CONCEPTUALIZING ALTERNATIVE WAYS OF CURRICULAR TEACHING THROUGH TECHNOLOGY

By

KUMAR LAXMAN

University of Auckland, School of Curriculum, Pedagogy and Development, New Zealand.

ABSTRACT

Traditional methods of teaching algebra encourage students to identify algebraic notations and perform procedural computations without comprehending the underlying relationships between the different modes of algebraic representations involving graph, expression and tables. Such approaches might enable students to ace in standardized Mathematics examinations but they do not acquire robust conceptual understandings of algebraic principles and concepts. This has resulted in growing calls for reforms in traditional ways of algebra instructions to re-conceptualise and invigorate the learning of algebra content in schools. In this paper, an instructional intervention that implemented a function-based generative activity design approach supported by the capabilities of next-generation classroom networks as an alternative to traditional algebra pedagogy is described. The findings of the study examine the pedagogical efficacy and impact of this intervention on students' algebra learning performance.

Keywords: Algebraic Notations, Algebraic Representations, Standardized Mathematics, Curricular Teaching.

INTRODUCTION

Traditional algebra instructional methods have been criticized to encourage rote learning and memorization of facts rather than a deeper understanding of algebraic concepts involved. Mechanically learning the rules and procedures of algebra manipulation, students are unable to appreciate the conceptual connectedness of algebraic principles and their meaningful application in situated contexts. Such traditional algebraic methods of teaching enable students to identify algebraic notations and perform procedural computations without comprehending the underlying epistemic relationships between the different modes of algebraic representations of graph, expression and tables. To effectively function, students need to develop deeper understanding of algebra as it is the gatekeeper to higher-level mathematics (kaput, 2000). Hence, there have been calls for reforms in traditional algebra curriculum to re-conceptualise and invigorate the teaching and learning of algebra content in schools. One alternative approach to algebra pedagogy is a function-based approach that assumes the function to be the central concept around which the teaching of algebra can be meaningfully organized (Yerushalmy, 2000). This version of a function-based approach can be situated relative to generative activity design supported by the capabilities of next-generation classroom networks (Stroup, Ares & Hurford, 2005). In such a network-supported function-based algebra method, graphing calculators are used to examine representations of functions and their associations in symbolic, tabular and graphical forms, especially located real-world contexts. In the study described in this paper, the pedagogical impact of a curricular intervention involving network-supported function-based algebra implemented in a high school in Singapore is examined. This experimental design study looks at how the intervention influenced students' conceptual understanding of key algebra precepts and impacted their performance in an achievement test.

Defining Algebra and Challenges in Learning Algebra

Bass (1998) has defined school algebra to be involving the following four main elements:(i) basic number systems, such as integers and real numbers, (ii) arithmetic operations (+, -, x, ÷) on these number systems,(iii) linear ordering of numbers, and (iv) resultant algebraic equations. Carraher and Schliemann (2007) have argued that algebra is inherent to arithmetic and that arithmetic has an algebraic character to it. They have defined algebraic reasoning as the psychological processes involved in solving problems that can be expressed using algebraic notations. Motz and Weaver (1993) posit that algebra is important to learn since it is a mathematical bridge that connects all the branches of mathematics to each other. For many students, algebra eventually becomes a stumbling block to the study of higher-level mathematics (Bellisio & Maher, 1998). One reason for this is that the students do not make a smooth transition from concrete arithmetic to the more abstract ideas in algebra (Spang, 2009). Another reason is that traditional algebra, due to its terminology and use of symbols, has been likened to a foreign language and, therefore, is more difficult to learn (Usiskin, 1999).The importance of algebra is minimized when algebraic ideas are presented as separate independent facts or as "a collection of tricks". As a result, students frequently fail to see their relevance and connection (Thorpe, 1989). To encourage students to learn algebra within the context of the bigger picture of mathematics, algebraic concepts should be presented in relation to real-life (hyphen) situations (Spang, 2009).

When to Teach Algebra

According to Usiskin (1999), traditionally, algebra has been formally studied by students from grade levels as early as seventh grade and as late as college. He recommends that formal algebra be taught at the eighth grade level with a strong preparation course as a precursor.

Prior research brings to light the difficulties middle- and high-school students have with learning algebra: Students focusing on finding particular, fixed answers; not recognizing the commutative and distributive properties; not using mathematical symbols to express relationships among quantities; not comprehending the use of letters as generalized numbers or variables; having difficulty operating on unknowns; and, not understanding that equivalent transformations on both sides of an equation will not impact true value (Bednarz, 2001).

