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Existence Regions of Shock Wave Triple Configurations

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ABSTRACT

The aim of the research is to create the classification for shock wave triple configurations and their existence regions of various types: type 1, type 2, type 3. Analytical solutions for limit Mach numbers and passing shock intensity that define existence region of every type of triple configuration have been acquired. The ratios that conjugate intensities of three shock in triple configuration and flow turn angle on them are presented. The transition (boundary) shock wave triple configurations have been reviewed. The acquired results can be used to design shock wave structures with set properties in detonation engines, air collectors, technological plants, when analyzing shock wave influence on objects during an explosion. Triple configurations of type 1 are used in internal compression air intakes that are based on interaction of oncoming shocks. Triple configurations of type 2 can be found in supersonic gas jets, at Mach reflection of shock and detonative waves from solid walls. Triple configurations of type 3 are used in supersonics multishock air intakes of external or mixed compression.

KEYWORDS
Mach reflection, triple shock wave configuration, von Neumann criterion, Mach configuration, Mach stem

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Introduction

Recently, there has been going on an active debate on the various ways of organization of detonative combustion in promising air-breathing engines and rocket engines (Bulat, 2014), and also on organization of supersonic combustion in ramjets of hypersonic aircraft (Roy et al., 2004). In order to correctly understand the nature of these projects, its required to have a clear understanding of stationary and non-stationary gas-dynamic discontinuity (GDD), shock wave and what differs them from detonation wave which is also a gas-dynamic discontinuity (Bulat & Uskov, 2014; Omelchenko & Uskov, 1999).

To be certain, let's agree that the detonation wave is a shock wave that forms as a result of chemical oxidation or formed by an external source but has a region of very fast combustion behind its front. And just *shock wave* – it's a gas-dynamic discontinuity, which if formed upon the interaction of supersonic flow with a solid wall or in case of intersection (interference) of other GDD. A

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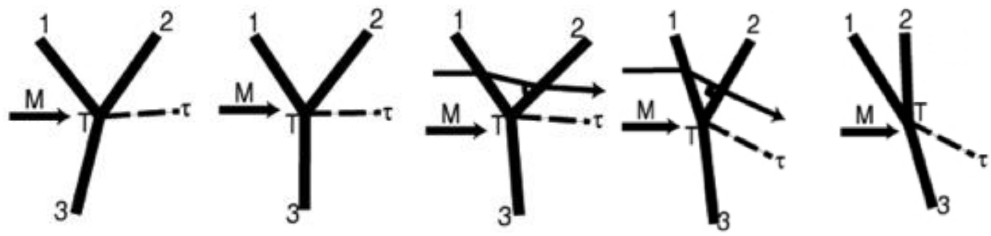
stationary shock wave is called compression shock. The a fuel-oxidizer mixture passes through a surface of shock wave, the shock wave can cause it to detonate, because of rapid increase of temperature on shock wave front. Shock wave and compression shock can be direct, when their front is perpendicular velocity vector of oncoming flow, or oblique. On the oblique compression shock and shock wave that are located at an angle σ to the flow, occurs not only compression but also flow rotation to an angle β (Courant & Friedrichs, 1948).

It is pertinent to point out that the interference of gas-dynamic discontinuities leads to the formation of *shock wave structures* (SWS). The SWS form as a result of interaction of shock waves or discontinuities with each other, with tangential or contact discontinuities, with boundaries between two mediums or solid surfaces (Landau & Lifshitz, 2003). The SWS formation can be differentiated as a result of overtaking or oncoming discontinuities, breakdown or branching of discontinuities.

A shock wave structure that consists of three shocks (1, 2 and 3 on Fig. 1) and tangential discontinuity (τ), that have a shared point (T) is called a triple shock configuration (Adrianov, Starykh & Uskov, 1995). Shock 1 and 2, through which line of flow passes consecutively, make a shock wave system, the third shock is called main and point T is called triple point.



a)TC-1, b) TC -1/2, c) TC -2, d) TC -2/3, e) TC -3.

