What can student work show?

From playing a game to exploring probability theory


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Rich learning tasks embedded within a familiar context allow students to work like mathematicians while making sense of the mathematics. This article demonstrates how 11–12 year-old students were able to employ all of the proficiency strands while demonstrating a deep understanding of some of the 'big ideas' of probabilistic thinking

Introduction

It is expected that teachers will use multiple sources of evidence to assess children's learning in mathematics. The evidence garnered may be informed by methods such as observations, conversations with learners and their recording of ideas (Neil & Fisher, 2010). Such practices assist teachers to gain insights into children's thinking. Multiple opportunities to learn, review and consolidate ideas over time are therefore required for learners to demonstrate their understandings. Suh, Johnston, Jamieson & Mills, (2008) suggest that students be encouraged to use different representations to justify their thinking and reasoning in mathematics.

When mathematical concepts are introduced to learners it can be helpful to ensure those ideas are embedded within a familiar context. Attard (2012) states that motivating and assisting children to engage with particular ideas can mean linking to their prior knowledge, encouraging cognitive development and offering opportunities to be actively involved.

Rich learning tasks can aid children to learn mathematics. Such tasks offer opportunities for children to speculate, hypothesise, discuss, make decisions and justify their responses (Breyfogle & Williams, 2008). Sullivan, Clarke & Clarke, (2013, p.14) state that "… the best tasks are those that provide appropriate contexts and complexity, that stimulate construction of cognitive networks, thinking, creativity and reflection; and that address significant mathematical topics explicitly."

Developing probabilistic thinking is an integral part of the mathematics curriculum. Teachers are expected to provide opportunities for learners to think about the chances of an event occurring and provide a range of tasks that encourage the emergence of children's thinking and explanations. When children engage in probability tasks misconceptions can be revealed that necessitate the exploration of experimental and theoretical probability of a particular event occurring (Barnes, 1998; Neill, 2010). Jones, Langrell, Thornton & Mogill (1997) argue also that children's thinking in probability will not necessarily follow an ordered progression and nor will it be uniform. While rich learning tasks can enable engagement, teacher intervention is often needed to extend learners' thinking.

There are a number of key concepts that learners need to explore and understand when developing probabilistic thinking. These include notions of sample space, possible outcomes, combinations, the probability of an event occurring and the assigning of a numerical probability measure when comparing different outcomes (Neil, 2010; Barnes, 1998).

Background

Research was conducted with a class of 19 children who were Year 7–8 (11–12 years old). They had explored a contextually based probability task that involved throwing two dice and adding the numbers to get a total. After playing the game several times, some children realised that particular outcomes were more likely to occur than others but could not explain why. Ensuing discussion revealed to the teacher that some learners needed further opportunities to focus, review and consolidate their thinking about quantifying chance. The following task was utilised to this end. Children's visual recordings and conversations enabled the teacher to gain further insights about their learning and thereby inform her assessments against curriculum expectations.

Task considerations

This task centres around an environmental context familiar to many children. It involves the saving of dolphins (six counters allocated to each player) that are stranded on sandbanks (labelled 1–6 on a shared sheet for both players. See Figure 1) and need to be 'saved'. Children play in pairs. Each pair requires two dice, one playing board between them (refer Figure 1), and twelve counters (six of one colour for Player One and six of a different colour for Player Two). Players take turns to throw the two dice and subtract the smaller number from the larger. The outcome will equate with a designated number on one of the sandbanks. Players are able to 'save' one of their dolphins if they have a counter located on that number sandbank. Before they start the game players have to decide where their six dolphins are stranded. The 'winner' is the player who saves their six dolphins first.

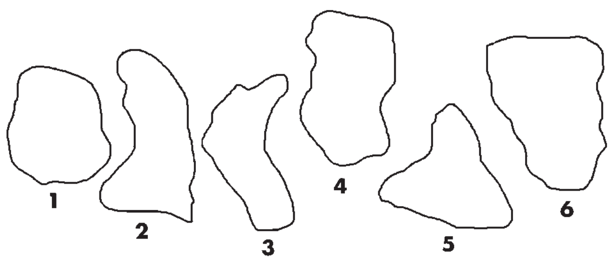

Figure 1: Playing board.

Pedagogical justification for the task

Presenting the task in a context that children can relate to and understand can support them to engage with it. Teachers in other locations might select an alternative context that better suits their environment.

Asking children to decide on which sandbanks they might locate their dolphin allows them to make a prediction. This practice is an important aspect of probability that can reveal possible misconceptions. For example, to some children it would not matter where the dolphins were located because they considered that the chance of each sandbank number coming up "will be the same". Teachers noticing any misunderstandings can take steps to address such thinking. They can ask children to place their counters according to their prediction and play the game. Following the game(s) they need to ask children, "What do you notice? Can you explain what you see?"

Finding the difference between the numbers on two dice means that it is likely that all children at Year 7–8 will be able to access and participate in the task. Children can begin by playing the game to see who wins. After playing a few times some children will notice that certain numbers seem to be appearing more often than others and can begin to formulate conjectures about why this might be occurring.

Posing and refuting conjectures provides an opportunity for the teacher to suggest that children could play again and collect data to consider whether or not they can justify their conjecture. Supporting children to collect and record data systematically is an important part of learning in probability (Hurrell, 2011). Constructing a tally chart can be a familiar method of recording for most children at this level as demonstrated in Figure 2.