However, in recent times many mathematics educators have argued that algebra should be introduced much earlier than its traditional appearance in high school courses (Carraher & Schliemann, 2007). The National Council of Teachers of Mathematics (NCTM) (2000) recommends that the teaching of algebraic ideas be included for all grades beginning from pre-kindergarten through high school (2000). NCTM suggests that teachers can then help students build a solid foundation of understanding and experience in preparation for more sophisticated algebraic work in the middle grades and high school (NCTM, 2000). Kaput (1995) advocates that algebra be weaved throughout the K-12 curriculum to add greater coherence, depth, and power to school mathematics and prevent late, isolated, and superficial high-school algebra courses. A deeper understanding of arithmetic concepts requires students to make mathematical generalizations that are algebraic in nature (Carraher & Schliemann, 2007). Bodanski (1991) is of the opinion that the algebraic method is the more effective and natural way of solving problems with the aid of equations rather than arithmetic methods.

Current Traditional Methods of Algebra Instructions

Algebra as a teaching subject in high school has been defined in a number of ways and viewed through different lenses, but primarily as follows.

Algebra is used to model reality (Moses & Cobb, 2001);

Algebra is symbolism (Usiskin, 1995);

Algebra is the study of functions (Chazan, 2000);

Algebra is multiple representations (Kaput, 1989);

Algebra is the study of structures (Rowen, 1994);

Algebra is the search for solutions through the solving of historical problems (Wheeler, 1996).

Kieran (1992) has listed some of the topics in a one-year course in high school algebra: (a) the properties of real and complex numbers; (b) the forming and solving of first- and second-degree equations in one unknown; (c) the simplification of polynomial and rational expressions; (d) the symbolic representation of linear, quadratic, exponential, logarithmic, and trigonometric functions, along with their graphs; and (e) sequences and series.

Those teaching algebra usually work from a curriculum that is based on pre-defined content (Pan, 2008). Teachers sometimes select the subject list from the table of contents of the selected textbook, or take the list from mathematics 'standards' documents. The curriculum is a list of topics the teacher must teach within a certain period of time. The common response to teaching such a curriculum is for teachers to mechanically “explain” the concepts and how they are used by giving examples. The explanation often involves considerable use of manipulation of symbols, which is followed by student practice on standard textbook questions in class or as homework.

Kieran (1992) has described that algebra has traditionally been taught as a cycle of procedural-structural steps. This approach focuses on memorization of rules and procedures, as well as the manipulation of symbols. Without a well-grounded understanding of why, the rules may seem meaningless to many students. Moreover, when the rules get mixed up or forgotten, students have no prior building blocks of understanding to refer back to. These rules are usually taught in bits and pieces in isolation from each other (Davis, 1994). Consequently, students are unable to assemble these small bits and pieces of ideas into a meaningful larger whole that will give mathematical power to their thinking (Davis & Maher, 1997).

Criticism of Traditional Teaching of Algebra

Kaput (2008) states: "The underlying goal of early algebra is for children to learn to see and express generality in mathematics." Unfortunately, many teachers who were taught algebra in the traditional procedural way adopted the same methods when they taught the subject in their classes as they know of no better alternative (Pan, 2008). The result of such an approach results in students resorting to memorizing rules and procedures to cover their lack of understanding, and they eventually believe that this activity represents the essence of algebra (Kieran, 1992).

Historically, the traditional ways of teaching high-school algebra have persisted with a few minor changes resulting from the advent of manipulative and computer technology (Dossey, 1997). Burrill (1995) offered the following explanation on why this trend continues: There is an institutional tradition about algebra. The school community and the public have expectations about the content of algebra, and they do not lightly accept changes that conflict with these expectations. In addition, tradition has imposed a sequence on the subject that teachers find hard to modify.

Need for Reforms in the Teaching of Algebra

Given the centrality of the topic of algebra in the implementation of secondary mathematics curriculum, improving students' learning outcomes related to the content of algebra is perhaps the single most need felt relative to secondary mathematics education. Hence there is a need to re-think the design of pedagogy in the teaching of algebra curriculum to move beyond addressing performance shortcomings with just-in-time remediation. Better approaches have to be identified to enhance students' outcomes in the learning of the subject of algebra (Stroup, Carmona, & Davis, 2005).

Kaput has outlined three dimensions of reforms for algebra to make it more accessible and relevant to students (Kaput, 1995). The three dimensions are breadth, integration, and pedagogy. Breadth refers to interweaving the many different facets of what constitutes algebra, i.e. modeling, working with functions, generalization, and abstraction. Integration refers to the need to embed algebra across other subjects. Pedagogy refers to making changes in the ways algebra is taught, especially by considering the affordances of new technologies.

However, implementing reforms in algebra instructions is not an easy task. Moses (2001) encountered both administrative and historical obstacles when he attempted to introduce algebra, usually taught in a ninth grade class, to the eighth grade curriculum. Dossey (1997) has elaborated on structures, representation, functions and relations, and modeling as models of understanding for algebra. However, the difficulty in terms of curricular enactment has been in merging these components to support a coherent program of study in algebra. This has led students to think that one chapter of the algebra textbook is not connected to the other chapters (Pan, 2008). Hence, it is hardly surprising that research has shown that students have difficulties with concepts involved in working with expression, equations, and inequalities. For example, many studies have demonstrated students' difficulties in understanding differences between expressions and equations as well as the many uses of the equal sign (Bednarz, Kieran & Lee, 1996).

