# The effectiveness of using the model method to solve word problems

**Lei Bao**
Leopold Primary School, Vic.
<bao.lei.l@edumail.vic.gov.au>

**The model (or bar) method provides students with an opportunity to demonstrate their interpretation of word problems and to see the relevant relationships in a visual way. This supports students to make sense of a problem, identify relationships and work out suitable strategies for solving word problems.**

## Abstract

The aim of this study is to investigate whether the model method is effective to assist primary students to solve word problems. The model method not only provides students with an opportunity to interpret the problem by drawing the rectangular bar but also helps students to visually represent problem situations and relevant relationships on the bar and choose the correct operations to solve the problem. Three Year 4 classes participated in a two week trial of the method. Comparing pre-test with post-test results showed that performance improved on every question. The findings suggest that drawing a bar to make a model assists primary students to solve word problems. Practical suggestions for using the model method are included.

## Introduction

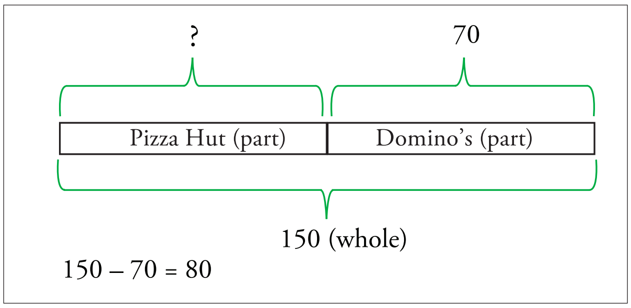
Word problems have always been a major part of primary school mathematics. However, anecdotally it appears that many primary students have difficulties with word problems, particularly multi-step word problems. Researchers (Chan & Foong, 2013; Englard, 2010; Fong & Lee, 2005; Ng & Lee 2009) have been seeking effective teaching methods and believe that the model method can address this need. Since the model method was introduced in Singapore in the early 1980s, it has made a great contribution to student learning. Although the model method has been practised in Singapore and other countries for more than a decade, in my own experience I have not seen this method used in Australian primary schools. This paper describes a study to examine the effectiveness of using the model method with Year 4 students.

## What is the model method?

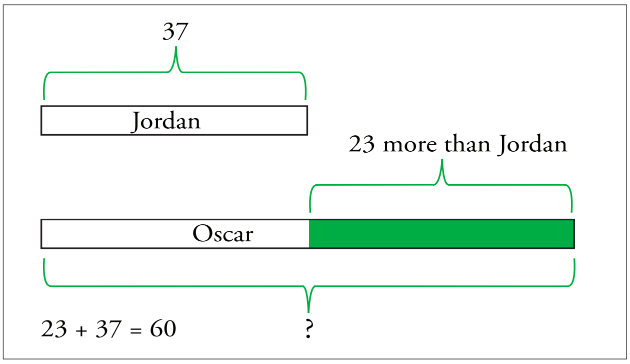
In this approach, students draw diagrams by using rectangular bars. They use the bars to represent and visualise the mathematical relationships in the problem, and record information on them as they solve the word problem. Rectangular bars are used because they are easy to draw, divide, represent numbers and display relevant relationships (Ng & Lee, 2009). Different lengths of rectangular bars are used to represent different numbers: a longer rectangular bar for a known bigger number, a short one for a known smaller number, and bars of arbitrary length for unknown numbers. In some problems one bar is divided into known and unknown sections; in others the lengths of two or more different bars are compared.

Several studies (Englard, 2010; Ng & Lee, 2009) used word problems involving the relationship of part-whole, additive and multiplicative structures to trial the model method. These models not only serve to explain and reinforce the concepts such as addition and subtraction but assist students to deal with multiplicative structure problems. Englard (2010) highlighted that with the visualised bar model, it is easier for students to identify the equal groups and partition the quantities into smaller equal parts, thus to overcome the barrier of the concept of multiplicative structure.

