# The complexities of teaching prime decomposition and multiplicative structure with tools to preservice elementary teachers

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Abstract:

Preservice elementary teachers often struggle with prime decomposition and other mathematical topics that correlate with number theory. This paper provides a framework for integrating prime factor tiles into their curriculum with a particular emphasis on prime decomposition. Using this framework, preservice teachers explored and evaluated numbers using prime factor tiles. The results of the exploratory inquiry showed that preservice teachers made some progress in their understanding of prime decomposition after exploring with the tools. However, they struggled with problems requiring the application of prime decomposition. More time to delve into this topic is probably needed in order to observe further gains.

**Introduction**

Prime decomposition is an important skill for preservice teachers to have as it connects to many different areas of mathematics including Fundamental Theorem of Arithmetic, Modular Arithmetic, Group Theory, Galois Theory and Number Theory among others. Unfortunately, prime decomposition and number theory are often ignored in relation to research (Zazkis, 2000; Zazkis & Campbell, 1996; Zazkis & Liljedahl, 2004).

Prime decomposition can be described as the multiplicative decomposition of numbers into their prime number components. Oftentimes, prime decomposition is presumed as difficult (Lenstra, 2000). Although presumed difficult, it is an important concept starting with third through fifth grade *Standards*. Students in these grades should "recognize equivalent representations for the same number and generate them by decomposing and composing numbers" (NCTM, 2000, p. 148). At the sixth through eighth grade *Standards*, students should "use factors, multiples, prime factorization, and relatively prime numbers to solve problems" (NCTM, 2000, p. 214). Because of this emphasis, it is imperative that preservice teachers have a robust awareness of prime decomposition and can work interchangeably between factors and the numbers they produce. It is important to help preservice teachers become exposed to some of the richness of prime decomposition and divisibility notions (Brown, 1968).

A common method for teaching prime decomposition involves the formation of factor trees (Griffiths, 2010) (see Figure 1). Factor trees help students visualize the multiplicative breakdown of a number but do little else in relation to the development of the concept. Preservice teachers can usually perform the procedure of creating a factor tree. When they have to apply the yielded factor tree in a context or work backward from the factors, difficulties often result (Zazkis, 1999; Zazkis & Liljedahl, 2004).

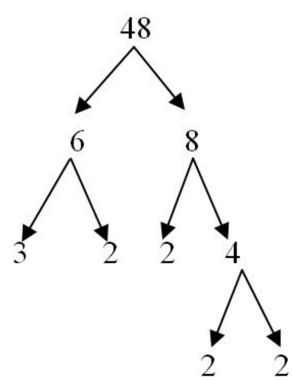


*Figure 1*. A demonstration of the typical prime factor tree format.

To address this, an alternative method to teach prime decomposition was developed (Kurz & Garcia, 2010; 2012). Using tiles, students decompose numbers to their prime factors. These prime numbers can then be used in a number of ways such as discovering the uniqueness of prime decomposition, simplifying fractions, finding the greatest common factor (GCF), finding the least common multiple (LCM), finding all factors and identifying roots. The purpose of this paper is to describe methods for teaching prime decomposition using tools and then present the findings of an inquiry that investigated preservice teachers' knowledge after exposure to the prime factor tiles. First, an explanation of prime decomposition along with preservice teachers' difficulties with the topic is described. Then, an alternative method for teaching prime decomposition using tools is explained with supportive lesson ideas. Preservice teachers explored these lessons using the tools. The following question was investigated: How did exposure to prime factor tiles influence preservice teachers' understanding of prime decomposition? Recommendations for teaching prime decomposition based on the results are provided.

**Prime Decomposition**

For preservice teachers, there seems to be real difficulty with understanding and applying prime decomposition in relation to factors (Zazkis, 1999). Several studies have outlined preservice teachers' difficulties. Zazkis and Liljedahl (2004) found that preservice teachers were able to define (or explain) what makes a number prime. Greater difficulty resulted when preservice teachers were asked to implement their knowledge of prime. Once they had to apply reasoning, the preservice teachers were unsuccessful (for the most part). Their understanding of primes was incomplete or inconsistent and focused primarily on the algorithm.

