem solving achievement. This finding coincides with the findings obtained in the studies by Elia et al. (2007). Panasuk and Beyranevand (2010) studied on students' skills of transforming verbal expressions into images and transforming the mathematical skills and revealed that those who could transform those expressions into each other had a higher level of problem solving achievement. A similar finding was obtained by Moreno and Mayer (1999) and Hegarty et al. (1995). Some of the students not being able to transform the verbal expressions into images in these studies coincide with the findings obtained in the earlier mentioned studies.

The multitude of the errors due to the use of images proves the necessity to investigate the reasons of the errors due to the use of images. In this context, the errors due to the use of images were categorized, and it was seen that the errors originated from lack of knowledge (18.92%), misinterpretation (18.35%), incorrect structuring (14.91%), incomplete structuring (11.81%), and misplacement of what is given (7.11%) respectively. In the studies performed by Schoenfeld, (1991), Verschaffel et al. (2000), Verschaffel et al. (1999) and Xin et al. (2007), it was emphasized that non-routine problems might require real-life knowledge and it was seen that the students who were lack of real-life knowledge might give incorrect answers. The fact that the students structured the images incorrectly due to lack of some real-life knowledge in this study shows that the case also applies to the transformation to images.

The case with the errors originating from mis-interpretation, incorrect structuring, incomplete structuring and misplacement of what is given in the problem differs from the errors due to lack of knowledge. The clinical interviews revealed that the students, who misinterpreted the illustration even though they drew it correctly, believed in the accuracy of their rates/proportions rather than of the illustrations they drew when the solutions they found in their minds did not coincide with the solutions they found using the illustrations through their rates/ proportions. While it was expected from the students to create their images independently from the formulary solution in their minds, the clinical interviews showed that the students drew their illustrations according to the rate/proportion in their minds in the errors due to incorrect structuring, incomplete structuring and misplacement of what is given. This indicates that some of the students were willing to benefit from the readily available schemas in their minds when drawing an image; however, the studies performed by Viennot and Moreau (2007) and Soylu and Soylu (2006) reveals that readily available schemas may be misleading for the problems which are encountered for the first time. Available solutions valid for the schema theory are addressed in the scope of procedural knowledge and it is recommended that students should be gravitated towards conceptual knowledge because available schemas may mostly fall insufficient in non-routine problems (Anderson, 2010; Brynes and Wasik 1991, Baroody, Feil, Johnson, 2007). The multitude of the errors due to the use of readily available schemas in this study proves that students should gravitate towards conceptual knowledge so that they can break taboos in their minds.

According to the results of the study, it was observed that the use of images decreased the number of blank answers and increased the number of correct answers; however, the students did not prefer to use the illustration much when they were not guided to use the images as they were in the verbal form of the test. Based on this finding, it is recommended that teachers should emphasize visual solutions in their problem solving activities and encourage students to use images. It is seen that the greatest reason for the errors due to the use of images is the formulary solutions in students' minds; in accordance with this finding, students should be encouraged to create their images independently from the schemas in their minds. It should be made felt that not only the solution will be fun but also the risk of error will be mitigated when the images are structured correctly.

Conflict of Interests

The authors have not declared any conflict of interests.

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