An Initial Investigation of a Modified Procedure for Parallel Analysis

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Abstract
Parallel analysis has been well documented to be an effective and accurate method for determining the number of factors to retain in exploratory factor analysis. Despite its theoretical and empirical advantages, the popularity of parallel analysis has been thwarted by its limited access in statistical software such as SPSS and SAS, especially in software that analyzes ordinal data. Among the few commonly used procedures, the Hayton, Allen, and Scarpello (2004) procedure requires manually computing the mean of eigenvalues from at least 50 replications. The O’Connor (2000) procedure overcomes that limitation, yet it has difficulties dealing with random missing data. To address these technical issues of parallel analysis for ordinal variables, we adapted and modified the O’Connor procedure to provide an alternative that best approximates the ordinal data by factoring in the frequency distributions of the variables (e.g., the number of response categories and the frequency of each response category per variable). Our procedure has a slightly different theoretical rationale from O’Connor’s as well as a practical advantage in dealing with missing data.

Key words: Categorical data, factor analysis, factor retention, parallel analysis, SAS program
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Parallel Analysis: An Overview

Factor analysis can be generally characterized as a set of multivariate statistical methods for data reduction and for establishing a more parsimonious relationship among measured variables (Fabrigar, Wegener, MacCallum, & Strahan, 1999). The American Educational Research Association, the American Psychological Association, and the National Council on Measurement in Education (1999) defined it as “any of several statistical methods of describing the interrelationships of a set of variables by statistically deriving new variables, called factors, that are fewer in number than the original set of variables” (p. 175). Factor analysis is also an important tool that is used to investigate the construct validity of many psychological measures. A critical component in the process of construct validation is to determine the number of factors on the basis of exploratory factor analysis (EFA). Compared to confirmatory factor analysis, EFA is particularly useful when there is little theoretical basis for specifying the number of factors or the relationship between the variables and latent factors (Hurley et al., 1997). The selection of the appropriate number of factors is one of the most important decisions in EFA, since underfactoring (selecting too few factors) or overfactoring (selecting too many factors) can have a deleterious impact on subsequent confirmatory analysis and item estimates (Brown, 2006). Ideally, the number of factors should represent qualitatively distinct constructs that conform to the underlying guiding theory.

Parallel analysis is one of the methods to help determine the number of factors in EFA (Carraher & Buckley, 1995; Horn, 1965). The underlying rational for parallel analysis is that the eigenvalues of the salient factors from real data with a valid latent factor structure should be larger than the eigenvalues of the corresponding factors generated from random data (Ford, MacCallum, & Tait, 1986; Lautenschlager, 1989). Parallel analysis requires the construction of a number of correlation matrices of random variables based on the same sample size and number of variables in the real data set. The mean eigenvalues from the random correlation matrices are compared to the eigenvalues from the real data correlation matrix. For example, the average of the first eigenvalues from the random data is compared to the first eigenvalue from the real data, and the average of the second eigenvalues from the random data is compared to the second eigenvalue from the real data. Namely, the eigenvalues observed from the real data should be larger than the corresponding average eigenvalues from the random data. Otherwise, the observed eigenvalues are considered a consequence of sampling error (Glorfeld, 1995; Horn,
Accordingly, a substantial and meaningful factor should account for more variance than expected by chance alone.

To perform parallel analysis, a number of \( k \) random data sets should be generated and the average of the eigenvalues from these \( k \) replications should be compared with the eigenvalues from the real data. There is no set rule regarding the number of random data sets that should be generated for parallel analysis. Horn (1965) suggested that the number should be reasonably large; other researchers suggested from 500–1,000 repetitions (Hayton et al., 2004). However, no significant difference has been found between using one random data set and 100 random data sets (Crawford & Koopman, 1979). Although there is no documented evidence, it appears that 50 data sets are reasonably large enough to perform parallel analysis (Hayton et al.). In terms of evaluating the results, a modification has been proposed by Glorfeld (1995) to Horn’s approach using the mean eigenvalues. The modified procedure for parallel analysis replaces the usual average of the eigenvalues by the upper 95th percentile (or any other reasonable percentile) in determining the number of factors. This reduces the tendency to overextract or extract poorly supported factors.