In another study, Zazkis and Campbell (1996) conducted research after the preservice teachers concluded their lessons on number theory. They found that preservice teachers demonstrated misunderstandings and misconceptions about divisibility and factors. They relied on procedural algorithms rather than applying logic and reasoning to determine divisibility. They concluded, “insufficient pedagogical emphasis has been placed on developing an understanding of the most basic and elementary concepts of arithmetic” (p. 562).

Bolte (1999) used concept maps to help preservice teachers voice their understanding of 20 terms related to number theory such as factor, prime, composite, prime factorization, multiple, divisible and other terms. In her analysis of two specific cases, one preservice teacher was able to use the concept map to show depth in relation to number theory. The participant was fluent in her connections and properly demonstrated the interconnectedness of the terms. Another participant created a weak map that showed superficial understanding of the interconnectedness of the 20 number theory terms. While Bolte specifically focused on concept maps, the showcased maps and supportive essays provide insight into preservice teachers’ connections and misconceptions regarding number theory.

With the documented difficulties preservice teachers have in relation to prime decomposition, it is important that they are provided with opportunities to explore number theory concepts. Alternative methods to teach prime decomposition using tools with an emphasis on discovering the meaning of number theory terms are illustrated. The goal was to implement tools to support preservice teachers’ conceptual understanding of prime decomposition so that they could better understand the multiplicative structure of numbers.

**Investigative Ideas**

Prime factor tiles were developed as an alternative method to teach prime decomposition with the goal of implementing the tools. These lessons go beyond the drill aspect of mathematics and instead focus on sense making and reasoning as is recommending in a reform-based curriculum (Cooney, 1999). Prime factor tiles are tiles with a specific prime number written on each tile. In order to build all primes and composites equal or less than 102, the tiles should contain the following numbers: $2^6$, $3^6$, $5^4$, $7^3$, $11^2$, $13^2$, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97 and 101. Other composites beyond 102 are possible. The tiles can be easily made in two ways. The tiles can be printed on cardstock, see Kurz and Garcia (2010) for the template. Using inch-flats, a permanent maker and blank stickers, a more permanent set of factor tiles can be made.

***Exploring the Fundamental Theorem of Arithmetic***

The Fundamental Theorem of Arithmetic assures that the prime decomposition of a given number is unique. The instructor gives a number to the class and asks the students to find the prime decomposition using the prime factor tiles. Call on a student and ask what did you get? Call on another student and ask what did you get? Repeat this process for a few more students. All students should have the same result. Repeat this process whole class with several more numbers. Ask, can two students have different prime factor tiles that equal the same number? Can they have tiles that are a little different? Why not? How can we explain what we discovered? What does this mean?

***Simplifying fractions***

Prime factor tiles can be used to simplify fractions. A fraction is given to the students, perhaps 24/42. A large fraction bar is drawn. The student decomposes the numerator and denominator into a product of primes using the tiles. Then, the student places the corresponding tiles above and below the fraction bar (see Figure 2). The student continues removing any shared tiles until there are no further common tiles to remove. By multiplying the numbers in the numerator and denominator, the student finds the simplified fraction. What happens when a common tile is removed from both the numerator and the denominator? What does that process do to the fraction? Is the value of the fraction the same once the common tile has been removed?

Explain. Can you create a fraction with no common tiles? Can you create a fraction with at least three prime factor tiles for both the numerator and denominator with no common tiles? Can you create a fraction with at least three prime factor tiles located in the numerator and four prime tiles located in the denominator that simplifies yielding a 1 in the numerator? Explain your process.


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*Figure 2.* Prime factor tiles demonstrating fraction simplification.