Figure 1. Triple shock wave configurations

Gas flows that passed through various compression shock systems (a sequence of shock 1 and 2 or singular shock 3, are split by tangential discontinuity τ). Triple configurations occur on irregular shock reflection from a solid surface and from symmetry axis in axially symmetrical flows, in some problems about interaction of oncoming shocks, and also during interaction of overtaking shock, for instance, in multishock air intakes (Fig. 2). The concept of TC was first introduced in a problem about shock wave oncoming onto a wedge. Later, the stationary case of non-regular shock reflection from a solid wall with formation of triple shock wave configuration was studied (Omelchenko & Uskov, 2002).

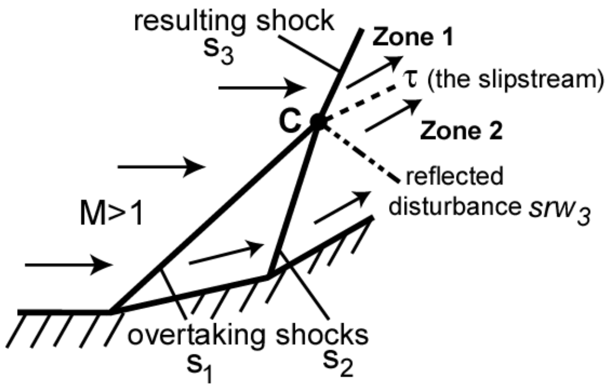


Figure 2. Interaction between overcoming compression shocks

A modern views about GDD interference in generalized form are presented in article (Uskov & Mostovykh, 2008; 2010). The known ratios that conjugate changes in gas-dynamic variables on shocks and waves in shock wave structures, have allowed to set and solve the problem of research of extreme SWS properties, including TC (Uskov & Omelchenko, 1995), and the problem of controlling shock wave processes in various technological plants (Uskov et al., 2006). During research the important advantages of TC, from perspective of various optimality criteria (Uskov & Chernyshov, 2006; Tao, Uskov & Chernyshov, 2005; Uskov, 1980), which allows to say about possibility of designing shock wave configurations that are optimal for each problem.

Aim of the study

The purpose of the study is to analytically investigate triple shock configurations in supersonic flow of and ideal gas, express in a compressed form all major fact about triple configurations, areas of their application, and method of calculation.

Research questions

What are the existence regions of wave triple configurations?

Method

In order to create the classification of TC we used the method of mathematical modeling. Analytical solutions for limit Mach numbers and passing shock intensity that define existence region of every type of triple configuration have been acquired.

Data, Analysis, and Results

The mathematical model and classification of TC

Shock's slope angle σ, intensity J and flow deflection angle β on shock, at set parameters of flow before compression shock are reciprocally explicitly conjugated with each other. The dependency $\Lambda = lnJ(\beta)$ at set Mach number is called a shock polar or heart-like curve. The conditions of dynamic compatibility (CDC) are ratios that conjugate variables on each side of the gas-dynamic discontinuities. The conditions of dynamic compatibility on tangential discontinuity τ (Fig. 2) conjugate shock parameters in TC and are written in a form

$$J_1 J_2 = J_3 \text{ or in logarithmic form - } \Lambda_1 + \Lambda_2 = \Lambda_3, \quad (1)$$

$$\beta_1 + \beta_2 = \beta_3. \quad (2)$$

The J_i (i=1..3) – is an intensity of shock number i, β_i –is a flows angle of turn on its surface. The flow turn angles β_i can be positive or negative, depending on the direction of flow's turn on the shock. It is convenient to study the properties of triple configurations by building shock polars. The intersections of shock polar, that were build using Mach numbers before corresponding shocks, correspond to ratios (1-2). For shocks 1 and 2 (Fig. 2) it's a number M of main flow, for shock 2 – the Mach number before shock 1.