Besides parallel analysis, other factor retention methods are available to determine the number of factors, including the Kaiser-Guttman rule (Guttman, 1954; Kaiser, 1960, 1970), the scree test (Cattell, 1966), the minimum average partial (MAP) method (Velicer, 1976), and the maximum likelihood (ML) estimation. The Kaiser-Guttman rule and the scree test are probably the two most widely used methods based on eigenvalues. The Kaiser-Guttman rule is very straightforward. It specifies that the number of factors to retain equals the number of eigenvalues larger than one. The logic that underlies this rule is that when an eigenvalue is less than one, the variance explained by the factor is smaller than that explained by a single indicator (Brown, 2006). The Kaiser-Guttman rule is the default option in many popular software programs such as SPSS, due to its simplicity. However, the population-based characteristics of this method make it vulnerable to overestimating or underestimating the number of factors to retain (Horn, 1965).

In a population correlation matrix, the eigenvalues for uncorrelated variables are one. When it comes to looking at a finite sample, sampling error and least squares bias cause the eigenvalues to vary from these values (Hayton et al., 2004). Lance, Butts, and Michels (2006) raised the point that as a widely applied method, the Kaiser-Guttman rule has been documented repeatedly as retaining too many—sometimes far too many—factors, thus introducing
difficulties in determining a reasonable number of factors. In addition, the Kaiser-Guttman rule is intended to provide a lower bound for the rank of the correlation matrix. Therefore, it should be used to specify the upper bound for the number of factors to retain. However, it has been often used to determine the exact number of factors (Gorsuch, 1983).

The scree test is another popular approach in helping to determine the number of factors in EFA. Its central feature is a graph, with a horizontal axis representing the factors and a vertical axis representing the eigenvalues. The graph’s function is to help locate the last substantial decline in the magnitude of the eigenvalues. This provides an indication that a major limitation of this approach lies in its subjectivity. It is less of a problem when the sample size is large and the data contain well-defined factors (Gorsuch, 1983). However, when the sample is small, it is less likely that there are apparent or clear shifts in the slope. In this case, the scree test method is prone to subjectivity and possibly ambiguity.

The MAP method advanced by Velicer (1976) is based on partial correlations. This method calculates the average of squared partial correlations after each component has been partialled out. When the minimum average squared partial correlation is reached, no further components are extracted. In this case, the residual matrix closely resembles an identity matrix. Noted by Velicer, this method can be applied with any covariance matrix and can be conceptualized as factors that represent more than one variable. Some FORTRAN programming is available to conduct the MAP procedure (Reddon, 1985).

Compared to the methods based on eigenvalues, the ML method of factor extraction is gaining popularity. The issue of determining the number of factors can be conceptualized as choosing the most appropriate model from a series of factor analysis models that differ in their structure (Fabrigar et al., 1999). The goal is to select a model that accounts for substantially more variance than alternative models. An advantage of the ML method is that it offers a wide range of goodness-of-fit indices that can be utilized to determine the number of factors to retain. Some commonly used fit measures include the likelihood ratio statistic (Lawley, 1940), the reliability coefficient for ML factor analysis solutions (Tucker & Lewis, 1973), the root mean-square error of approximation fit index (RMSEA; Browne & Cudeck, 1992), and the expected cross-validation index (ECVI; Browne & Cudeck, 1989). A primary limitation of the ML method is that it requires a strong assumption of multivariate normality. Inaccurate results may occur if this
assumption is severely violated (Curran, West, & Finch, 1996). In addition, this method is heavily influenced by sample size (Hayton et al., 2004).