What is a Function-Based Approach to Teaching Algebra?

In part because of availability of technology such as graphing calculators that enable links to be built between input-output tables, Cartesian graphs, and algebraic symbols, one popular alternative to the traditional teaching of algebra curriculum is a functions-based approach (Kieran, Boileau and Garancon, 1996). Rather than organize school algebra around the continued study of numbers and a long list of symbolic manipulations, this approach organizes introductory algebra concepts and skills around functions, their representations, and operations on functions (Chazan, 1999). The function becomes the central concept around which algebra curriculum is meaningfully organized (Yerushalmy, 2000). Taking such an approach, terms typical to the study of traditional algebra courses such as symbols, expressions, and equations assume new meanings (Kieran, Boileau, & Garancon, 1996). The function approach to teaching algebra does not mean moving the function chapter from near the end to near the beginning of a textbook. The true definition of a function approach implies that function is an underlying theme throughout a course in algebra – not just studied as a chapter or as part of a "content" or "concept" list as might be found in standards documents. Such an approach usually involves the use of a tool such as a graphing calculator for students to explore different representations of the functions (Laughbam, 2007).

In tying this approach to the teaching of algebra, Chazan (1999) felt better equipped to help students understand what the course was about, how the parts of the course were connected, and how algebra related to the world around them. Making functions and their standard representations central to the course, this helped him to express the problems posed to students in a way that allowed them to better understand the desired goals of the lesson. However, Yerushalmy (2000) cautions that the shift in focus from procedures to operations on functions and the availability of varied representations for expressing generality doesnot mean that the function approach to algebra offers a simpler path for understanding the complexities inherent in algebra than traditional instruction. Rather, what it suggests is that such an approach offers more opportunities for students and teachers to engage in inquiry-based learning.

Classroom Network Systems

Different types of synchronous mobile technologies are available to support network-based learning where students are situated in the same physical classroom location. Alternatively called a classroom communication system, it can make students' thinking visible to. and help teachers provide meaningful learning experiences, metacognitive scaffolding of mathematical concepts, and tailored instruction to meet specific needs of students (National Research Council, 1999). Roschelle, et al. (2004a) defined a classroom communication system as a combination of networking hardware (computers or hand-held devices) and software to provide displays that show what students are doing, thinking and understanding, and as a tool to expand the interaction loops between teacher and students in the classroom. This type of communication system enhances questioning and feedback, stimulates the participation of all students, supports discussions of important concepts, and empowers students' thinking.

An example of a classroom communication system is the HubNet discussion tool (Wilensky & Stroup, 1999).This software is used with networked computers in the classroom to enhance students' learning. It allows teachers to prepare questions in advance or introduce new questions during class. Teachers can collect students' text or numerical responses, and display the whole-class responses in various formats including lists, text, and histograms of numerical or letter responses.

Another technology that leverages upon the capabilities of a networked classroom is Group Scribbles (GS) 2.0 co-developed by SRI International and the National Institute of Education, Singapore. Group Scribbles enables the generation, collection, and aggregation of collaborative ideas through a shared space based upon individual effort and social sharing of notes in graphical and textual form. It also supports the agency of instant formative feedback from students and teachers through which students learn from one another and help each other improve on the quality of ideas in the construction and re-construction of knowledge (Chen & Looi & Ng, 2009).

SimCalc MathWorlds® software allows students to create mathematical objects on graphing calculators and to see the dynamic representations of these functions through the animations of characters whose motion is driven by the defined functions. Students are then able to send their work to a teacher's computer. Calculators are connected to hubs that wirelessly communicate to the teacher's computer via a local access point. It was found that using SimCalc in high-school classrooms in the U.S. resulted in improved learner motivations, learning gains, and increase in students' knowledge of algebra concepts related to linear functions (Tapper, Brookstein, Dalton, Beaton & Hegedus, 2009).

Audience Response Systems (ARS) are being widely used in both large and small educational institutional settings (Caldwell, 2007). These handheld devices are commonly called clickers or key-pads in the United States and handsets or zappers in the United Kingdom. Clickers are small transmitters that students utilize to transmit their responses by pressing appropriate buttons (Simpson & Oliver, 2006). Usually clickers come with 10-digit numeric key-pad and some accessory buttons such as a power switch, a send button, or function keys that permit text entry (Barber & Njus, 2007). Clickers have been reported to be helpful in sustaining attention and breaking up contiguous content. Using clickers to emphasize key concepts are useful in checking student understanding and enables students to focus better in class. Middendorf and Kalish (1996) state that periodic breaks, which clicker questions can provide, may help relieve student fatigue and "restart the attention clock."Clicker questioning enables learning to be more active and handson. Clickers also help reveal student misunderstandings and allow for prompt remediation to be done by the teachers. They also increase the ease with which teachers engage their students in frequent formative assessment and model effective instructional approaches (Roschelle, Penuel & Abrahamson, 2004b).

