AMOVA ["ACCUMULATIVE MANIFOLD VALIDATION ANALYSIS"]: AN ADVANCED STATISTICAL METHODOLOGY DESIGNED TO MEASURE AND TEST THE VALIDITY, RELIABILITY, AND OVERALL EFFICACY OF INQUIRY–BASED PSYCHOMETRIC INSTRUMENTS

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ABSTRACT

This monograph provides an epistemological rational for the Accumulative Manifold Validation Analysis [also referred by the acronym "AMOVA"] statistical methodology designed to test psychometric instruments. This form of inquiry is a form of mathematical optimization in the discipline of linear stochastic modelling. AMOVA is an in–depth statistical procedure for the internal testing of research instruments based on the metrics from the novel "Taxonomy of Process Education". The Taxonomy of Process Education (TPE) is based off of the Process Education (PE), four–level measures designed to measure self–growth. The PE four levels in particular are viewed as sequential stages (or phases) of professional development. The four levels are also constructed to build towards the highest level of content knowledge or subject matter expertise (Pacific Crest, 2015). The TPE metric has universal applicability and is ideally suited for weighted mathematical measurement of content (subject matter), knowledge (cognitive), disposition (affective), and capability (psychomotor). This original methodology is a novel approach to advanced statistical post hoc data analysis. It adds considerable value to the methods designed to assess instrument validity and reliability especially when said instrumentation is researcher–designed. A sequential AMOVA mathematical model is provided (for sample data "Crosswise–Validation Analysis") along with its associated PE Taxonomy and measurement metrics in a step-by-step fashion that illustrates the entire process of advanced instrument validation inquiry.

Keywords: AMOVA, AMOVA Cluster Axiom for Manifold Consistency, Analysis, Accumulative, Accumulative Manifold Validation, Cluster, Crosswise–Validation, Linear Stochastic Modelling, Manifold, Mathematical Model, Mathematical Optimization, Mean, Outcomes, Process Education, Psychometrics, Research, Statistical Test, Statistics, Taxonomy of Process Education, Validation.

INTRODUCTION

Defining Accumulative Manifold Validation Analysis

The AMOVA model is a form of mathematical optimization in the linear stochastic modelling framework. In mathematics, computer science and operations research, mathematical optimization (alternatively, optimization or mathematical programming) is the selection of a best element (with regard to some criteria) from some set of available alternatives (INFORMS, 2015). In the field of mathematical optimization, stochastic programming is a framework for modeling optimization problems that involve uncertainty (Shapiro, A., Dentcheva, D. and Ruszczyński, A., 2009). In terms of linear stochastic modelling, AMOVA is similar in the selection of psychometric item outcomes as the selected parameters for the purposes of hypothesis testing. The acronym AMOVA literally means, "Accumulative Manifold Validation Analysis". The "A" portion of the acronym represents an abbreviated form of the term "Accumulative". The "MOVA" portion represents an abbreviated form of the term- "Manifold". Lastly, the "VA" portion represents an abbreviated form of the two terms-"Validation" and "Analysis". Accumulative Manifold Validation Analysis [AMOVA] can be used as a post hoc instrument item measure of precisely and how exactly the Weighted Qualitative Variables are truthful and to what extent are they consistentn their categorical and overall data analysis methodology. Thus, the AMOVA statistical test holistically provides rich data on research instrumentation in terms of psychometric strength that details both specific categorical group–based data and overall accumulative universal taxonomic analytics.

The Objective of the AMOVA Statistic

Accumulative Manifold Validation Analysis ["AMOVA"] is a specialized statistical methodology designed to test the internal and external validity of uniquely designed psychometric instruments. AMOVA uses a mathematically specialized form of inquiry that is an arithmetic form of natural mean optimization that is parallel to the discipline of linear stochastic modelling. AMOVA is an in–depth statistical procedure for the internal testing of research instruments based on the metrics from a novel taxonomy based on and grounded in "Process Education". This new taxonomy is referred to as the "Taxonomy of Process Education" ("TPE"). The TPE is based off of the Process Education ("PE") four–level measures designed to measure self–growth. The Taxonomy of Process Education (TPE) is based off of the Process Education [PE] (Pacific Crest, 2015) four–level measures designed to measure self–growth. The PE four levels in particular are viewed as sequential stages (as levels and/or phases) of professional development. The four–level measures are also constructed to build towards the highest level of content knowledge or subject matter expertise and are:

(1) Emerging (the lowest level);

(2) Developing (the next stage that arises from Emerging and illustrates a higher level of self–growth and authentically-based learning); followed by

(3) Proficient (the next level and second highest level of growth displaying the ability to adequately implement the task and/or skillset); and lastly followed by

(4) Accomplished (the highest level demonstrating mastery of the topic, concept, task, skillset, and/or requirement).

The PE four levels in particular are viewed as sequential stages (or phases) that through the TPE ideally measure "professional development".

Literature Review in Support of AMOVA

Cross-Validation is a statistical method of evaluating and comparing learning algorithms by dividing data into two segments: one used to learn or train a model and the other used to validate the model. In typical cross-validation, the training and validation sets must cross-over in successive rounds such that each data point has a chance of being validated against. The basic form of cross-validation is k-fold cross-validation. Other forms of cross-validation are special cases of k-fold cross-validation or involve repeated rounds of k-fold cross-validation (Refaeilzadeh, Tang & Liu, 2009). In addition, researchers Allen (1974), Stone (1974) and Geisser (1975), each independently introduced cross-validation as a way of estimating parameters for predictive models in order to improve predictions (Krstajic, Buturovic, Leahy & Thomas, 2014).

