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Improving Junior High School Students' Mathematical Analogical Ability Using Discovery Learning Method

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Abstract

The aim of this study was to identify the influence of discovery learning method towards the mathematical analogical ability of junior high school's students. This is a research using factorial design 2x2 with ANOVA-Two ways. The population of this research included the entire students of SMPN 13 Jakarta (State Junior High School 13 of Jakarta) taken by using cluster random sampling with two samples for each class. In this research, there were two learning groups; one with discovery learning method and the other one with expository. Class VII.6 was used as the experiment group, while Class VII.8 was used as the control group. Each group consisted of 36 students who were divided into three ability scales, namely high, medium, and low. The research data were gained from test, questionnaire, observation, and interview. The result shows that: (1) the improvement of the students' mathematical analogical ability using discovery learning method is considered better than the expository group; (2) There is significant improvement of the students' mathematical analogical ability based on higher, medium, and lower groups.

Key words: Discovery learning method; Mathematical analogical ability

Introduction

Education is at the forefront of preparing competent human resources. It is because education is believed to be able to encourage students to maximize the potential of human resources as a reliable candidate to be critical, logical and innovative in to face and to resolve any problems. This is relevant to the opinion of Sumarmo (2005) which states that mathematics education as a an active, dynamic, and generative process through mathematical activities (doing the math). Those activities provide an important contribution to the students in the development of systematic reasoning, logical thinking, thorough and critical, objective and open in dealing with various problems.

However, the results of The Third International Mathematics and Science Study (TIMSS) conducted on two junior high schools' students in Indonesia against the average math score show that most students achieved only 397 far below the international average which reached 500 TIMSS (TIMSS, 2008) . The value achieved by Indonesian students is also lower when it is compared to some other countries in Asia, such as Taiwan (with an average value of 598), South Korea (597), Singapore (593), Japan (570) and even Malaysia (474). While the PISA 2006 reported that Indonesia ranked 52 out of 57 countries. While the results of math scores on the National Exam, at all units and levels of education are always glued to the low number. At the first time of UN implementation in 2003, the government set a minimum standard for students passing score, which was 3.01, with an average graduation rate of junior high school and senior high school students at 71.55 %. Within several years of the UN implementation at SMP / MTs, the average value in mathematics courses in 2005/2006 is 7.08, with the lowest value of 0.67, and in the year 2006/2007 was 6.92, with the lowest value of 0.33 (Yunengsih, et al . 2008). This indicates that the learning process of mathematics in Indonesia needs to be improved so as to obtain better learning results in mathematics.

The low mathematical ability of students, one of which could be due to students' ability to perform mathematical reasoning is still low. According to the research result conducted by Rahman (2004), it is stated that the initial test results indicate that the ability of students' mathematical analogy are at less qualification. The same statement is also expressed by Suryadi (2005) who explains that the eighth grade students in the city and district of Bandung have difficulty in the ability to argue and find patterns and general forms of testing. Likewise with Herdian (2010), in his research he suggests that the ability of analogy and mathematical analogies of low ability students are at less qualification. The phenomena have happened because the process of learning through

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discovery method has been more difficult for weak students and vice versa for clever students. In addition, Yuliani (2011) has suggested that the ability of analogy and mathematical analogy of the medium and low ability students performed with guided inquiry learning model are at less qualification.

The low ability of the students' mathematical reasoning adversely affects the achievement of the learning. This is consistent with Wahyudin's findings (Herdian, 2010) in a study revealing that one of the tendencies that led to a number of students failure to master the fine points of discussion in mathematics was the lack of using logical reasoning in solving math problems or issues given. The results of Rifat's study (Suzana, 2003) also showed the weakness in mathematical ability of students seen in the performance of reasoning. For example, errors in mathematical problem-solving by mistake using deductive logic.

Reasoning and mathematics are two aspects that cannot be separated as understood through mathematical reasoning, while reasoning is understood and practiced through the learning of mathematics. This is reinforced by the results of research conducted by Prowsri and Jearakul (Priatna, 2003) that the Thai middle school students perform significant relationship between the ability of reasoning with their math learning outcomes. This suggests that the ability of reasoning plays an important role in the success of students. Those who have good reasoning skills are expected to have a good learning achievement.