In correspondence with ratios (1-2), the intensity J_1 of a branching shock and Mach number M of oncoming flow explicitly (except for few easily fixable cases) define the type of triple configuration, parameters of shocks and flows behind them. On the other hand, setting values of adiabat γ, Mach number M of flow before triple configuration and intensity J_1 of branching shock 1, does not always explicitly define properties of other shock in the system of equations (1-2). The same parameters γ, M and J_1, can have up to three physically justified roots of system of equations, with different values β_2 and β_3 (Fig. 3 for an example). The selection of a proper root is a task that must be dealt with individually for each case.

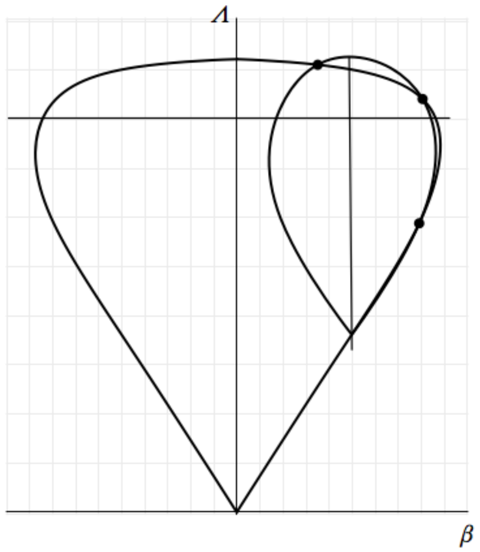
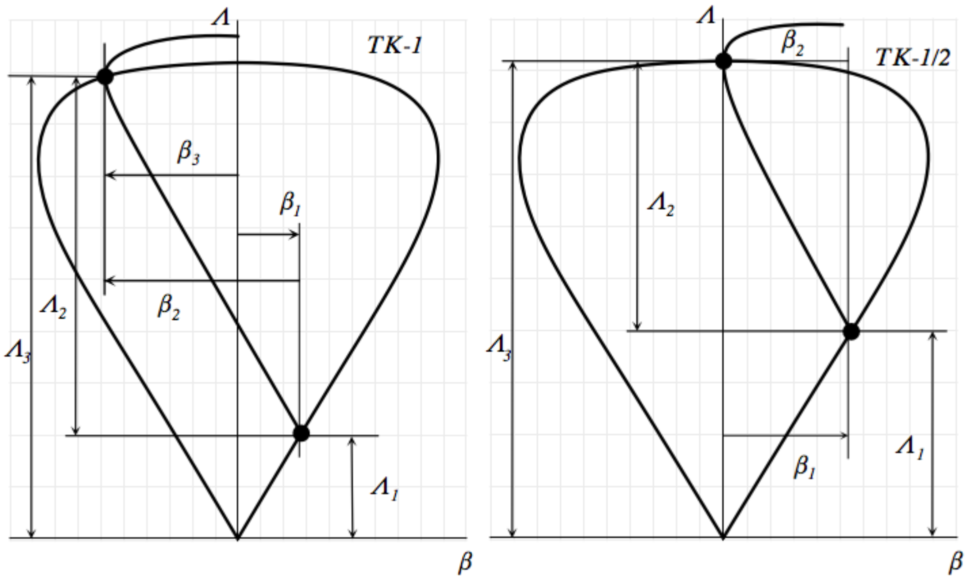


Figure 3. Three alternative solutions for TC on a polar plane

Based on ratios of flow turn direction on various shocks of shock wave structure 1-3, a three type of triple configurations can be defined.