Parallel analysis has been well documented to be a robust and accurate method for determining the number of factors to retain. Results from various studies have demonstrated that parallel analysis performed better than the Kaiser-Guttman rule (Silverstein, 1987) and many other procedures. For example, it has been found to be more accurate than the ML method (Humphreys & Montanelli, 1975). Additionally, parallel analysis and MAP were found to be the two most accurate methods when compared to the Kaiser-Guttman rule, scree test, and the chi-square test (Bartlett, 1950). More recently, parallel analysis was again proven to be the most accurate method for determining the number of factors to retain, followed by MAP, with the Kaiser-Guttman rule being the least accurate method (Eaton, Velicer, & Fava, 1999; Zwick & Velicer, 1986).

Glorfeld (1995) pointed out that there is little reason not to use parallel analysis, considering the wealth of evidence favoring this procedure. However, too many researchers rely on the Kaiser-Guttman rule, when parallel analysis should be used instead. Hayton et al. (2004) conducted a review of the studies involving EFA in two major journals between 1990 and 1999, the Academy of Management Journal and the Journal of Management. They found that among the 142 papers using EFA, 47.2% of the studies used some combination of the Kaiser-Guttman rule and the scree test; 25.4% of the studies solely relied on the Kaiser-Guttman rule, and 5.6% of the studies solely relied on the scree test. Surprisingly, none of the studies reported using other methods for determining the number of factors, such as parallel analysis. Fabrigar et al. (1999) pointed out that the lack of widespread use of this procedure includes a paucity of awareness by researchers, insufficient training in graduate school programs, inadequate coverage in textbooks introducing factor analysis knowledge, and simply the inclination to use the methods most frequently used (e.g., Kaiser-Guttman rule, scree test). Another major reason for the limited use of parallel analysis is the restricted access to readily available software programs that can perform parallel analysis. As stated by Brown (2006, p. 29), “A practical drawback of the [parallel analysis] procedure is that it is not available in major statistical software packages such as SAS and SPSS.”

Among the few available examples of the application of parallel analysis, Hayton et al. (2004) provided syntax for conducting the procedure in SPSS. The syntax takes into
consideration the characteristics of ordinal data represented by Likert-type scales, which is a popular format for psychological surveys. Being similar to the real data is a critical characteristic of parallel analysis. The more closely the random data approximate the real data, except for the correlational structure, the more accurate the results are likely to be. Particularly with ordinal data, the factors to be considered when conducting parallel analysis include the number of categories for each variable (e.g., 4-point, Likert-type scale) and the frequency of each response category per variable. The frequencies are important because the variance structure is, in general, not independent of the mean for nonnormally distributed random variables.

As thorough as the Hayton et al. (2004) procedure is, the syntax has a major limitation: It generates eigenvalues for only one random data set. As described above, in order for this method to be valid, at least 50 replications are required. This means that the user would need to run the syntax 50 times and average the results manually, which could be costly with respect to both time and labor.

O’Connor (2000, 2001) also provided syntax for conducting parallel analysis in SPSS, SAS, and MATLAB. Besides the programs that are available for data from normal distributions on this Web site, the “raw” programs listed on the same page are based on a permutation of the raw data, which is suitable for variables from nonnormal distributions. In particular, each column of the data matrix is permuted, so that the response vector of a new case consists of the responses of different old cases. This way, the dependence structure of the original data set is eliminated in the new data sets, but the marginal frequency distributions are preserved. In addition, the syntax is able to produce mean eigenvalues based on a flexible number of replications, thus relaxing the constraints of the procedure introduced by Hayton et al. (2004).

**Purpose of This Study**

To further refine the approach for parallel analysis, we adapted the SAS syntax provided by O’Connor (2000) and made some modifications to provide an alternative way of conducting parallel analysis, especially for ordinal data. There are some theoretical and practical differences between the O’Connor (2000) approach and our approach.