According Mundari (2000) there are two analogies, the inductive analogy and the declarative or explanatory analogy. Inductive analogy is the analogy drawn from the principal similarities between two different phenomena. By means of analogy cases, students are trained to see the extent to which they understand the concept and see the microscopic structure of the concept by examining the relationship between the concept of analogy and the case. The cases also open the students' minds about the application or the benefit of studying the concept. Hence, the students can control or monitor their understanding towards something that is being studied and will be aware of the advantages and limitations in learning. As a result, they will find the right solution to enhance their weaknesses in learning.

The method teachers often use is expository method which is used to explain the material and then give the example problems. It was caused by several possibilities, namely: 1) Schools already have props but not yet used optimally ; 2) School did not have props ; 3) The school has had adequate props in terms of places, quality and quantity (Asyhadi, 2005). To develop the mathematical analogy ability, a learning method that has the characteristics to build the category, determine the problem and create a supportive environment is strongly required. The learning method having these characteristics is Discovery Learning. It is based on the discovery learning process described by Veermans (Herdian, 2010) i.e. orientation, generating hypotheses, testing hypotheses, making inferences and evaluation (control).

Ruseffendi (1991) suggests that the discovery method is a method of teaching arranged to make children acquire knowledge that they previously did not already know was not through notification, in which some or all of the knowledge found himself with the help of a teacher. In line with Ruseffendi, Sund (Suriadi, 2006: 5) reveals that discovery is mental processes so that students are able to assimilate a concept or principle. Mental processes are, among others: observing, digesting, understanding, classifying, making allegations, explaining, measuring, making inferences and so on . It is expected that if the students are actively involved in finding a basic principle of their own, they will understand the concept better, remember longer and be able to use it into another context. Blake et. al. (Rochaminah, 2008) discusses the discovery method published by Whewell. Whewell files a discovery method with three stages, namely: 1) clarifying, 2) drawing conclusions inductively, 3) validating (verifying). According to the three stages, it can be seen that the students activeness in discovery methods is needed to put the idea of a mathematical problem. Hence, the students can clarify an issue, then to identify the facts, and finally to draw a conclusion. After the students draw conclusions, they can also prove the truth of the conclusion.

Ruseffendi (Darhim, 2004) also explains that in order to foster a positive attitude towards mathematics, among others is teaching mathematics according to the environment and knowledge of students. The discovery method is one of the progressive teaching methods and focuses on students activities in the learning process. Explicitly Amin (Yuliani, 2011) suggests that a "discovery or invention" activity means a learning activity designed to enable students in discovering the concepts and principles through their own mental processes. In this case, the discovery occurs when students perform mental processes, such as observing, classifying, making allegations, measuring, explaining, drawing conclusions, and so on to find some of the concepts or principles.

While Suryosubroto (2002) suggests that one of widely-used teaching methods these days at schools that has been developed is a method of discovery. It is because this method: 1. It is a way of developing active student

learning; 2. By finding his own, investigating itself, the results obtained will be glued and long-lasting in the memory, not easily forgotten by the students; 3. By discovering themselves, the understanding found by students can be really controlled and is easy to use or to be transferred to other situations ; 4. By using discovery strategies children learn to master one of the scientific methods that will be developed; 5. By this method, children learn to think and try to solve analytical problems by themselves. This habit will be transferred in the real life.

Moreover, Bicknell - Holmes and Hoffman (Herdian, 2010) portray discovery learning using three key properties: 1) exploring and solving problems to create, integrate, and generalize knowledge ; 2) interest -based activities in which students determine the phases and frequencies; and 3) activities that encourage the integration of new knowledge into the prior knowledge base. Discovery learning can be facilitated through a variety of strategies in the classroom. The use of discovery method means teachers try to improve the quality of the students' activity in the learning processes. Hence, the discovery method according Roestiyah (2001) has the following advantages: a) the technique is able to help students to develop, reproduce readiness, as well as master the skills within the students' cognitive/recognition processes; b) the students acquire knowledge that is highly personal / individual, so it can be solid or deep left in the students' memory; and c) It is to increase the excitement of the students' learning.