In configurations of type 1(TC-1, Fig. 4a), the flow on shock 1 turns into a direction different from those on shocks 2 and 3. For an example, at β_1<0 the angles β_2>0, β_3>0. The triple configurations of TC-1 type appear during interference of oncoming compression shocks. As the intensity of shock 1 increases, the intersection point shock polar moves towards the apex of main polar until they overlap (Fig. 4b). *A stationary Mach configuration* (SMC or TC-

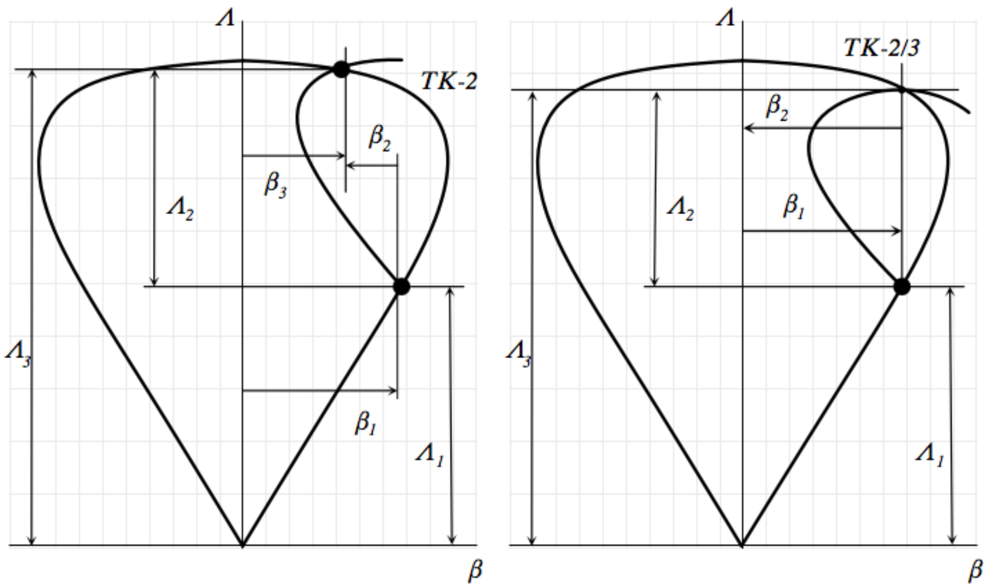
1/2) with straight main shock (β_3=0) corresponds to this case. For a special Mach number (*M=2.203* for air) a secondary polar that corresponds to SMC, at the intersection point with main polar does touch the ordinate axis but doesn't intersect it. (Fig. 4b).



a) b)

Figure 4. Triple configuration TC-1 (a) and intermediate configuration TC-1/2 (b)

In the configurations of type 2 (TC-2, Fig. 5) the direction of flow's turn on shock 2 differs from the direction of flow's turn on shocks 1 and 3 (the left branch of secondary polar intersects with the right branch of main polar).



a) b)

Figure 5. Triple configuration TC-2 (a) and intermediate configuration TC-2/3 (b)

It can be assumed, that configurations of type 2 correspond to Mach reflection of shock form a wall. As the intensity of shock 1 increases, the intersection point shock polar moves to the right until it overlaps with the apex of secondary polar (Fig. 5b). The configuration TC-2/3 with straight shock 2 (β_2=0) is a transition to TK-3.

In the configurations of type 3(TC-3, Fig. 6) flows on all shock turn into the same direction. The polars intersect with their right branches. Such SWS correspond to a special case of shock interaction of a single direction, which in analogy with one-dimensional moving shock waves, are also called overtaking compression shocks. In general case during interference of overtaking shocks, in addition to tangential discontinuity, at the interference point not one but two out coming discontinuity appears: main and reflected. The reflected discontinuity can be a shock and rarefaction wave. In this case the secondary polar doesn't intersect with the main one. The limiting point is a special intensity of shock 1, at which the secondary polar doesn't intersect with main polar, but only touches it.

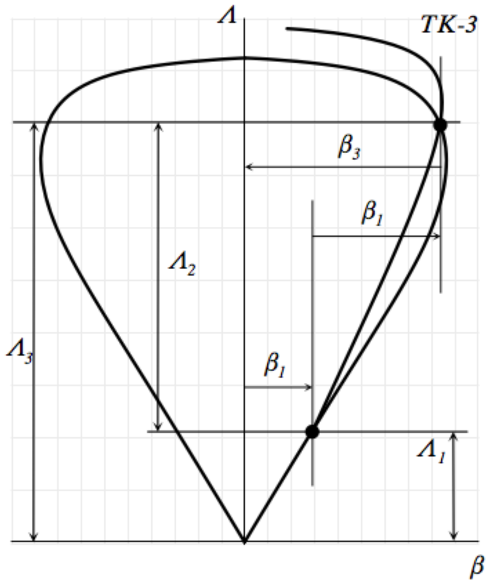


Figure 6. Triple configuration TC-3

Figure 7 shows existence regions I-III of triple configurations of various types, depending on Mach number M and slope angle of first shock σ_1.