Davis (2003) studied the educational implications of the design features of a classroom network of wireless handheld graphing calculator using TI Navigator software. Themes explored were anonymity of data submission to the group, the ability for students to see their data displayed in the group space, the ability for the teacher to instantly assess how all students are doing at any time during a lesson, and the ability of the network to let all students answer all questions. Such synchronous classroom knowledge sharing systems were found to have the potential to allow for greater equity of input, reduce academic anxiety, increase teachers knowledge of student understanding, and improve student participation

Generative Design and Activities

Generative design looks at turning tasks that typically converge to one fixed outcome, e.g. simplify $2x + 3x$, to ones that are more divergent in nature and allow for the creation of a space of responses, e.g, generate functions that are the same as $f(x) = 5x$. The same content is handled in different ways: the first method, which forces students to conform to a single solution, now opens up a space of diverse options (Stroup, Ares, & Hurford, 2005).

Generative design can be viewed as a way of changing the topology of classroom activities to fully utilize the classroom space and leverage upon diversity (Stroup, Ares, Hurford, & Lesh, 2007). The instructional tasks in generative design do not prescribe a pre-defined method of working towards the solution or a set of fixed answers. Rather, it encourages students to generate a range of responses that serve as the context to exploring the topic of learning. Generative design attempts to address the biggest shortcoming of traditional mathematics instruction of doing mathematics by following the rules laid down by the teacher and knowing mathematics by remembering and applying the correct rule when the teacher asks a question (Lampert, 1990). Therefore, instead of having students simplify $\frac{25x+7x}{16}$ they are challenged to come up with three functions the same as 2x. Through such a generative activity, students do mathematics as a process involving experimentation, creativity, and even failure (what doesn't work can be a potent source of learning) and know mathematics by using underlying structural concepts to find answers that meet targeted criteria. Generative activities facilitated by classroom networks foster a classroom environment that is an authentic group space where all individuals interact to form rich and unique digital artifacts using manipulatives.

In the pedagogical approach of generative design, there are four key principles to be noted in conceptualizing the generative space as the collective avenue for students to brainstorm mathematical ideas. Activities should have a dynamic structure, open up a space for mathematical play, allow greater agency for students, and increase participation. Dynamic structure refers to the impact the emerging artifacts have on how the activity will progressively proceed. It is people-dependent in nature and gives learners varied opportunities for participation to impact the direction of instruction. The artifacts serve as an indicator of learners' current levels of conceptual understanding. Closely tied to dynamic structure is space-creating play, which is task dependent. This means that activities have to be structured to allow for multiple, valid ways of participation. Agency refers to students' identity in class, the value-laden perceptions they assign to these identities, and the amounts of influence they can assert on the content of the class. In generative design, ownership of content and learning in class is determined by the students themselves. Anonymity of responses in the generative space allows students to expand their agency to play different roles. All individual answers become everyone's answers and provide the field for a diverse range of discussions to occur in class. Agency is directly co-related to participation levels as increased agency leads to increased participation. By asking questions that have more than one correct answer, students are invited to participate in ways more meaningful to them and explore the validity of the rationale behind the different solutions offered. All generated responses are seen as learning opportunities – either they are exemplars of mathematical attributes or, in the case of misconceptions, they provide the lenses for students to analyze the incorrect understandings (Stroup, et al., 2005).

Networked Generative Activities Using Function Based Algebra

Networked generative activities using function-based algebra is one approach among many promoting function-based algebra that has been developed as part of the on-going efforts to reorganize school-based mathematics to focus more on modelling. In enacting such an approach in the classroom the formal set-theoretical approach to conceptualize function is downplayed and replaced by an approach highlighting how functions can be used to model co-variation. Computing technologies such as the graphing calculator are used to support engagement with and movement between representations of functions in symbolic, tabular and graphical forms. Greater meaning is brought to these multiple representations of functional relationships by situating them in real world contexts (Stroup, Carmona, & Davis, 2005).

Nearly seventy percent of a standard introductory algebra curriculum focuses on three major topics: equivalence (of functions), equals (as one kind of comparison of functions) and a systematic engagement with aspects of the linear function. Instead of prescriptively remembering rules for simplifying and solving, function-based algebra affords a consistent intepretation of both equivalence and equals. If the expression $x + x + 3$ is equivalent to the expression $2x + 3$, then the function $f(x) = x + x + 3$ and the function $g(x) = 2x + 3$ will have graphs in the calculators that are the same with all points in the domain coinciding. This equivalence can then be differentiated from equals refers to the value(s) of the independent variable where the the given functions intersect. This means that students will realize by looking at the graphs of $f(x) = 2x$ and $g(x) = x + 3$ that they are not equivalent but rather have one value of x where the 2 functions have the same y-value, i.e., are equal at a certain point (Stroup, Carmona, & Davis, 2005).