Dr. Jon Starkweather in his research provides ongoing support for statistical "cross validation" research models in his 2013 article entitled, "Cross Validation techniques in R: A brief overview of some methods, packages, and functions for assessing prediction models." (Starkweather, 2013). Starkweather states, "Cross validation is useful for overcoming the problem of over-fitting. Over-fitting is one aspect of the larger issue of what statisticians refer to as shrinkage (Harrell, Lee & Mark, 1996). Over-fitting is a term which refers to when the model requires more information than the data can provide. For example, over-fitting can occur when a model which was initially fit with the same data as was used to assess fit. Much like exploratory and confirmatory analysis should not be done on the same sample of data, fitting a model and then assessing how well that model performs on the same data should be avoided. When we speak of assessing how well a model performs, we generally think of fit measures (e.g. R^2, adj. R^2, AIC, BIC, RMSEA, etc.); but, what we really would like to know is how well a particular model predicts based on new information. This really gets at the goals of science and how we go about them; observation yields description, experimentation yields explanation, and all of those utilize statistical models with the goal of explanation and/or prediction. When predictions are confirmed, evidence is born for supporting a theory. When predictions fail, evidence is born for rejecting a theory." (Starkweather, 2013). Furthermore, researchers Bengio and Grandvalet state in "No unbiased estimator of the variance of k-fold cross-validation" the following, "Most machine learning researchers perform quantitative experiments to estimate generalization error and compare the performance of different algorithms (in particular, their proposed algorithm). In order to draw statistically convincing conclusions, it is important to estimate the uncertainty of such estimates." (Bengio and Grandvalet, 2004). Such a statement supports the mathematical arithmetic natural mean use found in novel statistical metrics such as AMOVA.

"K-Fold Cross Validation" estimates of instrument performance are idealistic measures of psychometric performance that can be associated with efficacy of instrument metrics. In general, cross-validation is a computer intensive technique, using all available examples as training and test examples. It mimics the use of training and test sets by repeatedly training the algorithm K times with a fraction 1/K of training examples left out for testing purposes. This kind of hold-out estimate of performance lacks computational efficiency due to the repeated training, but the latter is meant to lower the variance of the estimate (Stone, 1974). In addition, "Two-fold cross-validation" has been advocated to perform hypothesis testing (Dietterich, 1998). In 2014, researchers Krstajic, Buturovic, Leahy & Thomas demonstrated that there are datasets where the interval of nested cross-validation error is wide, and in which cases the user must assess the suitability of the model for the task in hand (as in the AMOVA statistical methodology). They also state the following regarding the specific use of cross validation procedures, "these situations point to the inadequacy of the dataset itself, rather than inadequacy of the nested cross-validation method. In such cases the application of repeated nested cross-validation points to the need to collect additional samples/compounds and/or alternative descriptors" (Krstajic, Buturovic, Leahy & Thomas, 2014). The review of the literature supports the use of cross validation statistical methods such as AMOVA as sound research measures that have both validity and viability in research hypothesis testing.

The Origin of AMOVA and its Unique Methodology

The infrastructure of Accumulative Manifold Validation Analysisis similar to the "k–Fold Cross–Validation" statistical procedure. In the "k–Fold Cross–Validation" model, the original sample that is under analysis is randomly partitioned into identified "k" equal size subsamples. The "k" represents the number of subsamples within the k–Fold Cross–Validation model. A single subsample is retained within the "k" equal size subsamples as the validation data for testing the model under analysis. The remaining "k – 1" subsamples are used within the model as measurable training data. The cross–validation process is then repeated "k" times (this is referred to as the "folds" part of the Cross–Validation statistical procedure). Each of the "k" subsamples is used a single time as the validation data. The k results from the folds can then be averaged (or otherwise combined) to produce a single estimation to determine research instrument validity. This process is essential to psychometric evaluative analytics. The process of designing instruments for the purposes of assessment and evaluation is universally called "Psychometrics". Psychometrics is broadly defined as the science of psychological assessment (Rust and Golombok, 1989). Psychometrics is the field of testing essential to determining the validity and viability of a system in socio–behavioral and traditional sciences.

AMOVA Methodology Measuring Reliability: Introducing theAMOVA Cluster Axiom for Manifold Consistency

The "AMOVA Cluster Axiom for Manifold Consistency" (and thereby "research reliability") is a measurable logical–mathematical statistical procedure that is designed for measuring the efficacy of psychometric research instrument items and has the following content–area specifications: (a.) "Similarity of Content"; (b.) "Directly Applicable Utility and Purpose"; and lastly (c.) "An Exhaustive Placement of Researchable Items that have a Specified and Holistic Meaning". The AMOVA Cluster Axiom is constructed from the threefold [Manifold] notion that:

(1) The "Psychometric Research Instrument" (identified by the acronym "psyri") items are grouped together based on relevance information;

(2) Each and every "Psychometric Research Instrument"

item has timely and relevant subject matter aligned to the initial research–design method of inquiry (i.e., research hypotheses and/or research questions); and

(3) All "Psychometric Research Instrument" items exhaustively belong to some particular categorical cluster based on similarity of content, measurement, and data gathering procedure.