Some publications (Chan and Foong, 2013; Cheong, 2002; Englard, 2010; Ng & Lee, 2009) report that drawing the model not only permits students to visualise the abstract information given in the problem but also helps students to represent relationships between known and unknown numerical quantities and hence to solve problems. Chan and Foong (2013) explain that using the model shifts the focus from results to working




Sample 1

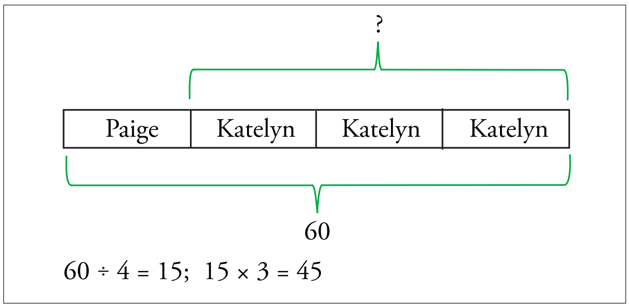



Sample 2

process and from calculations to relationships between known and unknown quantities. Moreover, it helps students identify the correct operations and steps that are needed to solve a problem (Englard, 2010). It is believed that if students can draw the bars realistically and represent all the relevant quantities, thus to identify the relationships between known and unknown quantities, they have already understood the structure of the problem (Cheong, 2002; Ng & Lee, 2009).

## The study

Three Year 4 classes from a public school in Victoria participated in a two-week trial on the model method. Students had not been taught the bar model method before. The normal way to teach word problems at the school was to understand the question and devise a plan. This trial began with training the teachers involved during the weekly planning and Professional Learning Team (PLT) sessions.




Sample 3

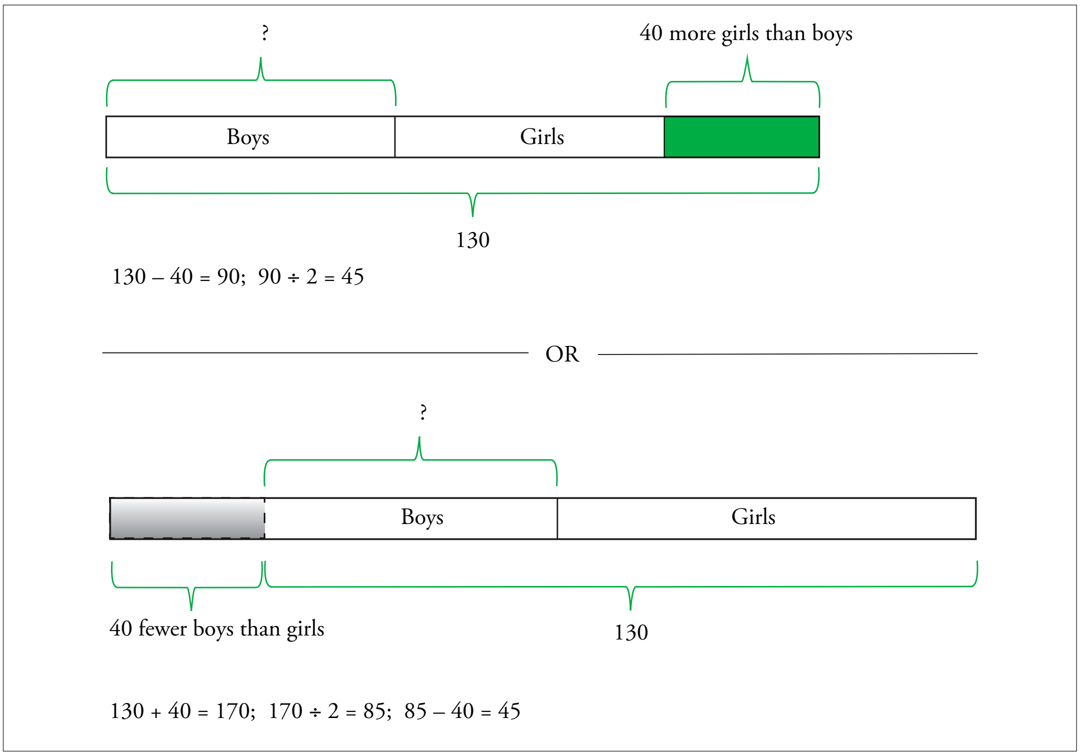
First, teachers were provided with a *PowerPoint* presentation based on the findings of key research papers. Then, a unit of work was planned together based on the learning sequence: part-whole relationship, comparison and multiplicative structure. Students' responses were anticipated and scaffolding questions were prepared during planning. A pre-test with five questions involving part-whole relationships, additive and multiplicative comparisons was administrated before the trial and a post-test with five parallel questions given in Table 1 was administrated at the end.