On a theoretical level, instead of permuting the columns of the original data set, we proceed by estimating the relative frequencies for each variable in the original data set and subsequently simulate the data for this variable in each new data set by drawing from a multinomial distribution for which the probabilities are given by these relative frequencies.
Contrary to O’Connor’s (2000) raw programs, the marginal frequency distributions of the variables in the data sets simulated using this procedure will not be exactly the same as those of the original data, due to sampling error. A consequence is that in our procedure, each case is an independent draw, whereas in O’Connor’s (2000) raw program, dependencies between cases are introduced by conditioning on the marginal frequencies (i.e., the values for the last case are determined by the values of the other cases). Since the dependencies vanish with increasing sample size, this theoretical distinction should result in negligible differences when the sample size is large enough.

On a practical level, our procedure deals with random missing data in a more straightforward manner than the O’Connor (2000) approach. For our approach, the frequencies for each variable are computed on all available cases. The simulated data will not contain missing values, which is required by the subsequent SAS IML (interactive matrix language) code. In contrast, O’Connor’s (2001) permutation-based approach will yield missing values in simulated data sets, thus violating the requirement of the subsequent SAS IML code. Of course, one could get around the missing data issue through, for example, imputing data for the missing observations. The most obvious way to do so would be to use the marginal relative frequencies of the variables, which is similar to our approach. Furthermore, imputing data will no longer assure that the marginal frequencies of the simulated data exactly match those of the original data. In conclusion, the theoretical difference between these two approaches is very small as sample size approaches infinity. However, our approach has a practical advantage in terms of how it deals with random missing data. The SAS syntax for the parallel analysis procedure developed in this study is described in the following section.

**The Current Procedure for Parallel Analysis**

The procedure we developed, represented by the SAS syntax in the appendix, is comprised of four major components. Each component is described in detail, followed by an example illustrating the output from the syntax at the very end.

**Part I: Importing Data and Generating a Frequency Table**

1. Import the real data through specifying the file path and file name.
2. Specify the type of data file using the keyword DBMS. An Excel file was used here.
3. Specify the output file using the keyword OUT.
4. Calculate the frequencies of response categories of the real data using PROC FREQ.
5. Generate the frequency table for the real data.

Part II: Generating Random Data From the Multivariate Distribution and Computing Eigenvalues

1. Specify the number of random cases using Ncases.
2. Specify the number of random variables using Nvars.
3. Specify the number of random data sets using Ndatsets.
4. Specify the number of response categories using Nrespcat.
5. Specify the probability of each response category using Prob.
6. Specify the kind of parallel analysis using Kind, where kind = 1 referring to principal components analysis and kind = 2 referring to principal axis or common factor analysis.
7. Generate random variables from the multinomial distribution.
8. Specify the frequency function that corresponds to the frequency of each response category for the real data.
9. Specify the algorithm for conducting principal component analysis (PCA) or specify the algorithm for factor analysis.
10. The default number of categories for the random data is set to four. Change the ELSE IF command accordingly if the number of categories is some other number.

If the eventual goal is to conduct a PCA on the real data, then parallel analyses using the principal components option in the SAS codes should be used. However, when the eventual goal is to conduct a common factor analysis or principal axis factor analysis on the real data, then the decision whether to use PCA or factor analysis for parallel analysis becomes controversial. As noted by O’Connor (2000), experts do not agree on whether principal component eigenvalues or common and principal axis factor eigenvalues should be used to determine the number of factors. Some experts (e.g., Gorsuch, 1983; Humphreys & Montanelli, 1975) have argued that if the
eventual goal is to conduct a common factor analysis or principal axis factor analysis, then
communalities should be placed on the diagonal of a correlation matrix, before the eigenvalues
are extracted based on the correlation matrix. Others (e.g., Cattell, 1966) have extracted and
examined eigenvalues based on the correlation matrix of PCA and have used these eigenvalues
to determine the number of factors for the common and principal axis factor analysis. The latter
is the procedure used in the scree tests and in SPSS and SAS factor analysis. Here in our SAS
codes, we provide both options for the user to make the decision whether to use PCA or factor
analysis.