Mathematically the "AMOVA Cluster Axiom for Manifold Consistency" is represented in the following manner:

$$psy_{[ri]} = \left\lceil \frac{n}{m} \right\rceil \quad (1)$$

where, m<n always for any and all specified psychometric research instrumenst ("$psy_{[ri]}$") items.

The AMOVA Cluster Axiom for Manifold Consistency is mathematically defined as follows:

(1) $psy_{[ri]}$ = The *psy*chometric *r*esearch *i*nstrument;

(2) [–] = Concentration on the quotient of…;

(3) n = Total number of "psychometric research instrument items"; and

(4) m = Total number of research categories (indicated by the term "m–fold" = "Manifold").

This is also indicative of the number of "psychometric research instrument items contained within manifolds".

The maximum number of psychometric research instrument items within Manifolds (this may differentiate from Manifold to Manifold thereby creating the inequalities from group to group and thusly creates "Manifold Unequal Groups").

The Unique Accumulative Manifold Validation Analysis Research Hypotheses Structure

Similar to mathematical optimization's linear stochastic modelling problem–solving in the arena of uncertainty. The AMOVA hypothesis test is based upon the metrics of the Taxonomy of Process Education (TPE). The cutoff scale for acceptance or rejection of the null hypothesis is based upon the amount of evidence that can be derived from the research that directly correlates and aligns with the "floor and ceiling nearest integer function" (indicated by "AMOVA ⌊x⌉") of TPE. The Accumulative Manifold Validation Analysis has a specific cutoff score for the null hypothesis region of rejection which starts with a score of 2.45 (see Table 2 and Figures 1 and 2 respectively for further insight into specific TPE numerical metrics). The score of 2.45 rounds up to a holistic integer of 3.00 (using AMOVA ⌊x⌉). This means that the AMOVA nearest integer function for the AMOVA null hypothesis evidence rejection is equal to AMOVA ⌊x⌉= AMOVA Round (x) = 2.45 ≅ 2.50 ≅ 3.00. This function is used for all AMOVA calculations and the final research outcome as a cumulative overall score to determine if the null hypothesis can be accurately rejected (or conversely accepted). As a result of the aforementioned, an "AMOVA hypothesis structure" has the following format (and corresponding mathematical structure as defined by the "Taxonomy of Process Education as a measure of Self–Growth" [thusly, indicated in research inquiry by the acronym: "TPES–G(x)"]):

H_0: There is no significant difference in terms of a deviation from base zero score (as "0.00" = Non–Existent) to 2.44 (Developing) [2.44 to 2.00, as the taxonomic nearest integer left "⌊floor⌋" (down) and right "⌈ceiling⌉" (up) rounding or nearest integer function TPES–G(x) = "⌊x⌉"] on the Taxonomy of Process Education, thereby indicating that there exists: (1) no measurement; (2) non–validity; (3.) non–reliability; (4) non–efficacy; (5) and an overall lack of professional development in terms of self–growth within and between the items as categorized on the psychometric instrument under Accumulative Crosswise –Validation Analysis.

H_1: There is a significant difference in terms of a deviation from base zero as a score of 2.45 [which actually rounds to 2.50]indicating (Proficient) [2.45 to 3.00, as the taxonomic nearest integer left "⌊floor⌋" (down) and right "⌈ceiling⌉" (up) rounding or nearest integer function TPES–G(x) = "⌊x⌉"] to 4.00 (Accomplished) on the Taxonomy of Process Education, thereby indicating that there does exists: (1) a high level of measurement, (2) validity of instrument items; (3) reliability of measurement; (4) overall instrumentation efficacy; and (5) overall professional development in terms of self-growth within and between the items as categorized on the psychometric instrument under Accumulative Crosswise–Validation Analysis.

The AMOVA Research Hypotheses Mathematical Form

H_0: [H_0: AMOVA= 0] = Accumulative Manifold Validation

Analysis= 0 [thereby indicating no overall instrument validity or efficacy in terms of measurement] = (via a mathematically defined mean) =H0: $\left[\mathrm{T}_{i=1}^{m}\bar{x}_{w_m}:m\right]$ m_{value}= [extracted from: $psy_{[ri]}$ = [$\bar{m}$] = 0; and alternatively an opposing hypothesis that is written.

H_1: [H1: AMOVA≠ 0] = Accumulative Manifold Validation Analysis≠ 0 [thereby indicating a high level of overall from Proficient to Accomplished based on the Taxonomy of Process education (in terms of Self–Growth) instrument validity or efficacy]= (via a mathematically defined mean) = H1: $\left[\mathrm{T}_{i=1}^{m}\bar{x}_{w_m}:m\right]$mvalue≠0. The AMOVA research hypotheses have a parsimonious mathematical form (for purposes of inquiry) is detailed in the example that follows.

An Example of Two–Tailed AMOVA Analysis Mathematically Defined Hypotheses is given below.

(a) H_0: mvalue= [$\bar{m}$]= 0; or

(b) H_1: mvalue=[$\bar{m}$]≠0.