Question 1 is a one-step problem involving a part-whole model. Students would draw the bar for the whole quantity (marked as 150), and divide it into two sections (Pizza Hut of unknown length and Domino marked as 70). A suitable bar model is shown in Sample 1. Questions 2, 3, 4 and 5 require quantities to be compared. Sample 2 shows two bars (one for Jordan marked as 37 and a longer one for Oscar). Oscar's bar then needs to be divided into one part as long as Jordan's and the extra 23. The problem has then effectively been solved.

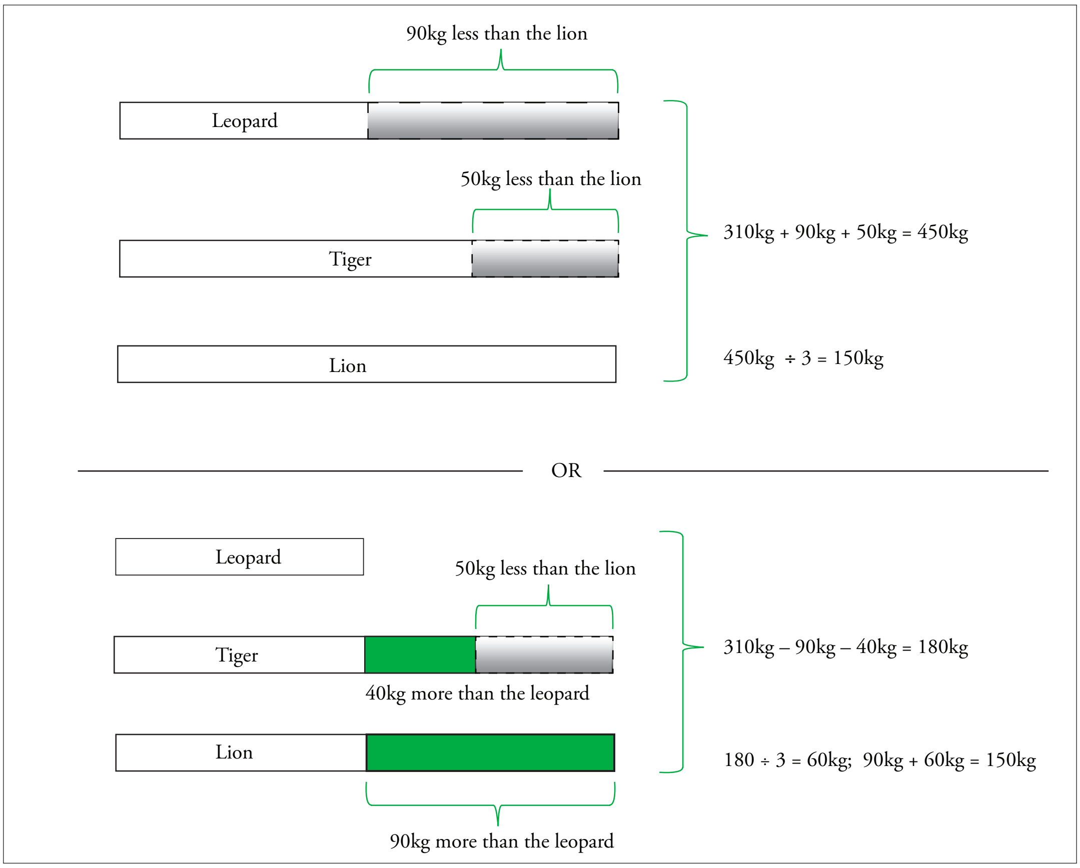
The comparison model is used in question 2, 3, 4 and 5. It shows the relationship between two or more quantities when they are compared. The varying lengths of the rectangles show that one quantity is more than another and the difference between the quantities is indicated by the difference in lengths of rectangles. Question 4 is a two-step problem involving the concept of an equivalent quantity. Students would draw 4 equivalent bars, 3 for Katelyn and 1 for Paige and mark the total 60 (Sample 3).