**Part III: Output Options**

1. Generate frequency tables.

2. Generate the mean and percentile of the eigenvalues for each component for the \( n \)
random data sets.

An Example of Parallel Analysis for the Learning Strategies Scale

The Learning Strategies Scale developed by the Center for New Constructs at ETS was
used as an example to illustrate the use of parallel analysis. This scale consists of 52 items,
represented by a 4-point Likert scale. This instrument was developed on the basis of an extensive
literature review and was designed to measure multiple aspects of student behavioral, cognitive,
and metacognitive strategies in learning. Students were asked to select the response (strongly
agree, agree, disagree, or strongly disagree) that best described how often in the past year each
statement had been true for them. A sample question is, “I use headings when I’m taking notes
so that I can find information when I look through them again.” Respondents were 238 middle
school students from three schools in New Jersey, which constituted sixth graders (39%),
seventh graders (24%), and eighth graders (37%). To examine the construct validity of the
Learning Strategies Scale, EFA was conducted, using the parallel analysis procedure.

Figure 1 illustrates the results from the EFA using the new procedure for parallel analysis
based on 50 replications. Both the mean and the 95 percentiles of the eigenvalues from the
simulated random data were included. As shown in Figure 1, the mean eigenvalues and the 95
percentiles are very close. Four factors from the real data had larger eigenvalues than the factors
generated from the random data.
Figure 1. Results from parallel analysis (PA) for the Learning Strategies Scale.

The O’Connor (2000) procedure for ordinal data was also tested, and the same number of factors emerged from the results. Table 1 shows the correlations among the eigenvalues from the real-data, mean eigenvalues generated using the O’Connor procedure and the new procedure. In addition, the eigenvalues ordered by size and estimated using the newly developed procedure and the eigenvalues ordered by size and estimated from the O’Connor procedure were in close correspondence, as evidenced by the almost perfect correlation.

Table 1

Correlations Between Real-Data Eigenvalues and Generated Eigenvalues

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Real-data eigenvalues</th>
<th>O’Connor procedure</th>
<th>New procedure: mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real data</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>O’Connor procedure</td>
<td>0.6831</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>New procedure: mean</td>
<td>0.6845</td>
<td>0.9998</td>
<td>1.0000</td>
</tr>
<tr>
<td>New procedure: 95th percentile</td>
<td>0.6956</td>
<td>0.9996</td>
<td>0.9997</td>
</tr>
</tbody>
</table>

Given the fact that the magnitude of the 4th factor from the real data was slightly greater than the magnitude of the corresponding factor from the random data (Figure 1), a 4-factor
model might be indicated. However, results of a 4-factor EFA model indicated that only two items loaded on the fourth factor, and these two items also loaded saliently on another factor using the larger than .30 loading rule. On the basis of both content analysis and model parsimony, 3 factors were retained for the Learning Strategies Scale, measuring (a) effective strategies, (b) bad habits, and (c) metacognition. It is notable that 17 factors emerged when the Kaiser-Guttman rule was used, which was not very informative in determining how many factors to retain.

**Conclusion and Discussion**

We hope that this newly developed procedure can provide some practical value in determining the number of factors when conducting EFA. As described above, parallel analysis has been demonstrated to be an effective method for determining the number of factors to retain. The SAS procedure we developed offered a more straightforward solution for categorical data with missing cases. Readers are encouraged to use parallel analysis for factor retention when appropriate, in conjunction with content analysis and other types of evidence. The decision about factor structure should be made on the basis of consistent evidence from multiple sources reflecting both exploratory and confirmatory approaches.

A further refinement of the parallel analysis procedure for ordinal variables would be to compute the eigenvalues on the basis of the polychoric correlation matrix, rather than of the Pearson correlation matrix. However, the computation of a polychoric correlation matrix is computationally intensive, so that conducting a parallel analysis would become cumbersome. Nevertheless, once the number of factors has been selected, further confirmatory factor analyses should be conducted using an estimation procedure that is most appropriate for the data at hand.
References


Appendix

The procedure we developed, represented by the SAS syntax below, is composed of four major components. This procedure is specially tailored for questionnaires using ordinal scales. In this procedure, the variables are generated from a multinomial distribution, the number of categories can be freely specified, and the frequency of each response category is approximated.