Sample AMOVA Calculation Procedures

In terms of the manifold "m" in this example 1 through 7+ and 1 through 8+as shown in Table 1 that end in an even and an odd number of items respectively the Manifold = m = "The Overall Total Number of Groups". In these two modular examples, the total number of psychometric research instrument items per "Category" is determined by the researcher and are placed in "Categorical Clusters" at the outset of the research design. The Weighted scores are determined from the AMOVA TPES–G (see Table 2, Figure 1, and Figure 2 respectively). Note that the Table 1 AMOVA for "Crosswise–Validation" of research outcomes in terms of the respective variables "n" and "m" are as follows: n_1 = Even n^{th}= "8" (in this particular case, providing a n = 8 for the weighted Mean calculation of the Categories in the "n_1" Group).; n_2 = Odd nth = "9" (in this particular case, providing a n = 9 for the weighted Mean calculation of the Categories in the "n_2" Group). Where, TPES–G = The "Taxonomy of Process Education Self–Growth"; and Weighted Mean = The Accumulative Manifold Validation Analysis 'Individual' Unequal [by "Item"] Weighted Categorical Mean = $\frac{1}{n}\mathrm{T}_{i=1}^{n} w x_n$ based upon the instrument "n"= number of overall weighted items per "n^{th} Within Categorically Clustered Items"). Thereby providing the data to accumulatively calculate the "Accumulative 'Between' [All Items per Group] Weighted Group Mean = $\frac{1}{m}\mathrm{T}_{i=1}^{m}\bar{x}_{w_m}$(based upon the total identified instrument "Manifold" or "m–Fold" (or "m") = the specifically designated number of instrument Categorized Groups). Table 2 follows the more detailed narrative on Table 1 and details "The AMOVA Continuum of Self–Growth".

1 2 3 4 5 6 7 Even n^{th}
Weighted Mean Based Off of $TPE_{S–G}$
1 2 3 4 5 6 7 8 Odd n^{th}
Weighted Mean Based Off of $TPE_{S–G}$

Table 1. An example of the UnequalManifold [For items 1 through Seven Even and Odd to the nth Integer, in this particular example]

Table 1 provided examples of the "Unequal Manifold" [for items 1 through seven even and odd to the n^{th} Integer, in this particular example]. Each of the Table 1 "blocks or sections represent an item within the psychometric research instrument that are grouped together as one "Categorical Cluster" in "Tabular Blocks". The tabular blocks are individually "Weighted" according to the "Taxonomy of Process Education $_{Self–Growth}$ Metrical Scale" or "$TPE_{S–G}$". Each of the items on the research instrument is individually measured according to the $TPE_{S–G}$ as a specific score [which is now indicated by the acronym and associated variable "$TPE_{S–G}$ (x)"] this is the AMOVA procedure for weighting each instrument item according to the TPE. Accumulated Manifold Validation Analysis in terms of the methodology of "Psychometric Research Instrument Itemization" is a means of itemizing for "Crosswise–Validation" (which is defined as a measure of validity that is taken across items in rows representing "within" categorical clusters of data and down columns of data in groups representing an overall "between" items validation analysis). Thusly, the Accumulative Manifold Validation Analysis analytic can be most accurately defined arithmetically in the following series of sequential steps that carefully define and relate the following three mathematical methods that can be used to determine the final AMOVA calculated value. They are:

(1) The "Final AMOVA Calculated Value" (or "m_{value}");

(2) The "AMOVA Cluster Axiom for Manifold Consistency" ("$psy_{[ri]}$"); and

(3) The "Analytic AMOVA Total Notation Calculation"

("[1:m] $\sum_{i=1}^{m}\sum_{i=1}^{n} wx$ [1:n]").

Accordingly the sequences of related AMOVA equations are:

(1) The Final AMOVA Calculated Value = AMOVA as a "concentration" on the "Manifold Mathematical Mean" (or the "Concentrated Manifold Arithmetic Average")= $[\bar{m}]$= m_{value}=$psy_{[ri]}$=$\left[\frac{n}{m}\right]$=[1:m] $\sum_{i=1}^{m}\sum_{i=1}^{n} wx$[1:n]. (2)

This is holistically and accumulatively calculated as:

(2) The AMOVA Cluster Axiom for Manifold Consistency= $[\bar{m}]$ = mvalue=psy[ri] = $\left[\frac{n}{m}\right] = \frac{\sum_{i=1}^{m} \bar{x}_{w_m}}{m} \Rightarrow \frac{\sum_{i=1}^{n} wx_n}{n}$ (3)

This can be parsimoniously represented as:

(3) The Analytic AMOVA Total Notation Calculation= $[\bar{m}]$ = mvalue=psy[ri] = $\left[\frac{n}{m}\right] = \left[\frac{\sum}{[m]}\frac{\sum}{[n]}\right]$. (4)

Mathematically Defining the AMOVA Arithmetic Algorithmic Procedure

Much like the TRINOVA (Trichotomous Nomographical Variance) statistical measure (Osler, 2014b) AMOVA is a post hoc test of research outcomes. It is designed to test manifolds (in this case the term "manifolds" refers to the many different parts of the psychometric research instrument) as psychometrical test items using statistical test tables that are internal to the Taxonomy of Process Education as a metric of levels of expertise (as indicated in Figures 1 and 2 respectively). Similar to the measures that are used to test Triostatistical multiple group hypotheses within and between the post hoc Standard Tri–Squared Test 3 × 3 Table (Osler, 2014a) the AMOVA measure has its strength in the precision of its nearest integer function embedded within the long substantiated efficacy of the Process Education unidimensional progression metrics (0 through 4). As a result of the utility of the Taxonomy of Process Education as a comprehensive measurement framework, specific calculations can be used to determine the overall effectiveness of a psychometric research instrument. Thus, the calculation parameters of the Accumulative Manifold Validation Analysis in terms of specific, novel, and unique statistical formulae are as follows:

The initial definitive mathematical calculation for AMOVA is [1:m] $\sum_{i=1}^{m}\sum_{i=1}^{n} wx$ [1:n] (that can be rewritten with a small change in quotient as, [1/m]$\sum_{i=1}^{m}\sum_{i=1}^{n} wx$] [1/n] (for the purposes of differentiation in terms of arithmetic calculation). It can also be holistically mathematically defined using a more comprehensive "Totality Notation" to illustrate the algebraic relationships between "Total Instrument Item Manifolds and Total Instrument Items in Categorical Clusters" in the following manner:

$$\left(\frac{1}{m}\right)\sum_{i=1}^{m}\sum_{i=1}^{n} wx \left(\frac{1}{n}\right) = \frac{1}{m}\sum_{i=1}^{m} \bar{x}_{w_m} \frac{1}{n}\sum_{i=1}^{n} wx_n = \left(\frac{1}{m}\right)\sum_{i=1}^{m} \bar{x}_{w_m} \left(\frac{1}{n}\right)\sum_{i=1}^{n} wx_n \quad (5)$$

Thus, [1:m] $\sum_{i=1}^{m}\sum_{i=1}^{n} wx$ [1:n]= The "Overall Accumulative Group Mean ['m' as 'm–Folds' or Manifold(s)]" is a "Logical Biconditional ("⇒ ")" (meaning "if and only if") there is the complete summation of each of the "Individual Instrument Unequal Group Means" ('n' per group), mathematically this defined as follows (and in turn presents the following mathematical operation):

$$\left[\frac{1}{m}\right]\sum_{i=1}^{m}\sum_{i=1}^{n} wx \left(\frac{1}{n}\right) = \frac{1}{m}\sum_{i=1}^{m} \bar{x}_{w_m} \Rightarrow \frac{1}{n}\sum_{i=1}^{n} wx = \left[\left(\frac{1}{m}\right)\sum_{i=1}^{m} \bar{x}_{w_m}\right] \Rightarrow \left[\left(\frac{1}{n}\right)\sum_{i=1}^{n} wx\right] \quad (6)$$

For the overall "Accumulative 'Between' Weighted Group Mean" directly based upon each of the calculated "Individual 'Within' Weighted Group Means". Table 2 which highlights the AMOVA Continuum of Self–Growth immediately follows.

The AMOVA Continuum of Self–Growth is designed to display the sequential (left to right) relationship between the instrument values for the purposes of validation. In this manner the individual weighted outcomes have a multiple manifold applicable rubric that illustrates how scores were obtained, their relative value, and their expression in terms of the Taxonomy of Process Education in terms of Self–Growth. Table 3 follows and highlights "The Accumulative Crosswise–Validation Analysis Table".

The Accumulative Crosswise-Validation Analysis Table is designed to yield sequential (left to right) instrumentation validation outcomes similar to the critical values used in the one factor Analysis of Variance [ANOVA] F Test statistical

Repetitively Assigned Mathematical Weight	"Mathematically Equal to"	Repetitive Weight Assignment Based on the Taxonomy of Process Education $_{Self-Growth}$	"Mathematically Identical to"	Taxonomy of Process Education$_{Self-Growth:}$ Weighted Accumulative Outcome
0	=	Never	=	Non-Existent
1	=	Seldom	=	Emerging
2	=	Occasionally	=	Developing
3	=	Often	=	Proficient
4	=	Consistently	=	Accomplished

Table 2. The AMOVA Continuum of Self–Growth

Type of Validation	Total Number of AMOVA Items	Total Number of Items	Weighted Mean Formulae	The Calculated m_{value} as the [Manifold] AMOVA Result
Across = [Within]AMOVA Rows Validity ▶	Items (Groups)[Within]	$n^{th} = n$ (No. of Items Per Group)	$\frac{1}{n}\sum_{i=1}^{n} wx$ =**[Per Group]▼**	$\left[\frac{\sum\sum}{[\boldsymbol{m}][\boldsymbol{n}]}\right] =$
All = [Between]AMOVA Total Validity ▶	Categories (Groups)[Between]	m–Fold = manifold applicability = m (Total No. of All Groups)	$\frac{1}{m}\sum_{i=1}^{m} \bar{x}_{w_m}$ =**[Total of All Groups]▸**	$[1{:}m]\sum_{i=1}^{m}\sum_{i=1}^{n} wx\,[1{:}n] =$ $\frac{\sum_{i=1}^{m}\bar{x}_{w_m}}{m} \Rightarrow \frac{\sum_{i=1}^{n} wx}{n} =$ $\boldsymbol{m}_{value}$

Table 3. The Accumulative Crosswise–Validation Analysis Table

critical values charts. The F Test chart is designed to analyze multiple group research design variance as the "spread of scores" (Searle, Casella, & McCulloch, 1992). Note: The AMOVA mvalue is mathematically equal to the "AMOVA Cluster Axiom for Manifold Consistency". This is represented by arithmetic definition in the following manner:

$$psy_{[ri]} = \left[\frac{n}{m}\right] = m_{value} = [\bar{\boldsymbol{m}}]. \quad (7)$$

Figure 1 follows and presents, "The Model of the Taxonomy of Process Education in Terms of Self–Growth".