According Sumarmo (2003), mathematical reasoning includes:1) drawing logical conclusions; 2) providing explanations using models, facts, properties, and relationships; 3) estimate answers and solution processes; 4) using patterns and relationships to analyze mathematical situations; 5) formulating and testing conjectures; 6) formulating opponent example; 7) following the rules of inference, checking the validity of the argument; 8) making the argument valid; 9) arranging direct and indirect evidences and using mathematical induction. Deductive and inductive reasoning can be used to acquire scientific knowledge. Inductive reasoning is a procedure stemming from special events as empirical observations and ends in a conclusion or new knowledge of a general nature. For example, number $4 = 3 \times 1 + 1$, $9 = 3 \times 3$, $16 = 3 \times 5 + 1$, $25 = 3 \times 8 + 1$, $36 = 3 \times 12$, and so on. Based on the events or facts, it is concluded that every perfect square number a^2 will meet the form $3k$ or $3k + 1$ for all integers k members. From these examples, it can be seen from the facts we can draw a conclusion for the specified generality.

Moreover, Sumarmo (1987) says that inductive reasoning consists of three types: generalization, analogies and causal relationships (cause and effect). Inductive reasoning involves the perception of regularity. Regularity is seen for example in drawing conclusions from the cases of a special nature then finding patterns/rules underlying or in obtaining the similarity/likeness of different examples. Inductive reasoning is divided into 3 parts, namely generalization, analogy, and causality. The analogy is used to compare two different things based on likeness. Besides finding the similarity between two different things, the analogy also draws conclusions on the basis of the similarity. Thus, the analogy is used as an explanation or as a basis for reasoning.

According Mundiri (2000), there are two kinds of analogy, namely inductive analogy and declarative or descriptive analogy. Inductive analogy is an analogy which is based on a different principle of equations on two phenomena. It can be further concluded that what is contained in the first is also the phenomenon of the second one. The analogy of declarative or descriptive is a method to describe something that is not known or is still vague, using terms that are already known. Suherman (2001) explains that the roles of mathematics at schools are: 1) to prepare students to be able to face changes in life circumstances in a changing world, through the practice of acting on the basis of logical thinking and rational, critical and careful, objective, effective, and synthetic analytically calculated; 2) to prepare students to use mathematics functionally in everyday life and in coping with other sciences.

In addition, Sumarmo (1987) also gives an overview of indicators to measure reasoning ability analogy, namely: a) Students can observe the pattern (from a picture or a number); b) Students can determine the relationship between the patterns of the images or numbers; and c) Students can estimate the proficiency level rules that make up the patterns. The mathematical analogy in understanding the ability of this research is the process of drawing conclusions on the basis of similarity by comparing two different things. The conclusions are drawn from the similarity so that they can be used as explanatory or as the basis of reasoning. Analogical ability should fulfill the indicators: Students can observe the pattern (from a picture or a number); students can determine the relationship between the image patterns or numbers; and students can estimate or predict proficiency level rules that make up the pattern.

Method

Samples in this study were the seventh-grade students of State Junior High School 13 Jakarta as much as two classes, namely classes VII.6 and VII.8 with each class selected 36 students as the samples. The reason for the selection of the sample was because they were considered to be able to adapt to the new learning (other than usual) and they did not interfere with the school program to prepare the national examination. The grouping of the students was based on mathematical skills according to the previous learning math results (from daily and midterm tests) as well as the classification done by the class teacher. The division of the student ability was divided into three different groups categories, namely higher, middle and lower, respectively 30%, 40% and 30% (Dahlan, 2004).

The results of grouping categories of students in the experimental and control class capabilities were the same, namely 11 students included in higher category, 14 students included in middle category, and 11 students, included in lower category. The design which is used in this study is "factorial design" which takes into account the presence of the control variables that affect treatment (independent variable) on the outcome (dependent variable). This study was conducted on the students of the two classes chosen with particular consideration. The study design is in the form:

$$\begin{array}{c} A \\ A \end{array} \quad \begin{array}{c} O \ X \ O \\ \hline O \ \ O \end{array}$$

where : O : pretest posttest (testing the students' mathematical abilities analogy)

X : Treatment of learning by discovery method

This study used a 2x1x3 factorial models, where 2 is the number of learning factors (method of discovery learning and expository teaching methods); 1 factor is the number of students' mathematical ability (ability of mathematical analogy); and 3 is the number of students' initial ability (high student, the student is and low student).