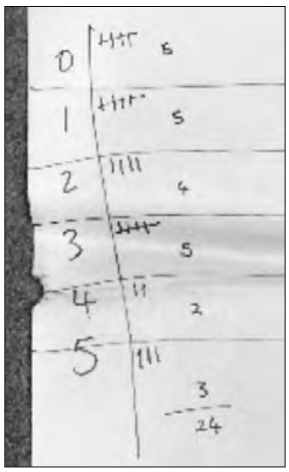

Figure 2: Tally chart.

This data collection can lead to children discussing the frequency of particular outcomes, for example noticing that "five out of twenty-four are ones." Acknowledging the frequency of events is important for children with regard to experimental probability. The frequency of a number occurring in a particular game will invariably be different in subsequent games. This situation then provides a natural opportunity to explore the theoretical probability of any number occurring. Such occasions create additional learning opportunities for children who need to understand that it is not possible to accurately predict the result of the next throw. Assistance from the classroom teacher can help children to understand, reason and explain that they can only predict the likelihood of an event occurring because the result of each throw is independent of those before it.

Exploring the theoretical probability

When asked to consider why some outcomes might appear more times than others after throwing two dice and finding the difference, children can begin an exploration of the theoretical probability.

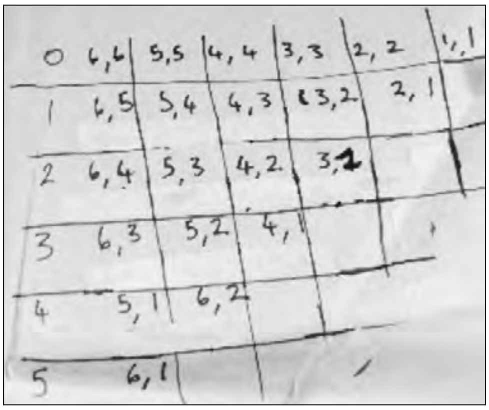

Figure 3: Recording of initial attempt to find the sample space for 36 possible outcomes.

Supporting children to make use of a visual structure introduced from a previous task (where throwing two dice and adding the numbers was required) can be helpful. Children can use this structure for organising their data. In the dolphin task one such example of a recording is seen in Figure 3 above.

Recording 36 possible outcomes

When finding all members of the sample space for throwing two dice and subtracting the numbers, it can be tempting for children to decide that the 21 shown above in Figure 3 is the total number of possible outcomes. Teacher assessment followed by intervention can support learners to ascertain how the 36 possible outcomes are derived.

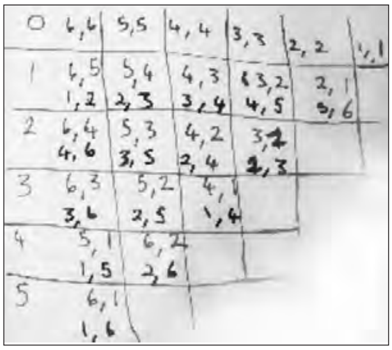

Figure 4: Revised recording of the sample space for showing 36 possible outcomes.

Using two different coloured dice will help children to realise for example, that a difference of two is possible from a red six with a white four as well as from a red four and a white six. Understanding and accepting that such combinations result in two separate possible outcomes is crucial for appreciating the need to add further combinations to the recording. Asking learners if there is any other way of getting however, a three and a three with two dice, will help them to see and understand that when doubles are thrown, there is only one possible combination for each. This confirms that there are only six ways of getting a zero. Consequently, adding the reversed combinations to their chart can confirm how the 36 possible outcomes are derived.

Creating a tree diagram

Another strategy to support children to record the total possible outcomes for this task is a tree diagram.This particular recording system can assist children to represent and justify their thinking in a visually explicit way and provide an additional artifact if required, for assessment purposes.

Figure 5: Tree diagram illustrating the 36 possible outcomes.

Using an array structure

When children have had experiences using an array structure in mathematics, for example, calculating multi-digit multiplication (Young-Loveridge & Mills, 2010), they can learn to appreciate how data can be recorded using a similar format as shown in Figure 6.

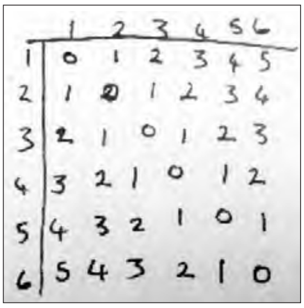

Figure 6: Using an array to show the outcomes when throwing two dice and subtracting the numbers.

For the dolphin task the structure of an array provides an avenue for displaying the 36 possible outcomes when throwing two dice (one noted along the top and the other along the side) and finding the difference. Some children will easily recognise a pattern when filling in the array and have no need to make each calculation. A completed array also offers children a quick reference point to ascertain the chances of a particular outcome occurring especially when compared with other possibilities.

Role of the teacher

Teachers require sufficient content knowledge so that they can ask a range of questions (for example, why do you think there are more zeros than fours? [Refer Figure 2]. Do you think a red six and a white four is different to a white six and a red four? [Refer Figure 4]). Teachers need to listen to children's responses and act appropriately (Higgins & Parsons, 2011). For the dolphin task, teachers have to know the probability ideas that the task affords and the pedagogical knowledge required to support children to move beyond subjective probability towards a theoretical perspective. It may be necessary to teach particular ways of representing data and making sense of it so that appropriate probabilistic thinking can be developed and extended.

Conclusion

The dolphin task proved accessible to these students and provided another opportunity for them to review and consolidate the probabilistic thinking they had recently encountered. Appropriate teacher intervention stimulated children's thinking beyond the initial playing of the "game". Significant probability ideas could be drawn out and made explicit, to challenge and enhance learning. As well as providing material for assessment, the visual representations acted as referents for discussion between the children and their teacher. The recordings and subsequent conversations provided multiple sources of evidence to inform assessment of children's learning against curriculum expectations.

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