The Accumulative Manifold Validation Analysis Figure above is the Taxonomy of Process Education in Terms of Self–Growth. It is designed to illustrate the sequential hierarchal (from bottom to top) steps that one matriculates through from "No Experience" (i.e. "Non–existent") to a maximized "Accomplished" Level indicating the penultimate level of achievement of "Professional Development". This particular taxonomy has universal applicability. The terms and associated values can be used to assess growth, disposition, content mastery, level of expertise, value of particular items, analysis of skillsets, the power relative to performance, the building of a specific set of measurement data (as in the course design "4A Metric" from Techtonics) (Osler, 2010), the creation of implicit goals and objectives, and the amount of assigned value to a particular criterion. The quantitative numerical equivalent of these "indices" or "indicators" can be found in Table 2 which displays the holistic "Learn to Learn Continuum Rubric" specifically for the Itemization of Accumulative Crosswise–Validation Analysis for the purposes of research instrumentation psychometric analysis. Figure 2 follows and presents, "The Explicative Model of the Repetitive Weight Assignment Based on the Taxonomy of Process Education in Terms of Self–Growth".

The above Accumulative Manifold Validation Analysis Figure 2 is designed to explain Figure 1 in terms of mathematical weighted outcome yield. It is sequential (from bottom to top) in terms of Professional development and associated Self–Growth. The base has an overall outcome of "Never" (equivalent to a mathematical term of "0.00"). Built into the weighted assessment of instrument item efficacy based on this diagram is the mathematical rounding of values to the nearest whole number (using the nearest integer function for the floor and ceiling function values to determine outputted

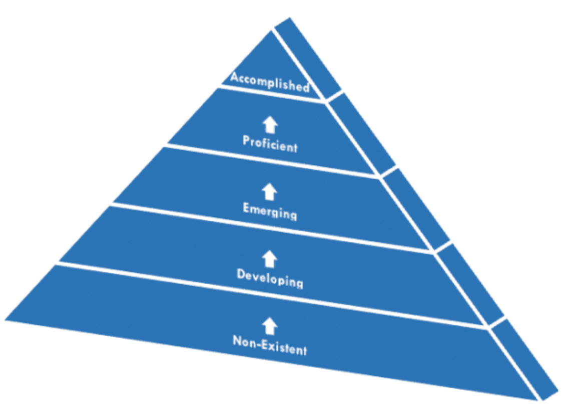

Figure 1. The Model of the Taxonomy of Process Education in Terms of Self–Growth as Used to Measure Professional Development

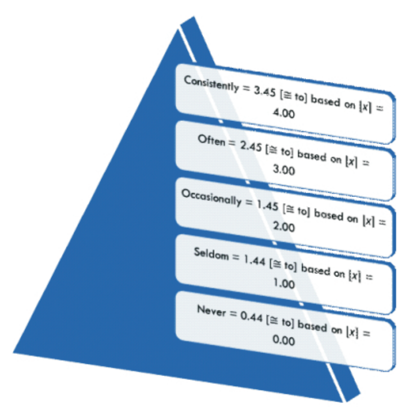

Figure 2. The Explicative Model of the Repetitive Weight Assignment Based on the Taxonomy of Process Education in Terms of Self–Growth as Used to Measure Instrument Item Efficacy

weights per research instrument categorical cluster). This provides the pure value needed to determine each individual group (or categorical) quantifiable value that will be eventually used to determine the overall instrument efficacy as an "Accumulative Manifold Validation" Coefficient based on the above numerical values. It is important to note that the diagram above is a continuation of the Taxonomy of Process Education from [PE] (Pacific Crest, 2015) and is specifically designed in deference to Table 2 (in terms of listed sequential titles and their associated mathematical weighted instrument item values). It is also important to note the floor and ceiling values in the model from the AMOVA $\lfloor x \rceil$. figure 3 illustrates, "A Sample of Weighted Means in Terms of Statistical Accumulative Manifold Validation Analysis for Unequal Size Groups".

In terms of manifold, m = 11 [1 through 11 (Manifold = Total Number of Groups), [in this particular example] Per Category via Tabular Blocks of Weighted TPES–GScores for AMOVA. In addition, Table 4 is a series of Accumulative Manifold Validation Analysis outcomes in terms of "n" and "m" and are sequentially listed as follows: $n_1 = 9$; $n_2 = 13$; $n_3 = 8$; $n_4 = 6$; $n_5 = 13$; $n_6 = 6$; $n_7 = 6$; $n_8 = 8$; $n_9 = 7$; $n_{10} = 9$; $n_{11} = 5$; and m = 11, respectively.