Results and Discussion

The data in this study were obtained from the pretest scores, as well as data scale showing the students' attitudes toward math. Pretest scores are used to determine the ability of students before being given treatment, whereas to see the improvement obtained from the difference between pretest and posttest scores as well as analogy and capabilities ideal score of mathematical students analogy expressed in normalized gain scores. The following is a statistical description of the scores from pretest, posttest, and normalized gain (g) in the form of a table.

Table 1. Descriptive statistics of mathematical analogy ability scores

Mathematical Analogy Ability	N	Minimum	Maximum	Mean	Standard Deviation	
Experiment	Pretest	36	2.00	15.00	8.2222	3.67315
	Posttest	36	8.00	19.00	15.2778	3.36886
	Gain	36	.20	.89	.6308	.19009
Control	Pretest	36	1.00	14.00	8.2500	3.21047
	Posttest	36	6.00	19.00	13.9167	2.94109
	Gain	36	.18	.88	.4925	.16712
Ideal Score			20			

Mathematical Analogy Ability Pretest Results

To identify that the initial ability between the experimental and control classes is not significantly different, the analysis of mean equality results test of the pretest is conducted. The steps that should be taken to conduct mean similarity are by doing the tests of data distribution and homogeneity of variance. If the data met the requirements of normality and homogeneity, the mean equality test is t-test; whereas the data are not normal, the non - parametric test is used.

Table 2. Mean similarity test of students' mathematical analogy ability pretest

Mathematical Analogical Skill			t	dk	p-value (2-tailed)	Kes.
Pretest	Variance Assumption	Similarity	-.034	70	.973	Terima H ₀
	Variance Assumption	Difference	-.034	68.768	.973	

Results of Normalized Gain Mathematical Analogy Ability

By using SPSS 16 for Windows, the statistical description of the data obtained and the normalized gain mathematical analogy capability is as follows:

Table 3. Descriptive statistics gain mathematical analogies ability normalized according to the method of student learning and capability category

Students' Initial Skill	Statistics	N-Gain			N-Gain Mean Difference
		MPD	MPE	Total	
High	Mean	0.7136	0.5555	0.6345	0.1581
	Std. Dev.	0.17557	0.13560	0.17317	
	N	11	11	22	
Medium	Mean	0.6836	0.5229	0.6032	0.1607
	Std. Dev.	0.17557	0.13560	0.17674	
	N	14	14	28	
Low	Mean	0.4809	0.3909	0.4359	0.0900
	Std. Dev.	0.18273	0.14896	0.16908	
	N	11	11	22	
Total	Mean	0.6308	0.4925	0.5617	0.1383
	Std. Dev.	0.19009	0.16712	0.19087	
	N	36	36	72	

Hypothesis Testing

The normalized gain of both experimental and control classes has a homogeneous variance and normal distribution. Then, the gain value is used to determine the significance of differences in mean of both groups by using two-path variance analysis (ANOVA). This analysis was conducted to see the direct influence of two different treatments given to the ability of students based on mathematical analogy learning methods and categories of students' abilities. The results of analysis of variance test calculated using SPSS General Linear Model 16 solid (GLM) performed at significance level $\alpha = 0.05$, while the summary is presented in Table 4.