In their interventionist study involving a treatment group of 127 students from algebra classes at a highly diverse school in central Texas who were taught using the concepts of a network-supported function-based algebra approach, Stroup, Carmona and Davis found that students' have a better structural understanding of algebra concepts and performed better than their peers in the control group in relation to learning based upon algebra objectives assessed on the ninth grade Texas Assessment of Knowledge and Skills.

The Singapore Context

The mathematics curriculum in Singapore places an overt focus on calculation skills and rote learning, with drills and practice being central to the process of teaching and learning. Although derivations of mathematical concepts and results are provided in students' handouts, mathematical proofs are usually omitted in learning and examinations (Ho, 2008). Tan & Forgasz (2006)) found the Singapore mathematics teachers in their study to be not very enthusiastic about using graphical calculators in their classes and, therefore, used them less frequently and in less varied ways compared to teachers from Victoria, Australia. Another significant observation noted was that even if Singapore students are able to plot the points of a graph, they lack the conceptual understanding underpinning the graphing process (cited from a conversation with a Maths HOD at the treatment site of the study described in this paper). Mathematical textbooks used in Singapore schools have largely remained the same, both in terms of content and instructional approaches over the last decade (Ang & Lee, 2005). Ang and Lee attribute this trend to the heavy focus the educational system in Singapore places on student performance in high-stakes standardized exams, which results in textbook writers priming their readers for excellence in these exams and thus deviating little from traditional norms.

Research Methodology

A quasi-experimental study design involving nonrandomized, pre-post intervention study was utilized in this study. This methodology was adopted since it was easier to set-up as it was not logistically or access wise feasible to conduct a randomized controlled study (Gribbons & Herman, 1997).

Two government schools in Singapore participated in this study. The school (A) that was the control school was selected by the Ministry of Education, Singapore as it was deemed to be identical to the experimental school (B - where the intervention was carried out) in terms of student academic caliber and achievements. From school B, a total of 8 classes participated (6 classes from the Express Stream and 2 classes from the Normal Academic Stream) and from school A, a total of 7 classes participated (5 classes from the Express Stream and 2 classes from the Normal Academic Stream). In Singapore's education system, students who academically perform well in the standardized national Primary School Leaving Examinations (PSLE) at the end of primary schooling are placed in the express stream classes in secondary schools to complete secondary schooling in 4 years to sit for Singapore-Cambridge GCE 'O'(Ordinary) Level examinations. On the other hand, students who are channeled to the normal academic stream undergo 4 years of secondary schooling leading up to Singapore-Cambridge GCE 'N' (Normal) Level examinations, with the possibility of another additional year of study to qualify for GCE O Levels. Class sizes for all the classes that participated in this study were about 40 students each.

For both the control and experimental groups, a pre-test was administered at the beginning of the school term and a post-test at the end of the school term. The control group underwent traditional instructions on Algebra whereas the experimental group were taught the same Algebra content but using the interventionist model based upon function-based networked generativ activities. The pre- and post-tests had identical question items and they covered a range of topics in Algebra. There were multiple choice questions, open-ended questions, as well as graphical/plotting questions in the test.

A coding key was developed as a guide to coding the test responses. Coding was done by a small group of coders - all using the same coding key, and was later verified by one chief coder in the research project to maintain inter-coder reliability. A total score was computed for each test and each student, and this was used in the ANOVA analysis that was later carried out. A gain score (= posttest totalscore- pretest totalscore) was computed and used as the dependent variable. Pretest totalscore and PSLE scores were used as covariates in the statistical analysis.

Results

ANOVA test results on 2010 test score gains (Table .1)

The ANOVA test showed a significant treatment effect of the intervention on 2010 test score gains ($F(1, 527) = 59.2$, $p < 0.001$, partial Eta Square = 0.101. The covariate also had a significant impact on test score gains, $F(1, 527)$, $p < 0.001$, partial Eta Square = 0.047.

One-way ANOVA test results on PSLE scores of students in the two schools (Table .2)

The one-way ANOVA test showed no significant difference in the PSLE scores of the two schools participating in this study ($F(1, 595)$, $p > 0.05$).

ANCOVA test results on 2009 post-test scores (Table .3).

The ANCOVA test showed a significant treatment effect of the intervention on 2009 post-test scores ($F(1, 382) = 44.6$, $p < 0.001$, partial Eta Square = 0.105. The covariate also had a significant impact on post-test scores, $F(1, 382)$, $p < 0.005$, partial Eta Square = 0.027.