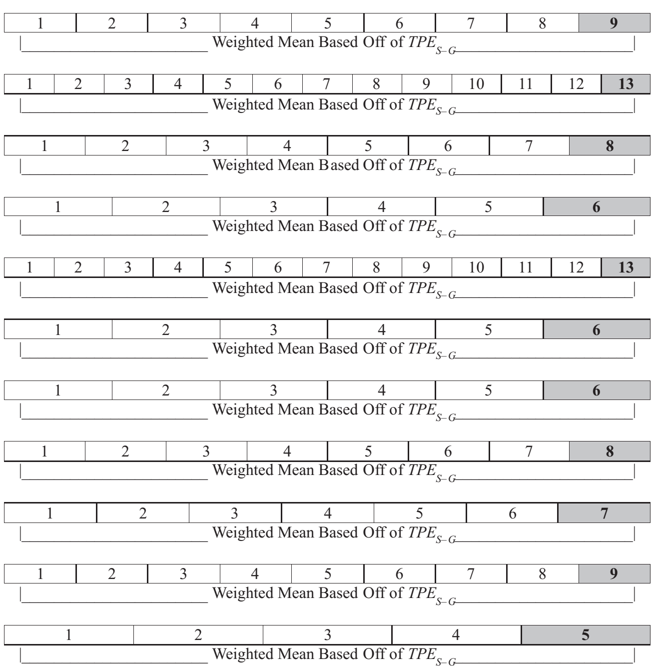

Figure 3. Sample Data Illustrating the Unequal Items from the Example Instrument

Using AMOVA: Sample Data Methodology and Final Results

The Accumulative Manifold Validation Analysis Table is designed to yield sequential (left to right) instrumentation validation outcomes for the Sample Data: n and m = 11. The final mathematical quantitative outcome yields the following: $31 \div 11 = 2.818 \cong 3.00$, which thereby indicates that in terms of Accumulative Manifold Validation Analysis that the total instrument in terms of efficacy is ["Proficient"] according to the Taxonomy of Process Education in terms of Self–Growth. The data finally yielded a value of 3.00 = a "Proficient" Level of Accumulative Crosswise–Validation Analysis Instrumentation Efficacy.

Summary

The sample data previously illustrated has a hypothetical AMOVA hypothesis structure as a method of mathematical optimization linear stochastic modelling in terms of psychometric instrument testing that can be applied in a variety of disciplines such as digital signal processing, instructional design, data optimization testing (and a host of others). The sample data hypothesis is presented in the narrative (in the previously displayed format) with an associated hypothesis mathematical structure. The hypothetical hypothesis mathematical structure can now be tested to determine the overall research instrument item efficacy as defined within the confines of a specific inquiry and investigation into the efficacy of faculty training using "Taxonomy of Process Education as a measure of Self–Growth" ["$TPE_{S–G}(x)$"] in terms of faculty professional development. The sample data research hypotheses are:

H_0: There is no significant difference in terms of a deviation from base zero score (as "0.00" = Non-Existent) to 2.44 (Developing) [2.44 to 2.00, as the taxonomic nearest integer left "$\lfloor$floor$\rceil$" (down) and right "$\lfloor$ceiling$\rceil$" (up) rounding or nearest integer function $TPE_{S–G}(x)$ = "$\lfloor x \rceil$"] on the Taxonomy of Process Education in regards to faculty professional development and training. This thereby indicates that there exists: (1) no measurement; (2) non–validity; (3) non–reliability; (4) non-efficacy; (5) an overall lack of professional development and training in terms of self–growth within and between the items as categorized on the psychometric research instrument under Accumulative Manifold Validation Analysis. This research hypothesis is rejected according to

Type of Validation	Total Number of AMOVA Items	Total Number of Items "n" [within] Categorical Clusters	Calculated Weighted Mean "wx" Formulae according to "n" within Categorical Clusters	The Calculated m value as the [Manifold] AMOVA Result
		n_1 for Category $_1$ = 9	$\frac{1}{9}\mathrm{T}_{i=1}^{9} wx_9$ = 2.55 ≅ 3.00[using the AMOVA ⌊X⌉]	$AMOVA = m_{value}$
		n_2 for Category $_2$ = 13	$\frac{1}{13}\mathrm{T}_{i=1}^{13} wx_{13}$ = 3.00 [using the AMOVA ⌊X⌉]	$= \left[\frac{\mathrm{T}\ \mathrm{T}}{[m][n]}\right] =$
		n_3 for Category $_3$ = 8	$\frac{1}{8}\mathrm{T}_{i=1}^{8} wx_8$ =3.63 ≅ 4.00 [using the AMOVA ⌊X⌉]	
		n_4 for Category $_4$ = 6	$\frac{1}{6}\mathrm{T}_{i=1}^{6} wx_6$ = 2.33 ≅ 2.00 [using the AMOVA ⌊X⌉]	$[1{:}m]\ \mathrm{T}\mathrm{T}_{i=1\ 1=}^{mn} wx\,[1{:}n]$ =
		n_5 for Category $_5$ = 13	$\frac{1}{13}\mathrm{T}_{i=1}^{13} wx_{13}$ = 2.23 ≅ 2.00 [using the AMOVA ⌊X⌉]	
Across = (Rows) = Clusters = [Within]A MOVA Rows Validity u	Items (Groups) [Within] = n	n_6 for Category $_6$ = 6	$\frac{1}{6}\mathrm{T}_{i=1}^{6} wx_6$ =1.83 ≅ 2.00 [using the AMOVA ⌊X⌉]	$\frac{\mathrm{T}_{i=1}^{m} \bar{x}_{w_m}}{m} \Rightarrow \frac{\mathrm{T}_{i=1}^{n} wx}{n} =$
		n_7 for Category $_7$ = 6	$\frac{1}{6}\mathrm{T}_{i=1}^{6} wx_6$ = 3.66 ≅ 4.00 [using the AMOVA ⌊X⌉]	$31 \div 11 =$
		n_8 for Category $_8$ = 8	$\frac{1}{8}\mathrm{T}_{i=1}^{8} wx_8$ = 3.25 ≅ 3.00 [using the AMOVA ⌊X⌉]	2.8 181818... ≅ 3.00 [using the AMOVA ⌊X⌉"] =
		n_9 for Category $_9$ = 7	$\frac{1}{7}\mathrm{T}_{i=1}^{7} wx_7$ = 2.14 ≅ 2.00 [using the AMOVA ⌊X⌉]	$[\bar{m}] = m_{value}$ = **3.00** =
		n_{10} for Category $_{10}$ = 9	$\frac{1}{9}\mathrm{T}_{i=1}^{9} wx_9$ = 3.22 ≅ 3.00 [using the AMOVA ⌊X⌉]	Proficient Level / [See Table 2] .
		n_{11} for Category $_{11}$ = 5	$\frac{1}{5}\mathrm{T}_{i=1}^{5} wx_5$ = 2.80 ≅ 3.00 [using the AMOVA ⌊X⌉]	Thus, for this particular research instrument in terms of
			Comprehensive Total = $\mathrm{T}_{i=1}^{11} \bar{x}_{w_{11}} = 31$	$[\bar{m}] = m_{value} =$
Down = Column) = All = Between] A MOVA Total Validity ▶	Holistic (Groups) [Between] = m	m = 11	Thus, $\frac{1}{m}\mathrm{T}_{i=1}^{m} \bar{x}_{w_m} = [(^1/_{11}) \cdot 31] = 31 \div 11 =$ 2.8 18 ≅ 3.00 [using the AMOVA ⌊X⌉"] ▶	$psy_{[ri]} = \left[\frac{n}{m}\right] = 3.00.$