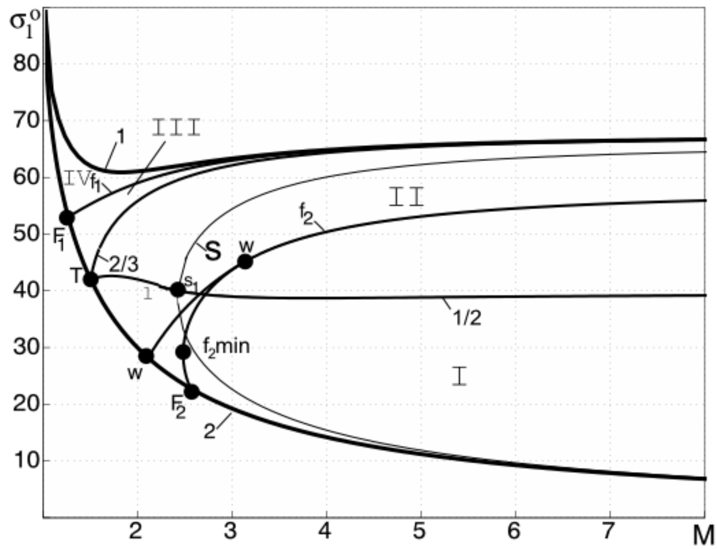


Figure 7. Existence regions of TC-1, 2, 3

On the figure 7 - I –an existence region of TC-1, II - an existence region of TC-2, III - an existence region of TC-3, IV- region in which TC cannot exist, 1/2 - a boundary between regions TC-1 and TC-2, 2/3 - boundary between regions TC-2 and TC-3, S-sonic line, f_2 - a line, that limits uncertainty region on the left, in which the conditions of dynamic compatibility on tangential discontinuity allow for existence of main solution for TC-1 or TC-2, and additional that corresponds to overtaking shocks TC-3, the curved triangle and f_2 - is an uncertainty region in which two additional solutions with overtaking shocks can exist (on the polar plane such case is shown on Fig, 3).

Let's investigate existence regions in more detail. The obvious condition for triple shock wave configuration's existence is a supersonic flow before shock 2 (Fig. 3). Mach number M_i before shock i is conjugated with Mach number after it, and with its intensity in a form of ratio:

$$M_i = \sqrt{\frac{(J_i+\varepsilon)M^2-(1-\varepsilon)(J_i^2-1)}{J_i(1+\varepsilon J_i)}}\ , \tag{3}$$

from which outcomes the equation for intensity of shock 1, at which Mach number before it is equal to one

$$J_{s1}(M) = \frac{M^2-1}{2} + \sqrt{\left(\frac{M^2-1}{2}\right)^2 + \varepsilon(M^2-1)+1}\ . \tag{4}$$

$$\sigma_{s1} = \arcsin\sqrt{\frac{J_{s1}+\varepsilon}{(1+\varepsilon)M^2}}\ . \tag{5}$$

The equation (5) sets curve 1 on figure 7. The line 2 is an opposite boundary that corresponds to $J_1=1$, i.e. the slope angle of shock 1 is equal to slope angle of characteristics α=arcsin $1/M$ (Mach angle). In the equations (3)-(5) $\varepsilon=(\gamma-1)/(\gamma+1)$, where γ – is a ratio between heat capacity at constant pressure and heat capacity at constant volume.