Discussions

The data analysis results inform that the intervention did have a significant impact in improving students' performance in the given algebra test. Since there was a concern that participant students from the two schools may not entirely be of equal academic abilities as the covariate of PSLE score did have an impact on test scores in the computations (though the effect size was considerably small), an ANOVA test was run on the PSLE scores of the students from the two schools and the results showed no significant difference in the PSLE scores. This could act as a confounding variable in influencing the reliable measurement of the treatment effects of the intervention on students' algebra test scores. To mitigate the possible effects of this factor and increase statistical power, ANCOVA tests were carried out by applying PSLE scores as covariate in the computations. The tests evidently indicate an improvement in students' algebra test outcomes after having undergone the revamped curriculum designed by the intervention. This implies that the network-based generative activities embedded in the intervention have had a positive impact in enhancing students' conceptual understanding of key algebra concepts. The findings of this study add further weight to the argument that reforms in the pedagogical enactment of algebra curriculum can enhance the effectiveness of the learning of algebra content. Specifically, the research of this study has shown that networked generative function-based approach is one effective alternative method of teaching introductory algebra through a series of modeling activities that leverage diversity and utilize the collective cognitive space of the classroom. Unlike traditional instructions on algebra which focus on rote learning of algebraic rules and prescriptive methods of reaching a fixed solution, the centrality of the use of function-based algebra supported by generative design activities in a next-generation classroom networked technology via IT navigator system promotes the learning of algebra in the classroom in more meaningful ways.

In traditional algebra instruction, both in Singapore and worldwide learning begins with students doing work on expressions – simplifying, expanding, factoring, solving, etc. Students frequently are doing these things as purely a computational exercise and they rarely have opportunities to understand that the seskills actually have other representational meanings, such as graphs and tables. The network-supported function-based algebra approach implemented as the curricular intervention for the experimental group of this study explored the social learning algebra by focusing on the multiple multimodal

Tests of Between-Subjects Effects

Dependent Variable:Gain

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	1467.928[a]	2	733.964	39.755	.000	.131
Intercept	102.262	1	102.262	5.539	.019	.010
PSLEScore	482.992	1	482.992	26.161	.000	.047
School	1093.824	1	1093.824	59.247	.000	.101
Error	9729.590	527	18.462			
Total	23227.000	530				
Corrected Total	11197.519	529				

a. R Squared = .131 (Adjusted R Squared = .128)

Table 1. ANOVA Test Results on 2010 test score gains

Tests of Between-Subjects Effects

Dependent Variable:PSLEScore

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	809.514[a]	1	809.514	1.168	.280
Intercept	3.149E7	1	3.149E7	45412.948	.000
School	809.514	1	809.514	1.168	.280
Error	412540.549	595	693.345		
Total	3.198E7	597			
Corrected Total	413350.064	596			

a. R Squared = .002 (Adjusted R Squared = .000)

Table 2. ANOVA test results on PSLE scores

Tests of Between-Subjects Effects

Dependent Variable:PostTestScore

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	740.176[a]	2	370.088	22.360	.000	.105
Intercept	37.917	1	37.917	2.291	.131	.006
PSLEScore	175.045	1	175.045	10.576	.001	.027
School	737.962	1	737.962	44.587	.000	.105
Error	6322.473	382	16.551			
Total	181712.000	385				
Corrected Total	7062.649	384				

a. R Squared = .105 (Adjusted R Squared = .100)

Table 3. ANCOVA Test Results on 2009 post-test scores

representations of functions i.e. expressions, tables and graphs in an integrated manner. By learning algebra through the lens of situating algebraic concepts in the context of functions students were able to make better sense of the structural and epistemological aspects of the algebra syllabus. Approaching algebra instructions in such a systematic manner provided students with a conceptual framework to contextualize the problem-solving application of the myriad of rules of algebraic manipulation. By projecting in real-time students' responses to the generative activities on the view panel on the overhead projector in the classroom, students could make visible their thinking processes and engage in joint mathematical knowledge construction.

Conclusion

In this experimental design study the treatment effects of an interventionist instructional approach involving function-based networked generative activities on the learning of algebra content prescribed in the Singapore Mathematics curriculum for lower secondary levels were examined. This was done in response to calls that the traditional ways of teaching algebra engendered rote learning in class and the mechanistic memorization of facts in isolation. The findings of this study offer promise that a pedagogical orientation embedding functions-based learning supported by networked technologies could revitalize the conceptual learning of 'dry' algebraic concepts in more meaningful and engaged ways. Students in the classes that underwent the intervention program test-scores wise outperformed their peers who learnt the same algebra topics by traditional methods of instructions. It is hoped that reform-minded educators who hope to make the learning of Mathematics intellectually more stimulating and relevant within a socially participative classroom milieu can draw useful insights from this study on how the algebra curriculum can be enacted by applying the framework of networked generative activities based upon function-based algebra principles.

References

[1]. **Barber, M., and Njus, D. (2007).** Clicker evolution: seeking intelligent design. *CBE—Life Sci. Educ*. 6, 1–8.

[2]. **Bass, H. (1998).** *Algebra with Integrity and Reality*. Paper presented at the The nature and role of algebra in the K-14 curriculum: Proceedings of a National Symposium Washington D.C.