Table 4. Normalized capability analysis of variance gain mathematical analogies according to methods of learning and student ability category

	Square SUM (JK)	Dk	JK Mean	F	p-value	Conclusion
Learning	.330	1	.330	12.737	.001	Tolak H ₀
Students' Skill	.513	2	.257	9.899	.000	Tolak H ₀
Learning * Students' Skill	.019	2	.009	.357	.701	Terima H ₀
Mistakes	1.711	66	.026			
Total	25.300	72				

Furthermore, based on the table in above, the research hypothesis testing was conducted. The hypothesis to be tested is as follows:

Hypothesis 1:

Criteria for testing H_0 is rejected, if $\text{Asymp.Sig} (1 - \text{tailed}) < \alpha = 0.05$. According Widiarso (2007) test the relationship significance values and two one -way direction of the output is $\text{Sig} .(1 - \text{tailed}) = 1/2 \text{Sig} .(2 - \text{tailed})$. After calculating the two- lane ANOVA results can be seen in Table 4. Retrieved sig. (1 - tailed) of $0.0005 < \alpha = 0.05$. Therefore, the result is the null hypothesis is rejected, it means an increase in the ability of students receiving mathematical analogy with the method of discovery learning is significantly better than the students who received learning with expository method. So it can be said that learning learn with discovery methods contribute significantly to the improvement of the ability of mathematical analogy.

Hypothesis 2:

Criteria for testing H_0 is rejected, if $\text{Asymp.Sig} (1 - \text{tailed}) < \alpha = 0.05$. After calculating the two- lane ANOVA results can be seen in Table 4:21 . Retrieved sig. (1 - tailed) of $0.000 < \alpha = 0.05$. Therefore, the result is the null hypothesis is rejected. It means that meaning that there are differences in students' mathematical analogy upgrades seen from the category of high –ability students, medium, and low.

Discussion

The learning activities administered using discovery method to discuss materials about triangle, square and rectangular was considered a new way for students. It can be seen when most students answered the interview sheet by stating that the use of discovery methods prepared by teachers was a novelty because the learning had not previously been done. They also felt happy with the learning that had been done. Their reason was that by the methods they learned and practiced using the props like origami paper and HVS paper. According to the students, they had been studying while practicing more easily because by using the method they could directly see the object and then concluded it through the easy questions existing on the worksheet (Students Worksheet) which had been provided by the teacher.

In addition, because the discovery learning method was done in groups, collaboration or interaction between students could run well. However, at the beginning of the learning at the first meeting it did not run smoothly. It was because there were some students who did not want to be divided into groups with a particular student. Consequently, the teacher gave the student chance to choose which group he/she intended to exchange with. Finally after the second and subsequent meetings they could interact with their peers smoothly.

Moreover, through the interaction between groups, they could express their opinions to solve a given problem in the worksheet. They also felt happy because they could ask friends about subject matters they did not understand yet and they also could work together. By the cooperation among students, the presented problems could be resolved easily and effectively. Problem solving activities also stimulated the group's mental activity in finding concepts, procedures and principles of mathematics which were heavily dependent on the questions presented in the student worksheet. As a result, the questions posed should encourage students to make the process of conducting analysis, finding analogies, and making generalization.

In addition, by learning with discovery methods students were given the opportunity to explore the material by using a tool such as origami paper. By using tools like the example, the students could identify the area of a triangle formula by folding paper to form a triangular shaped rectangle. The students, then, concluded that there was a relationship between the broad formulation of a rectangle and a triangle area formula. Then the students discussed how the relationship between the formulation of the broad rectangular area to the area of the triangle is. The activity greatly assisted students in understanding the concept to make identification and then draw conclusions.

Guidance by the teacher in the learning with discovery method was given in questions forms existing within the worksheets or during learning teacher walked around in each group to monitor and provide reinforcement (scaffolding) when the students asked questions. Consequently, the need for the didactic pedagogical anticipation should have been made in the lesson plan before the learning process was carried out. Prior to the learning process was to identify students' predictions of response to the problem which had been available in the worksheet. Anticipation is very helpful didactic pedagogical aspect when there were some students' responses were appropriate to the prediction. As an example, in the study of the properties of a square where the students were provided with two rectangles that had different sizes, then the students were asked "Are there any similarities and differences between the two pieces of the square? ". In this case, the anticipation is that the researcher had made no difference response prediction that the two long sides of a square were different and had

in common, but two rectangular had the same side's length and the same angles, which were 90°. In the learning process, the students took measurements and formed angles side. Then the students were asked the same questions and answered that both shapes had the same the same side length but different shapes had different lengths. Based on this response, prediction had been made equal to the students' answers.