Table 4. The Accumulative Manifold Validation Analysis Table for Sample Data: n1–11 and m = 11

the final research outcomes that yielded a 3.00 = "Proficient Level" as a final AMOVA calculation.

H_1: There is a significant difference in terms of a deviation from base zero as a score of 2.45 (Proficient) [2.45 to 3.00, as the taxonomic nearest integer left "⌊floor⌉" (down) and right "⌊ceiling⌉" (up) rounding or nearest integer function TPES-G(x) = "⌊X⌉"] to 4.00 (Accomplished) on the Taxonomy of Process Education in regards to faculty professional development and training. This thereby indicates that there exists: (1) a high level of measurement, (2) validity of the instrument items; (3) reliability of measurement; (4) overall instrumentation efficacy; and (5) overall positive professional development in terms of self-growth within and between the items as categorized on the psychometric research instrument under Accumulative Manifold Validation Analysis. This research hypothesis is accepted by virtue of the final research outcomes yielding a 3.00 final AMOVA calculation which indicates that there was a "Proficient" level of Self–Growth as indicated by the participants in the research study (according to the metrics of Taxonomy of Process Education).

AMOVA Hypothesis Results: Mathematical Research Hypotheses Outcomes

The sample mathematical research hypotheses were defined as follows: H_0: [H_0: AMOVA= 0] = Accumulative Manifold Validation Analysis= H_0: $\left[\mathrm{T}_{i=1}^{m} \bar{x}_{w_m} : m \right]$ = m_{value}= 0. The alternative opposing hypothesis is: H_1: [H_1: AMOVA$\neq$ 0] = Accumulative Manifold Validation Analysis$\neq$0H_1 $\left[\mathrm{T}_{i=1}^{m} \bar{x}_{w_m} : m \right]$ = H_1: = m_{value} $\neq$ 0. The AMOVA research hypotheses have a parsimonious mathematical form which is:

H_0: [$\bar{m}$= 0; or

H_1: [$\bar{m}$]$\neq$0.

The final result of the sample data study yielded:

[Rejected at $\lfloor x \rceil$ = 3.00];

H_1: [$\bar{m}$]$\neq$0 [Accepted at $\lfloor x \rceil$ = 3.00].

Thus, the sample data research outcomes are shown to be psychometrically viable in terms of between and within the AMOVA research instrument item validity. The positive Accumulative Manifold Validation Analysis final value ([([$\bar{m}$ or concentrated "m_{value}" = 3.00 = Proficient Level for $TPE_{S-G}(x)$ = $\lfloor x \rceil$), is a positive indicator that further investigations conducted under the same conditions will most likely yield further positive results. The initial research investigation psychometrically holds that the sample data on faculty professional development is positive in terms of perceptions of self–growth. The result of the sample data research also provides a good foundation for further inquiry into the same (or similar) researchable areas.

Recommendations

The psychometric efficacy of the AMOVA statistical test is confirmed in the sample data results presented in the detailed account of the faculty professional development research investigation. The utility of Accumulated Manifold Validation Analysis allows a variety of disciplines (digital signal processing, instructional design, data optimization testing, and educational ergonomics) to analyze test instruments to determine each test item's research viability, verifiability, and validity in terms of 'within'"categorical clusters" based on item similarity and 'between' item "groups" to ascertain overall item usability. The author recommends the following:

- That more research be conducted with model to substantiate its applicability;
- An assortment of psychometric research test instruments use the AMOVA model in a variety of research approaches and research disciplines to see if the methodology yields new arenas of application beyond the traditional uses of the model; and
- That the researchable applications and discoveries regarding this particular test are documented so that the novel research innovations can be readily applied.

Conclusion

The AMOVA itemized psychometric test efficacy increases the viability of research instrumentation by specifically determining the authenticity of psychometric instrumentation research outcomes. As such, this makes the AMOVA statistics a valuable resource the researcher who is now able to determine instrument efficacy item by item based on the research instrument design, the specific research questions under investigation, and the precise level of instrument item accuracy (using the Taxonomy of Process Education as a measure of efficacy). This new statistic aids the researcher by making the development of research instrumentation more viable, very precise, and rigorously determines their level of effectiveness. This ultimately insures that psychometric instrumentation and their associated research results are carefully analyzed, rigorusly studied, and are carefully considered before the research is presented or reported. Thus, AMOVA is a dynamic and effective addition to the world of statistical research designs.

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