/*****Part I: Data input******/

/*read data, specify correct location*/
PROC IMPORT OUT= WORK.example
   DATAFILE= "C:\E_Disk\LSAB_52.xls"
   DBMS=EXCEL REPLACE;
   SHEET="LSAB 52";
   GETNAMES=YES;
   MIXED=NO;
   SCANTEXT=YES;
   USEDATE=YES;
   SCANTIME=YES;
RUN;

/*compute frequencies of response categories and write to SAS dataset*/
ods trace on;
ods output OneWayFreqs=freqs ;
proc freq data=example;
tables V1-V52 ;/*specify variable names here*/
run;

data freqs;
set freqs (keep=Frequency);
run;

/*****Part II: Generating variables from a multinomial distribution and Computing Eigenvalues******/

options nocenter nodate nonumber linesize=90; title;
proc iml;
start RANDMULTINOMIAL( N, NumTrials, Prob );
mP = rowvec(Prob);
\begin{verbatim}
d = ncol(mP);
/* check parameters */
if N<1 then do;
    print "The requested number of observations should be at least 1:" N; stop;
end;
if NumTrials <1 then do;
    print "The number of trials should be at least 1:" NumTrials; stop;
end;
if abs(1 - sum(Prob))>1e-8 then do;
    print "The probabilities must sum to 1:" (sum(Prob)) [label="Sum"]; stop;
end;
if ncol(loc(Prob>0)) < d then do;
    print "Each probability should be positive:" Prob; stop;
end;

b = mP;
order = d + 1 - rank(mP);
mP[order] = b; /*For efficiency, sort probabilities in a descending order*/
X = j(n,d,0);
z = 0;

do i = 1 to N;
    if d = 1 then do;
        X[i] = NumTrials;
    end;
    else do;
        m = NumTrials;
        q = 1;
        call randgen(z,'BINOM',m,mP[1]);
        X[i,1] = z;
        do j = 2 to d-1 by 1 while ( m > 0 ); /* m should be positive */
            m = m - X[i,j-1];
            q = q - mP[j-1];
            newp = mP[j]/q;
            call randgen(z,'BINOM',m,newp);
            X[i,j] = z;
        end;
        X[i,d] = m - z ; /* to have the sum of x_i's = s*/
    end;
end;
outX = X;
outX[ , 1:d] = X[ , order]; /* rearrange the order according to the given probabilities*/
\end{verbatim}
return(outX);
finish;
store module=RANDMULTINOMIAL;
reset noname; seed = 1953125;

/*load freqs of response categories into matrix F*/
use freqs;
read all into F;

/* enter your specifications here */
Ncases = 238;
Nvars = 52;
Ndatsets = 50;
percent = 95;
Nrespcat=4;/*all response categories have to be observed*/
/* Specify the desired kind of parallell analysis, where:
1 = principal components analysis
2 = principal axis/common factor analysis */
kind = 1;

/* set diagonal to a column vector module */
start setdiag(matname,vector);
do i = 1 to nrow(matname);
do j = 1 to ncol(matname);
if (i = j) then; matname[i,j] = vector[i,1];
end;end;
finish;

/* row sums module */
start rsum(matname);
rsums =j(nrow(matname),1);
do rows = 1 to nrow(matname);
dumr = matname[rows,];
rsums[rows,1]=sum(dumr);
end;
return(rsums);
finish;