[3]. **Bednarz, N. (2001).** *A Problem-Solving Approach to Algebra: Accounting for the Reasoning and Notations Developed by Students*. Paper presented at the The future of the teaching and learning of algebra: Proceedings of the 12th ICMI Study Conference, University of Melbourne.

[4]. **Bednarz, N., Kieran, C, & Lee, L. (Eds.). (1996).** Approaches to algebra: Perspectives for research and teaching (Vol. 18). Dordrecht: Kluwer.

[5]. **Bellisio, C. W., and Maher, C.A. (1998).** What kind of notation do children use to express algebraic thinking. *Psychology of Mathematics Education,* 1, 161–165.

[6]. **Bodanskii, F. (1991).** The formation of an algebraic method of problem solving in primary school. In V. V. Davydov (Ed.), *Psychological Abilities of Primary School Children in Learning Mathematics* (Vol. 6, pp. 275-338). Reston, VA: National Council of Teachers of Mathematics.

[7]. **Burrill, G. (1995, May 95).** *Algebra in the K-12 curriculum*. Paper presented at the Algebra Initiative Colloquium, Washington D.C.

[8]. **Caldwell, J. E. (2007).** Clickers in large classrooms: Current research and best-practice tips. *CBE-Life Sciences Education*, 6, 9-20.

[9]. **Carraher, D., & Schliemann, A. D. (2007).** Early algebra and algebraic reasoning. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning*: National Council of Teachers of Mathematics.

[10]. **Chazan, D. (Ed.). (2000).** Beyond formulas in mathematics and teaching: Dynamics of the high school algebra classroom. New York: Teachers College.

[11]. **Davis, R.B. (1994).** What mathematics should students learn? *Journal of Mathematical Behavior,* 13, 3–33.

[12]. **Davis, S. M. (2003).**Observations in classrooms using a network of handheld devices. *Journal of Computer Assisted Learning*, 19, 298-307.

[13]. **Davis, R. B., & Maher, C. A. (1997).** How students think: the role of representations. In: L. English (Ed.), Mathematical reasoning: analogies, metaphors, and images (pp. 93–115). Hillsdale, NJ: Lawrence Erlbaum.

[14]. **Dossey, J. A. (1997).** Making algebra dynamic and motivating: A national challenge. *Paper presented at the Nature and Role of Algebra in the K-14 Curriculum*, Washington, D.C.

[15]. **Ang, K. C., & Lee, P. Y. (2005).** Technology and the teaching and learning of Mathematics – the Singapore Experience. Technology Adoption in Mathematics Education: A Global Perspective. Retrieved from http://www.atcminc.com/mDevelopment/ShortArticleSeries/Singapore/5.2Singapore.html

[16]. **Gribbons, B. & Herman, J. (1997).** True and quasi-experimental designs. *Practical Assessment, Research & Evaluation*, 5(14).

[17]. **Ho, W. K. (2008).** *Using history of mathematics in the teaching and learning of mathematics in Singapore*. Paper presented at the 1st Raffles International Conference on Education, Singapore.

[18]. **Kaput, J. J. (1989).** Linking representation in the symbol systems of algebra. In S.Wagner & C. Kieran (Eds.), Research issues in the learning and teaching of algebra (Vol. 4, pp. 167-194). Reston: *National Council of Teachers of Mathematics.*

[19]. **Kaput, J. J. (1995).** A research base supporting long term algebra reform? *Paper presented at the Seventeenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Columbus*, OH.

[20]. **Kaput, J. J. (2000).** Teaching and learning a new algebra with understanding. U.S.; Massachusetts: *National Center for Improving Student Learning and Achievement*.

[21]. **Kaput, J. J. (2008).** What is algebra? What is algebraic reasoning? In A. H. Schoenfeld (Series Ed.), J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), Algebra in the early grades (pp. 5-18). New York and London: Lawrence Erlbaum Associates.

[22]. **Kieran, C. (1992).** The learning and teaching of school algebra. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 390-419). Toronto: MacMillan Publishing Company.

[23]. **Kieran, C., Boileau, A., & Garancon, M. (1996).** Introducing algebra by means of a technology-supported functinal approach. In N. Bednarz, C. Kieran & L. Lee (Eds.), *Approaches to Algebra: Perspectives for Research and Teaching* (pp. 257-294). Dordrecht, The Netherlands: Kluwer.

[24]. **Lampert, M. (1990).** When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal,* 27(1), 29-63.

[25]. **Moses, R. & Cobb, C. (2001).** Organizing Algebra: The Need to Voice A Demand. Social Policy, 31(4), 4-12.

[26]. **Moses, R. P. (2001).** Radical equations : math literacy and civil rights. Boston: Beacon Press.