Table 1. The five word problems of the post-test.

| | |
|---|---|
| 1 | Pizza Hut and Domino's sold 150 pizzas on Sunday night. Domino's sold 70 pizzas on that night. How many pizzas did Pizza Hut sell on Sunday night? |
| 2 | Jordan has 37 footy cards. Oscar has 23 more cards than Jordan. How many cards does Oscar have? |
| 3 | 130 students took part in an art competition. There were 40 fewer boys than girls took part. How many boys took part in the competition? |
| 4 | Katelyn and Paige have collected 60 coins. Katelyn has three times as many coins as Paige. How many coins has Katelyn collected? |
| 5 | A zoo keeper weighed some of the animals at Melbourne Zoo. He found that the lion weighs 90kg more than the leopard and the tiger weighs 50kg less than the lion. Altogether the three animals weigh 310kg. How much does the lion weigh? |




Sample 4




Sample 5

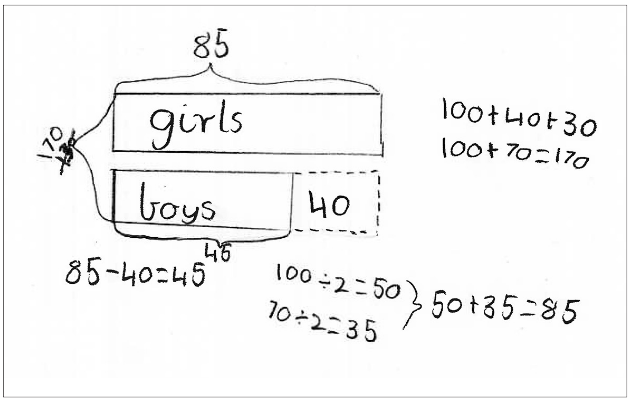
Table 2. Percentage of students successfully solving pre- and post-test items (N = 73).

| | Question 1 | Question 2 | Question 3 | Question 4 | Question 5 | Overall |
|---|---|---|---|---|---|---|
| Pre-test | 68% | 76% | 4% | 6% | 1% | 31% |
| Post-test | 97% | 97% | 7% | 51% | 9% | 52% |

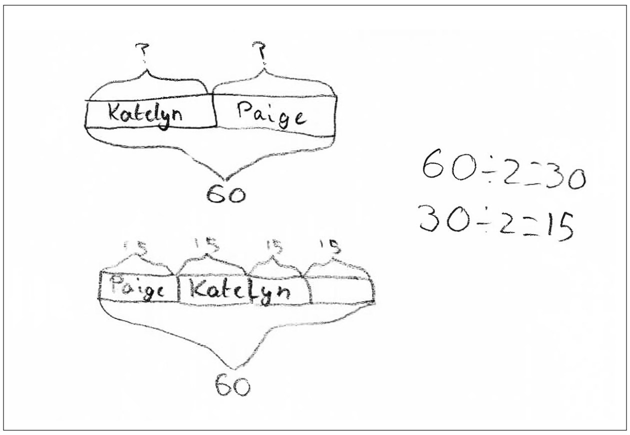
Two equivalent quantities can also be found in Question 3 and 5. After subtracting 40 from the girl's bar or adding 40 to the boy's bar (Sample 4). Like Question 3, Question 5 does not show equivalent quantities initially but they can be revealed after using the lion's weight as an equivalent quantity and adding fictitious weights 90kg onto the leopard and 50kg onto the tiger or using the leopard's weight as an equivalent quantity and subtracting 40kg from the tiger and subtracting 90kg from the lion (Sample 5).