However, there were also findings revealing that researcher had not made prediction, for example in the material about the triangle. The worksheet presented some questions to find the area of a triangle with a broad approach towards rectangular areas. In the worksheet, the students were told to make any triangle, then folded paper into triangles then formed a rectangle. There was one student made a right-angled triangle and an isosceles, but it was formed into a square, not a rectangular. Then, the researcher asked the students about the concept of square and rectangular. The student answered that square and rectangular was different if the same square sides and rectangles were not the same. Then the researchers asked the students to continue their activities and obtain the final conclusion that the area of a triangle is $\frac{1}{2} \times t$. The finding indicates that the students' activity in learning by using the method developed the discovery into the ideas of students. There is freedom of thought to remove the idea of rigid learning. By the students' freedom of expression, they can easily observe and identify problems contained in the worksheet so that they could integrate a generalization or conclusion.

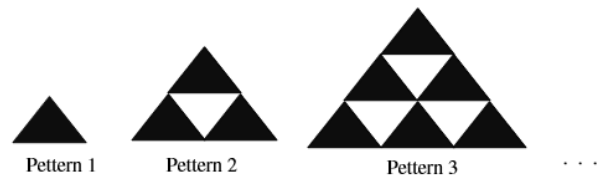
In constructing mathematical concepts either individually or in groups through the process of analysis, analogies, and generalization, students should receive help from the teacher. Assistance provided may form the questions that are simpler and more direct for the students to construct a mathematical concept. Discovery will become an effective learning method when the questions in the student worksheet are presented appropriately so that they can stimulate students' thinking process optimally. It means that the questions in the student worksheet should encourage students to make the process of discovery. The students' success or failure to discover concepts, procedures and principles in mathematics also depends on the shape of the questions posed orally by the teacher during the learning process. The questions posed should be able to be covered by the student's mind. This is in order not to make the students fail in finding the concept. It is aimed to make students not to feel frustrated, which can lead to them losing the spirit and confidence in finding mathematical concepts.

Based on data analysis, it can be concluded that learning by discovery methods significantly enhances the ability of mathematical analogy and generalization compared to the control group that received expository. The enhancement was based on their activities in the full meaning of learning by discovery methods so that students could understand the concept of mathematical analogy and generalization more easily. This is in accordance with the opinion of Bicknell-Holmes & Hoffman (Castronova, 2006: 2) about three main properties of discovery learning: (1) explore and solve problems to create, integrate, and generalize knowledge. (2) an interest-based activities where students set the stage and frequency, and (3) activities that encourage the integration of new knowledge into the prior knowledge.

The success of learning by discovery methods in enhancing the ability of mathematical analogy and generalization occurs because students conducted thinking processes, measurements, and observation towards the facts that existed then analyzed the facts and then drew conclusions. Students reacted with efforts to build meaningful patterns from the observations of others. As a result, students will stay longer given the concept of learning materials. This is reinforced by the opinion that discovery method has virtues as revealed by Suryosubroto (2002: 200) that this method helps students develop or augment supplies, mastery of skills and cognitive processes of students, student suppose it continues to be involved in the guided discovery. Knowledge gained from this strategy is very personal in nature and may constitute a very solid knowledge, in the sense of deepening the understanding of retention and transfer.

There are findings in the learning process where students generalized an activity to practice more easily as an example in the materials circumference and area of a rectangle in which students used the HVS paper then divided the sides into a two-centimeter lengths and then connected the dots marks so as to form squares. Connect young students with a number of the circumference or area of a rectangle is that $K = 2(p + l)$ and $L = p \times l$. Things practical activities with the help of the tool is very helpful in the generalization process, which in the context of the students found facts, then students integrate into a knowledge so gained generalization process. However, students had difficulties to determine the pattern of all n if the problem is patterned. Such examples expressed when a triangle was arranged as follows:

How many triangles are shaded in a pattern to- n ? There was one of the students who enrolled by arranging patterns of numbers 1, 3, 6, 10, 15, . . . , He found that every first term was added to the 2, 2nd rate plus 3, 3rd rate plus 4 onwards but could not conclude the n . To help the student, the researcher provided scaffolding to students by having students create a label just as below:



$$\text{pattern 1} = \frac{1 \times 2}{2}$$

$$\text{Pattern 2} = 3 = \frac{2 \times 3}{3}$$

$$\text{pattern 3} = 6 = \frac{3 \times 4}{3}$$

$$\text{pattern 4} = 10 = \frac{4 \times 5}{3}$$

....