TC-3 with overtaking shocks 1 and 2 of same direction (Fig. 1e) corresponds to region III. The top boundary of this region (curve f_1) is set by condition of transformation of second shock into a weak discontinuity $J_2=1$, which corresponds to second order tangency of polar at a starting point of second polar and is defined by condition

$$M=\sqrt{\frac{A\left(B\pm C\right)-1}{\varepsilon}+1},$$

$$A=\frac{1+\varepsilon J_1}{\left(1+\varepsilon\right)\left(J_1\left(1-3\varepsilon\right)-4\varepsilon^2\right)},\ B=J_1\left(1-2\varepsilon-\varepsilon^2\right)-2\varepsilon^2, \tag{6}$$

$$C=2\varepsilon\sqrt{\varepsilon\left(1+\varepsilon J_1\right)\left(J_1+\varepsilon\right)}.$$

If for equation (6) we assume that $J_1=1$, then we can acquire a limiting Mach number

$$M_f=\sqrt{\frac{2\left(1\pm\sqrt{\varepsilon}\right)}{1\pm2\sqrt{\varepsilon}}}. \tag{7}$$

The equation (7) has two roots. For air the lower root is $M_{F1}=1.245$. The second root (M_{F2}=2.540 for air) is related to the uncertainty region, when in parallel with configurations TC-2 and TC-1, one or two configurations TC-3 can exist (points on the right branch of polar on Fig, 3). At Mach numbers lower than M_{F1}, the triple configurations of shock waves TC-3, могут that correspond to interaction of overtaking compression shocks 1 and 2 of same direction, cannot exist.

On the lower boundary of region III (curve 2/3) the second shock – is straight (Fig. 5d), т.е. $J_2=J_m(M_1)$, which corresponds to intermediate TC-2/3 and defined by equation

$$M^4-rM^4+q=0,$$

$$\begin{aligned}&r=\left(J_1-1\right)\left(J_1+2-\varepsilon\right)/\left(J_1+\varepsilon\right)+\left(J_1+\varepsilon\right)/\left(1+\varepsilon\right)+\left(1+\varepsilon J_1\right)^2/\left(\left(1-\varepsilon\right)\left(J_1+\varepsilon\right)^2\right),\\&q=\left(J_1-1\right)\left(J_1+2-\varepsilon\right)/\left(1-\varepsilon\right).\end{aligned} \tag{8}$$

At the point T of curve intersection (2/3) and (1) intensity $J_1=1$ and shocks 2 and 3 transform into one straight shock wave. ТК-2 The curve (1/2) that limits existence region II of (1/2) from below, also arrives to that point (Fig.1b, 2b). The corresponding Mach number

$$M_T=\sqrt{\left(2-\varepsilon\right)/\left(1-\varepsilon\right)} \tag{9}$$

limits the range of Mach numbers $M<M_T$ ($M_T=1.483$ for air), in which triple configurations with oncoming shocks (TC-1 и TC-2) cannot exist.

On the limiting line (1/2) $J_3=J_m$ and second polar intersects with main polar at its apex (Fig. 4b), which is defined by an equation

$$\sum_{k=0}^{3} A_k J_1^k = 0\,,\quad \begin{aligned} &A_3 = 1-\varepsilon^2,\\ &A_2 = -\left(\left(1+\varepsilon-\varepsilon^2+\varepsilon^3\right)J_m + 1+\varepsilon^2\right),\\ &A_1 = \varepsilon\left(1+J_m\right)\left[\left(1-\varepsilon\right)J_m - 2\right],\\ &A_0 = \left(1-\varepsilon\right)J_m\left(J_m-1\right),\\ &J_m = \left(1+\varepsilon\right)M^2-\varepsilon. \end{aligned} \tag{10}$$

The shock wave structure transforms into intermediate TC-1/2. The direction of reflected shock 3 is reversed. Line S corresponds to a flow behind reflected shock 2 with velocity $M_2=1$. It split existence region of TC-1 and TC-2 into two sub-regions: to the left of line S the flow behind reflected shock 2 is subsonic, to the right - supersonic. AT $M_2>1$ the reflected shock is certainty reflected and outcoming because disturbances of subsonic flow behind Mach stem doesn't affect it. At $M_2<1$ it's uncertain however.