## Results

Overall 73 students completed the pre-test and post-test. They were more successful on every question at the end of the trial than at the pre-test and the overall success rates rose from 31% at the beginning of the trial to 52% at the end. Figure 1 provides students' overall success rate in solving the problems before and after the trial. Questions 1 and 2 are one-step questions and approximately one third of the students who were unsuccessful at the beginning learned to do these problems correctly during the trial. In both pre-test and post-test, students performed better in Question 2 than Question 1. This maybe because Question 1 is a subtraction problem and Question 2 is an addition problem.




Sample 6

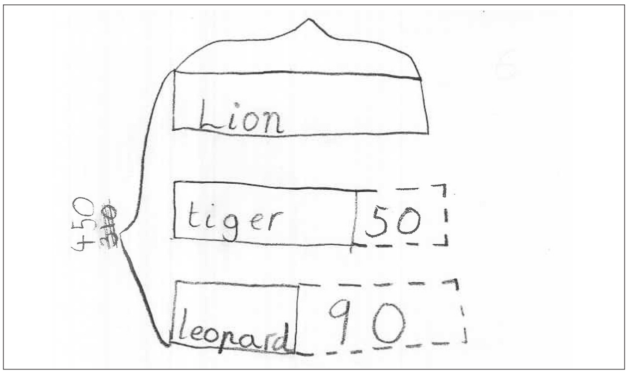



Sample 7

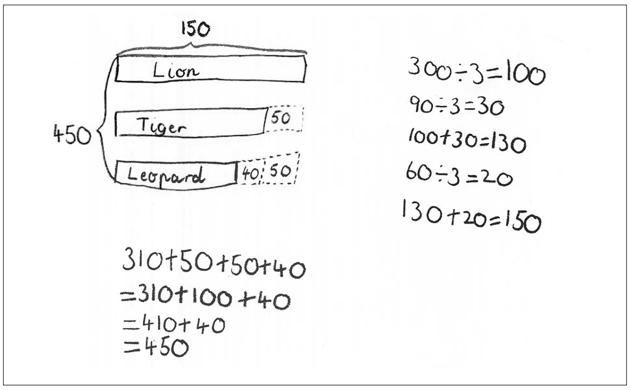
There was a very large improvement in question 4 which contains the multiplicative relationship 'three times as many as'. Most students drew 3 equivalent bars for Katelyn and a same size bar for Paige, showing the concept of "3 times as many as". Some students used 'guess and check' strategy to calculate the answer; others recognised 4 equivalent quantities so they halved 60 to get 30 and halved 30 to get 15 (Sample 7).

Questions 3 was difficult both before and after the trial. Many students were able to draw two separate bars in Question 3 and show the different lengths of bars between the boys and girls in the post-test. In contrast to the girls' bar, some students extended the length of the boys' bar, showing boys are 40 fewer than the girls (Sample 6), and made two equivalent quantities. However, most students failed to detect the relationship between the total number of students 130 and the related comparison unit 40 (130 – 40 = 90) when the boys' quantity is used as an equivalent quantity or (130 + 40 = 170) when the girls' quantity is used as an equivalent quantity as the two equivalent groups were not presented to students like Question 4.