From that data the students found a link between the numerals pattern with the next number multiplication then divided by 2 so the students could make a conclusion that the number of triangles shaded in a pattern to-n was

$$\frac{n(n+1)}{2}$$

The student will more easily be able to find a pattern to show the linkage of existing numbers but

teachers must continue to guide and provide scaffolding. Based on the analysis of pretest scores of the experimental and control groups, it is showed there was no significant difference. Furthermore, the two groups were given different treatments. The experimental group was treated with the method of discovery learning while gaining the control group learning with expository method. Based on normalized gain calculations, the experimental group showed significant mean increase in the ability of the control group analogy of 0.631 while the average for the control group mathematical analogy Based on the value of 0.493 ANOVA test, it is obtained significance Two Line $0.001 < 0.05$, which means an increased ability to obtain a mathematical analogy of learning by discovery method better than those gained by learning expository method. Overall increase in normalized gain abilities that students acquire mathematical analogy with the method of discovery learning with students receiving learning with expository method are at moderate classification.

From the calculation results also, it is also shown that the performance improvement gained mathematical analogy with the method of discovery learning is better than students who received the expository learning. Moreover, Suryosubroto (2002, 191) suggests that one method of teaching these days widely used in schools that have been developed is a method of discovery. This is because this method enables students to find their own meaning which is found quickly and easily used or transferred in any other situations. That is to analogize with the condition can make students easily to connect mathematics with material in other situations on the basis of a same - similarity principle or nature. In addition, the discovery stage of learning where students formulate principles and generalizations belong to the results of his findings (Suryosubroto, 2002). Hence, the students can deduce the material from the two similarity analysis which is presented in terms of student worksheets.

ANOVA statistical test is then performed two tracks to see differences in mathematical analogy seen an increase in the ability of the student's ability category. The test results indicate a rejection of H0 of the difference increased ability students 'mathematical analogy, between high, medium and low students' ability categories. It is shown that students indicated that significantly affect students' mathematical analogy upgrades. This is in line with the opinion of Galton (Ruseffendi, 1991) that there is a child of a group of some gifted children will be smart, moderate and less, which has the capability of individual differences. Problems frequently occurring in mathematics learning usually occur in less or low capable students. They tend not to follow the lessons as quickly and as best math students were much less capable of high rate capability. So, the learning process takes a slightly different treatment in the group of students who are less capable. Teachers should be more patient guide and inform any perceived lack of understanding by the students.

With the discovery method these problems can be minimized because the students who are grouped into low-ability students can exchange ideas with other students who have moderate or high ability students' understanding so that obstacles can be overcome. This is in accordance with the opinion of discovery method according to Roestiyah (2001, 20) who states that discovery method has the following advantages: a) the technique is able to help students to develop, reproduce readiness, as well as in the process of cognitive skills mastery/recognition of students; b) students gain knowledge that is highly personal / individual that can be solid or deep left in the soul of the student; and c) to increase the excitement of the students' learning.

From one of the observations at the first meeting to discuss the properties of triangles, students make observations of three triangles presented in LKS is an equilateral triangle with different sides different length. Students measure the length of the sides and angles major students can immediately conclude that the three triangles of the similarities and differences that have formed the third corner of the triangle is the same result that is 600 and the three pieces of the triangle have the same side. Then the students also found no difference in the three pieces of the triangle is the length of each side is different. After the student in question and what the terms say equilateral triangle? There are students who answered the same answer corners and there are students who answered all three sides equal. Thus, the ability of an analogy that is concluded on the basis of two kinds of situations students can run well.