Discussion and Conclusion

Figure 7 shows region IV, in which existence of triple configurations is impossible, because at any intensity of incoming shock the polar that was built by Mach number behind incoming shock is always located inside of main polar and doesn't intersect with it. At the same time such triple configurations are not observed in the experiment (Neumann paradox) (Isakova et al., 2012). One of the hypothesis, that was proposed by V.N. Uskov in his work in 1980, is a non-stationary nature of Mach reflection in those modes. It is partially proven by increased noise level from outflowing nozzles with low Mach number. The other hypothesis is K.G. Guderley (1962) model, corresponding to which, a fourth wave – a rarefaction wave is added to the triple point, thus this model is called quad wave.

To prove K.G. Guderley's (1962) hypothesis many years of research by A.N. Kraiko, Y.A. Bondar, M.S. Ivanov, G.V. Shoev, has been devoted to it, results of which, can be read in works (Ivanov, Bondar & Khotyanovsky, 2010; Isakova et al., 2012). At first glance, by using accurate calculation and various numerical methods in a setting of ideal gas model, the authors have managed to prove that reason behind inability to detect the rarefaction wave, is hidden behind lack precision of used methods. However, a detailed investigation of goals in aforementioned works, and results of calculations in a setting of a viscos gas model, show that it is to early to say about proof of K.G. Guderley's hypothesis, and the problem of Neumann paradox requires additional research.

Let's review in more detail the region of uncertain solutions. This question is quite important because oscillations and hysteresis occurrences to appear quite often in the region of uncertain solutions, which must be taken into account during design of SWS. The solution uncertainty was discovered by R. Courant & K.O. Friedrichs (1948) and thoroughly studied by L.F. Henderson & A. Lossi (1975; 1979). The line w-w on figure 7 corresponds to a contact of main and secondary polar at point on right branches. With increase of intensity of shock 1, the contact point breaks into two intersection points that are shown on right branches of polars on figure 3. A region limited by lines w-w и F_2-w forms, ТК in which various solutions to TC are possible. The solution on line F_2-w, that corresponds to lower point of polar intersection on figure 3, disappears. Thus, the line F_2-w-f_2 limits a region to the left, in which one main (TC-1 or TC-2) and one additional solutions can exist, that correspond to top point of polar intersection on figure 3.

Boundary of lines w-w - M_w = 2.089, 3.117. The lowest Mach number that corresponds to the boundary line F_2-w-f_2, is defined by an equation

$$\left(1-3\varepsilon\right)J_{1\Delta}^{3}+\varepsilon\left(1-11\varepsilon\right)J_{1\Delta}^{2}-\varepsilon\left(4+\varepsilon+9\varepsilon^{2}\right)J_{1\Delta}-\varepsilon\left(1+5\varepsilon\right)=0\,, \tag{11}$$

$$\begin{gathered}\left(1-3\varepsilon\right)^{2}M_{\Delta}^{6}-\left(3-7\varepsilon\right)\left(1-2\varepsilon+5\varepsilon^{2}\right)M_{\Delta}^{4}+\\ \left(1-\varepsilon\right)\left(3-23\varepsilon+25\varepsilon^{2}+27\varepsilon^{3}\right)M_{\Delta}-\left(1+10\varepsilon-27\varepsilon^{2}\right)\left(1-\varepsilon\right)^{2}=0.\end{gathered} \tag{12}$$

$M_{f2\min}$ = 2.462 for air.

The region in which TK-2, and TK-3 can exist simultaneously, is fully located to the right of line S, i.e. the flow behind shock 2 is always supersonic. It is more coplex with TC-2. In this case the uncertainty region is split y line S into two sub-regions.

In summary, all major ratios that allow to calculate existence region for triple configuration of every type are presented. The physical sense of intermediate (boundary) shock wave configurations have been demonstrated. The values of special Mach numbers in region below which triple configurations cannot exist have been presented.