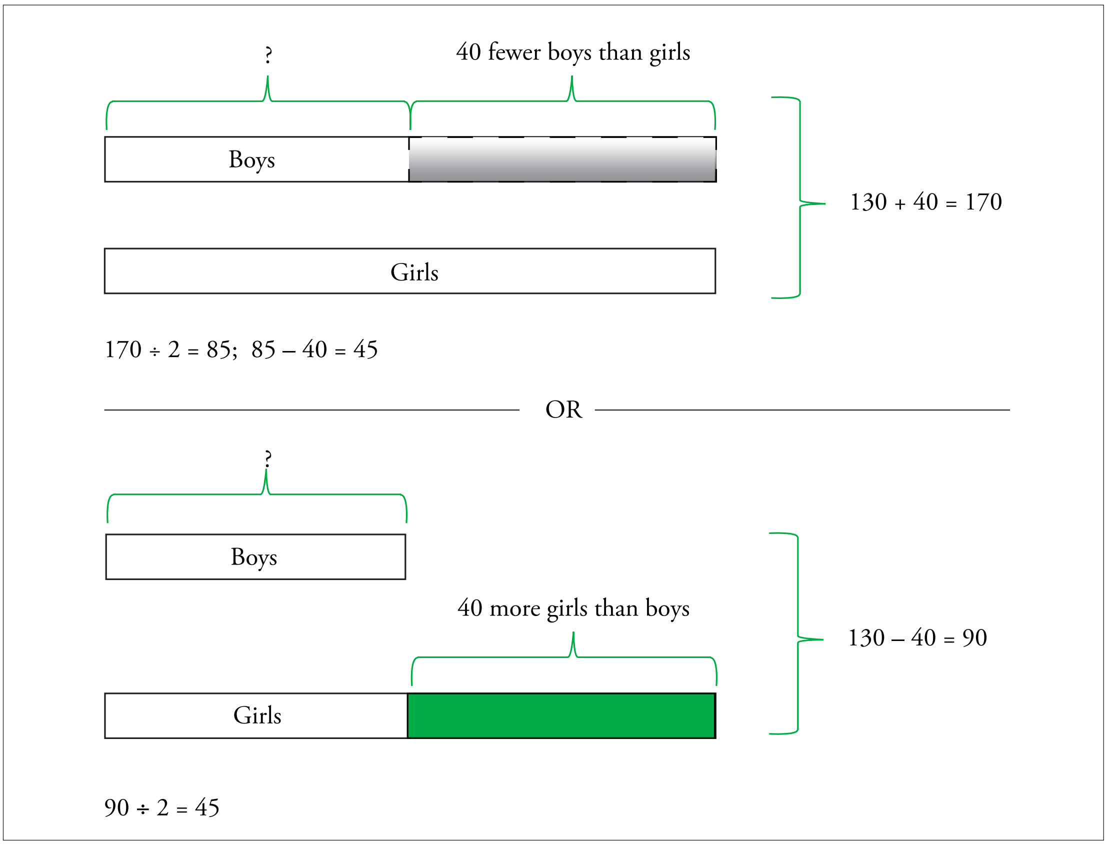
Question 5 was also difficult, with just a little improvement because the concept of equal quantities was not presented directly to students. A lot of students in the post-test correctly constructed the bars (Sample 8 and 9) and used the fictitious weights for the comparison units. Some of them identified the