[27]. **Motz, L. & Weaver, J.H. (1993).** The story of Mathematics. New York, NY: Pearson Addison Wesley. *National Council of Teachers of Mathematics* (2000). Principles and Standards for School.

[28]. **Mathematics. Reston, VA. Pan, R. (2008).** Teaching algebra in an inner-city classroom: Conceptualization, tasks and teaching. Doctoral thesis. University of Mitchigan. Louis Halle Rowen (1994). Algebra: groups, rings, and fields. A K Peters.

[29]. **Roschelle J., Penuel W. R., Abrahamson L (2004a).** Classroom Response and Communication Systems: Research Review and Theory. *Annual Meeting of the American Educational Research Association*; 2004; San Diego, CA. Retrieved from ubiqcomputing.org/CATAALYST_AERA_Proposal.pdf.

[30]. **Roschelle J., Penuel W. R., Abrahamson L. (2004b).** The networked classroom. *Educational Leadership*. 61(5), 50–54.

[31]. **Stroup, W., Carmona, L., & Davis, S. M. (2005).** Improving on Expectations: Preliminary Results from Using Network-Supported Function-Based Algebra. Conference Papers -- *Psychology of Mathematics & Education of North America*, 1-8.

[32]. **Stroup, W. M., Ares, N. M., & Hurford, A. C. (2005).** A dialectic analysis of generativity: Issues of network supported design in mathematics and science.

Mathematical Thinking and Learning, An International Journal.

[33]. Tan, H., & Forgasz, H. J. (2006). Graphics calculators for Mathematics learning in Singapore and Victoria (Australia): Teachers' views. Paper presented at the *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education*, Prague.

[34]. Thorpe, J. A. (1989). Algebra: What should we teach and how should we teach it? In Sigrid Wagner & C. Kieran (Eds.), Research issues in the learning and teaching of algebra (Vol. 4, pp. 11-24). Reston: *National Council of Teachers of Mathematics*.

[35]. Simpson, V., & Oliver, M. (2006). Using electronic voting systems in lectures. Retreived from http://www.ucl.ac.uk/learningtechnology/examples/ElectronicVotingSystems.pdf.

[36]. Spang, K. (2009). *Teaching Algebra to Elementary School Children*. Unpublished dissertation, Rutgers, the State University of New Jersey, New Brunswick, N.J.

[37]. Stroup, W. M., Ares, N. M., & Hurford, A. C. (2005). A dialectic analysis of generativity: Issues of network-supported design in mathematics and science. *Mathematical Thinking & Learning*, 7(3), 181.

[38]. Stroup, W. M., Ares, N. M., Hurford, A. C., & Lesh, R. A. (2007). Diversity-by-design: The why, what, and how of generativity in next-generation classroom networks. In R. A. Lesh, E. Hamilton, & J. J. Kaput (Eds.), *Foundations for the future in Mathematics Education*. (p.367-393). Mahawah, NJ: Lawrence Erlbaum.

[39]. Tapper, J., Brookstein, A., Dalton, S., Beaton, D. , & Hegedus, S. (2009). Relationship between motivation and student performance in a technology-rich classroom environment. Proceedings of the 2009 *Annual Meeting*, Retrieved from ERIC database.

[40]. Usiskin, Z. (1995). Doing algebra in grades K-4. In B. Moses (Ed.), *Algebraic thinking: Grades K-12 Readings from NCTM's School-Based Journals and other Publications* (pp. 5-6). Reston: National Council of Teachers of Mathematics.

[41]. Usiskin, Zalman (1999). Why is Algebra Important to Learn? *American Educator*, 30-37.

[42]. Wheeler, D. (1996). Rough or smooth? The transition from arithmetic to algebra in problem solving. In N. Bednarz, C. Kieran & L. Lee (Eds.), *Approaches to Algebra: Perspectives for Research and Teaching* (Vol. 18, pp. 151-154). Dordrecht: Kluwer.

[43]. Wilensky, U. & Stroup, W. (1999). HubNet. Evanston, IL. The CCL, Northwestern University. http://ccl.northwestern.edu/netlogo/hubnet.html

[44]. Yerushalmy, M. (2000). Problem solving strategies and mathematical resources: A longitudinal view on problem solving in a function based approach to algebra. Educational Studies in Mathematics, 43(2), 125-147.

ABOUT THE AUTHOR

Dr. Kumar Laxman is an Associate Professor of Education with the Faculty of Education, University of Auckland. He Graduated with a Ph.D in Instructional Design and Technology from Macquarie University, Australia. He has been actively promoting the use of technology to advance innovation in teaching and learning and he has served as a catalytic leader in participating in numerous e-learning and educational initiatives. He is knowledgeable in the various aspects of the field of Educational Design, having published widely in reputable Journals and presented at numerous International Conferences. He has also provided consultancy to a number of organizations in the domain of education, particularly enabling them to leverage upon technologies in enhancing teaching and learning. His areas of research interest include instructional design & technology, mobile learning and problem-based learning.