Implications and Recommendations

The submissions can be useful for design shock wave structures with set properties in detonation engines, air collectors, technological plants, when analyzing shock wave influence on objects during an explosion.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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References

Adrianov, A. L., Starykh, A. L. & Uskov, V. N. (1995). *Interference of Stationary Gasdynamic Discontinuities*. Novosibirsk: Publishing house "Nauka"

Bulat, P. V. (2014). About the detonation engine. *American Journal of Applied Sciences, 11*(8), 1357-1364.

Bulat, P. V. & Uskov, V. N. (2014). Shock and detonation wave in terms of view of the theory of interaction gasdynamic discontinuities. *Life Science Journal, 11(8),* 307-310.

Courant, R. & Friedrichs, K. O. (1948). *Supersonic Flow and Shock Waves.* New York Interscience, 376 p.

Guderley, K. G. (1962). *The theory of transonic flow*. Oxford, New York: Pergamon Press, 264 p.

Henderson, L. F. & Lozzi, A. (1975). Experiments on transition of Mach reflexion. *Journal of Fluid Mechanics, 68(1),* 139-355.

Henderson, L. F. & Lozzi, A. (1979). Further experiments on transition to Mach reflexion. *Journal of Fluid Mechanics, 94(3),* 541-559.

Isakova, N. P., Kraiko, A. N., P'yankov, K. S. & Tillyayeva N. I. (2012). The amplification of weak shock waves in axisymmetric supersonic flow and their reflection from an axis of symmetry. *Journal of Applied Mathematics and Mechanics, 76(4),* 451-465.

Ivanov, M. S., Bondar, Y. A. & Khotyanovsky, D. V. (2010). Viscosity effects on weak irregular reflection of shock waves in steady flow. *Progress in Aerospace Sciences, 46(3),* 89-105.

Landau, L. D. & Lifshitz, E. M. (2003). *Theoretical Physics*. Moscow: Publishing house "FIZMATLIT", 326 p.

Omelchenko, A. V. & Uskov, V. N. (2002). Interference of nonstationary oblique shock waves. *Technical Physics Letters, 28(6),* 491-493.

Omelchenko, A. V. & Uskov, V. N. (1999). Optimum overtaking compression shocks with restrictions imposed on the total flow-deflection angle. *Journal of Applied Mechanics and Technical Physics, 40(4),* 638-646.

Roy, G. D., Frolov, S. M., Borisov, A. A. & Netzer, D. W. (2004). Pulse detonation propulsion: challenges, current status, and future perspective. *Progress in Energy and Combustion Science, 30(6),* 545-672.

Tao, G., Uskov, V.N. & Chernyshov, M. V. (2005). *Optimal triple configurations of stationary shocks.* \ Direct access: http://link.springer.com/chapter/10.1007/978-3-540-27009-6_74.

Uskov, V. N. (1980). *Shock Waves and Their Interaction.* Leningrad: LMI press, 285 p.

Uskov, V. N. & Chernyshov, M. V. (2006). Special and extreme triple shock-wave configurations. *Journal of Applied Mechanics and Technical Physics, 47(4),* 492-504.

Uskov, V. N. & Mostovykh, P. S. (2008). Triple configurations of traveling shock waves in inviscid gas flows. *Journal of Applied Mechanics and Technical Physics, 49(3),* 347-353.

Uskov, V. N. & Mostovykh, P. S. (2010). Interference of stationary and non-stationary shock waves. *Shock Waves, 20(2),* 119-129.

Uskov, V. N. & Omelchenko, A. V. (1995). Optimal shock-wave systems. *Fluid Dynamics, 6,* 126-134.

Uskov, V. N., Chernyshov, M. V., Erofeev, V. K. & Genkin, P. (2006). *Optimal shock-wave structures and new ideas about supersonic gas jet noise generation.* Direct access: http://icsv13.tuwien.ac.at