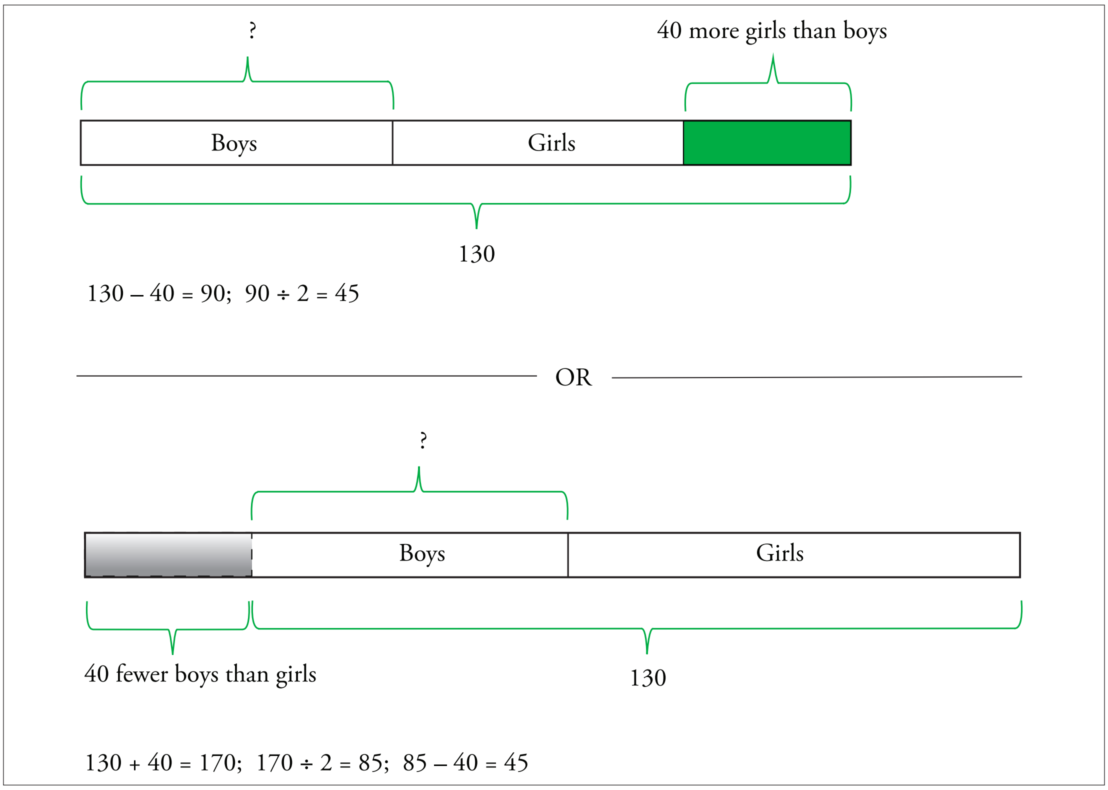

Sample 8




Sample 9




Sample 10




Sample 11

equivalent quantities, partitioned 450 into smaller numbers such as 300, 90 and 60 (Sample 9) and successfully solved the problem; nearly one fifth of students added the fictitious weights 50 and 90 onto 310 and found the new total 450 but did not progress further either because of failing to identify three equivalent bars with a new sum of weights or failing to calculate the division equation due to relatively large numbers involved in the problem and calculators were not provided.

## Findings

After the trial, students used the bar model and demonstrated their understanding of the part whole relationship in Question 1. Students illustrated their sound understanding of the concepts involving 'more than', 'less than' and 'as many as' in the post test but struggled to deduce the relationship with the concept of 'fewer than' in Question 3. This study confirms other findings (Englard, 2010, Ng & Lee, 2009) that multi-step word problems involving additive and multiplicative structure create more barriers for the students to succeed than one step problems.

The post-test results show that students are more likely to recognise the equivalent quantities in the multiplicative comparison model if the bars were drawn separately underneath each other (Sample 10) rather than being drawn next to each other (Sample 11) as it seems easier for students to see the comparison units added onto or subtracted from the total quantity when the bars are vertically aligned. Moreover, question 4 is more likely to be solved since the elements of variables such as the concept of equivalent quantities and the total number are presented in the problem. On the other hand, Questions 3 and 5 are hard to deduce as students need to use fictitious quantities in order to create equivalent bars. In Figure 7, for example, after adding the comparison unit 40 onto 130, two equivalent quantities are made. This study found that the use of dotted lines for the fictitious quantities or comparison units might help students identify the equivalent quantities; the use of coloured lines and braces might support student to visualise and deduce the relationship between known and unknown quantities and their comparison units, thus to help students to understand the structure of the problem and successfully solve the problem.

After comparing the models drawn in correct solutions with those that resulted in wrong answers, it is evident that most students who represented the information and relationship on the bars successfully solved the problems. On the other hand, students failing to draw the model correctly indicates they probably do not understand the problem.

Initially four classes were involved in the trial. However, one of the classes had to be taught by substitute teachers who had not attended the training. This class showed very little gain, and so these students have been excluded from the discussion above. This indicates that special training is probably needed for teachers to teach students to use the model method effectively.

## Conclusions

The model method not only provides students with an opportunity to interpret the problem by drawing the rectangular bar but also helps students to visually represent problem situations and relevant relationships on the bar and choose the correct operations to solve the problem. The purpose of drawing the models is not to teach students to follow specific rules but to provide them with a tool to support the understanding of the problem, identify the relationship and operations they need and hence work out a strategy for finding the answer.

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