/* principal components analysis */
if kind = 1 then do;
/* computing random data correlation matrices & eigenvalues */
evals = j(Nvars,Ndatsets,-9999);
nm1 = 1 / (Ncases-1);
do nds = 1 to Ndatsets;
x = j(Ncases,Nvars);
   do a_var = 1 to Nvars;
      b=(a_var-1)*Nrespcat+1; /* read the freqs of the response categories of variable a_var*/
   e=a_var*Nrespcat;
   Prob=F[b:e,1]/sum(F[b:e,1]); /* normalize, freqs into relative freqs */
   y = RANDMULTINOMIAL( Ncases, 1, Prob );
   do a_case = 1 to Ncases;
      if y[a_case,1] > 0 then 
         x[a_case,a_var] = 1;
      else if y[a_case,2] > 0 then
         x[a_case,a_var] = 2;
      else if y[a_case,3] > 0 then
         x[a_case,a_var] = 3;
      else if y[a_case,4] > 0 then
         x[a_case,a_var] = 4;
   end;
   vcv = nm1 * (t(x)*x - ((t(x[+,:])*x[+,:])/Ncases));
   d = inv(diag(sqrt(vecdiag(vcv))));
   evals[,nds] = eigval(d * vcv * d);
   end;
end;

/* principal axis/common factor analysis with SMCs on the diagonal */
if kind = 2 then do;
   /* computing random data correlation matrices & eigenvalues */
   evals = j(Nvars,Ndatsets,-9999);
   nm1 = 1 / (Ncases-1);
   do nds = 1 to Ndatsets;
      x = normal(j(Ncases,Nvars));
      /*x = normal(j(Ncases,Nvars));*/
      x = j(Ncases,Nvars);
      do a_var = 1 to Nvars;
         b=(a_var-1)*Nrespcat+1; /* read the freqs of the response categories of variable a_var*/
      e=a_var*Nrespcat;
      Prob=F[b:e,1]/sum(F[b:e,1]); /* normalize, freqs into relative freqs */
      y = RANDMULTINOMIAL( Ncases, 1, Prob );
   end;
end;
do a_case = 1 to Ncases;
   if y[a_case,1] > 0 then
      x[a_case,a_var] = 1;
   else if y[a_case,2] > 0 then
      x[a_case,a_var] = 2;
   else if y[a_case,3] > 0 then
      x[a_case,a_var] = 3;
   else if y[a_case,4] > 0 then
      x[a_case,a_var] = 4;
   end;
end;

vcv = nm1 * (t(x)*x - ((t(x[+])*x[+])/Ncases));
d = inv(diag(sqrt(vecdiag(vcv))));
r = d * vcv * d;
smc = 1 - (1 / vecdiag(inv(r)));
run setdiag(r,smc);
evals[,nds] = eigval(r);
end;
end;

/* identifying the eigenvalues corresponding to the desired percentile */
num = round((percent*Ndatsets)/100);
results = j(nvars,3,-9999);
s = 1:Nvars;
results[,1] = t(s);
do root = 1 to Nvars;
ranks = rank(evals[root,]);
do col = 1 to Ndatsets;
if (ranks[1,col] = num) then do;
   results[root,3] = evals[root,col];
col = Ndatsets;
end;
end;
end;
results[,2] = evals[+,] / Ndatsets;

/*-------Part III: Output options------*/

print, "Parallel Analysis:";
if (kind = 1) then;  print, "Principal Components";
if (kind = 2) then do;
print "Principal Axis / Common Factor Analysis";
print "Compare the random data eigenvalues below to the";
print "real-data eigenvalues that are obtained from a";
print "Common Factor Analysis in which the # of factors";
print "extracted equals the # of variables/items, and the";
print "number of iterations is fixed at zero (maxiter=0), as in:";
print "proc factor data=trial priors=smc maxiter = 0; run; ";
print "Or use the 'rawpar.sas program' to obtain the ";
print "baseline real-data eigenvalues.";
end;
specifs = (ncases // Nvars // Ndatsets // percent);
rlabes = {"Ncases" "Nvars" "Ndatsets" "Percent"};
print/ "Specifications for this Run:" , specifs[rowname=rlabes];
clabes={"Root" "Means" "Prctyle"};
print "Random Data Eigenvalues" , results[colname=clabes format=12.6];
quit;