However, from the findings of the students also have difficulty in concluding on the basis of similarity when students are faced with issues of material about the story about the circumference and area of a triangle. The difficulty is to link students to the elements of the problem and then find a solution to the first problem in the same way so that students can solve the problem on the second issue on the basis of the same way in the completion of the first problem. From these findings the researchers provide assistance on how to resolve the problem about the story on the circumference and area of a triangle by giving some problems. Then, after the students understand how to solve the problems of the story circumference and area of a triangle given the students about mathematical analogy contained in the worksheet and the students can identify problems and can solve the problem with the analogy to include what is used in solving the problem. It is within their opinion of Vygotsky (Muhammad Nur, 2004) the level of knowledge or knowledge of this cascade by Vygotskian describes as scaffolding that means giving to a large number of individual assistance during the early stages of learning and then reduce the effort and give the child the opportunity to take over greater responsibility as soon as able to do alone.

According to the average gain is equal to 0.630 normalized, there are 14 students or below the average of 38.89%. Of the 14 students, there are 11 students and 3 low-ability students are capable of being. This means that the discovery learning method was effective to improve students' mathematical analogy. However, there are shortcomings in its implementation such as the low-ability group of students who have difficulty in understanding the problems that exist on the worksheet. Besides, the students feel that there are problems in the worksheets so much that they cannot resolve all the issue contained in the worksheet. Besides that, it can be observed that the activeness of low ability students performed less when compared with students who have high or moderate capability.

The difference between the average normalized gain initial ability students showed that the higher the initial ability of the students the higher the increase in the ability of students' mathematical analogy. Which means that the students' initial ability is indispensable in learning. Since the beginning student a good ability of the material before the student can construct a new concept with its initial capabilities. Judging from the average normalized gain mathematical analogy capabilities in the category of high prior knowledge students acquire learning by discovery method is higher than the high-ability students, medium and low student to acquire learning expository method. This was due to the students' learning by discovery method presented a problem and the students themselves who determine the solution. In determining student solutions using tools so that they can identify directly the problems presented so that students can easily determine the solution.

When viewed from the difference in capacity building mathematical analogy, the largest difference occurs between the student is capable of learning was 0.161, high-ability students at 0.159 and 0.090 lower ability students. This is to indicate that the application of the MPD is better than the MPE application in improving the ability of students' mathematical analogy and students who have prior knowledge were greater benefit in improving the ability of students' mathematical analogy. It shows that students who experience learning was enabled by the discovery method can follow the lesson well. This is due to the clustering that is within a group of students are capable of high, medium and low. Hal shows the interaction between the student to determine the patterns of thought can go well. This is in line with Vygotsky expresses the opinion that in the four major principles such as Slavin (Muhammad Nur, 2004) is a social learning, ZPD (zone of proximal development),

cognitive apprenticeship and learning mediated). The essence of this theory is the emphasis on the interaction between the internal aspects of learning and emphasis on learning environment.

Conclusion

Based on research data and data analysis results, it is obtained some conclusions related to the research hypotheses, among others: 1. The increased ability mathematical analogy of the students who received learning methods discovery is better than the students receiving learning with expository method. 2. There are differences in students' mathematical analogy upgrades seen from the categories: a) the group of high- ability students; b) a group of students capable of being; and c) low-ability groups of students.

Recommendations

Based on the research results, it is obtained that the research recommendations are presented, among others, as follows: 1) Prior to the discovery learning method where the students in the group will create, within a group of students who are capable should be high, medium or low. This is for the guidance of activities undertaken by teachers to be more effective because the students having high or moderate ability will to help low-ability students, 2) The use of learning tools to make learning more meaningful so that the geometry of the material should be presented with the help of props. 3) Before using the discovery method, it is better for the teachers to identify the students' ability. It is aimed to create a condition where high and moderate students will help the low ones. 4) The students' analogical skill of junior high school students needs appropriate scaffolding by integrating it with the geometry shapes' presentation. 5) The learning materials in discovery method should be made after conducting pedagogical didactic analysis by connecting them to the analogical skill's obstacles on geometry lesson. 6. To develop the analogical skill in geometry, teachers can use algebraic patterns to present geometry shapes because junior high school students understand more about them.

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