***Finding the GCF and the LCM***

Two numbers are given, possibly 48 and 72. The student finds the prime decomposition of the given numbers using the prime factor tiles. A Venn diagram consisting of two circles that overlap is made. Prime factor tiles that correspond to the first number (48) are placed in one circle. The prime factor tiles that correspond to the second number (72) are placed in the other circle. If a prime factor tile is common to both numbers, the student picks up the tile from both circles and places one of the tiles in the overlapping section of the Venn diagram. The second duplicate tile is discarded. The overlapping circles demonstrate that both numbers share that prime tile in their prime decomposition (see Figure 3). The student continues to identify more common prime factor tiles and repeats the procedure if applicable. If a prime factor tile is not common, it remains in the circle of the Venn diagram corresponding to that number. Tell the student that the GCF is 24. How can the Venn diagram be used to find the GCF? How would you define GCF? The student can discover that when multiplying all of the numbers in the overlapping portion of the Venn, the GCF will result. What happens when all of the numbers in both circles are multiplied? What does that number represent? The student can find the LCM by multiplying all the prime factor tiles available in all the circles including the overlap. What would a Venn have to look like to yield a smaller LCM? What would a Venn have to look like to yield a large LCM? How do the numbers in the circles impact the size of the GCF and LCM? Can you build a Venn that has all of the prime factor tiles of one number in the overlapping portion of the Venn? What two numbers below 100 have the most factors in common? Build the Venn to show the breakdown of these two numbers.

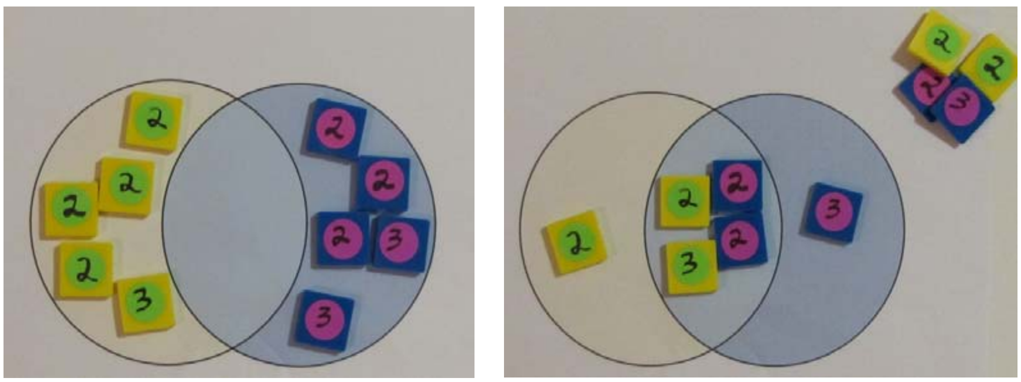

*Figure 3.* Venn diagrams demonstrating 48 and 72. Students may need to build the first diagram to yield the proper second diagram.

The previous activity can be generalized to three numbers. First, a Venn diagram with three circles is created. In the triple overlapping section, place a prime factor tiles that are common to the three given numbers (the two other tiles are discarded). This will yield the GCF for all three numbers. If a prime factor tile is only common to two numbers, we place that tile in the overlapping section of those two numbers. All of the prime factor tiles are then multiplied to find the LCM. Students can then investigate what the Venn represents where two circles overlap.

***Finding all the factors***

The student is given a number to decompose, 30 for example. The prime factor tiles that yield 30 are 2, 3, and 5. Using the prime factors, all the factors of a given number can be found.

Ask the student to place the numbers into two distinct groups (one of the groups might even be empty). The student then multiplies all the numbers in each group and records the results. The student continues this process until all distinct groups are found. The recorded results yield the factors of the given number (see figure 4). How many different groups of numbers can the student make? What do these numbers represent? Can you make a number with 10 factors? How about 9 factors? What kinds of numbers yield the least amount of factors? 20 is decomposed into 2x2x5 and 30 is decomposed into 2x3x5. Both numbers have 3 prime factors. Do 20 and 30 have the same number of factors? Why is that? What influences the number of factors a number has?

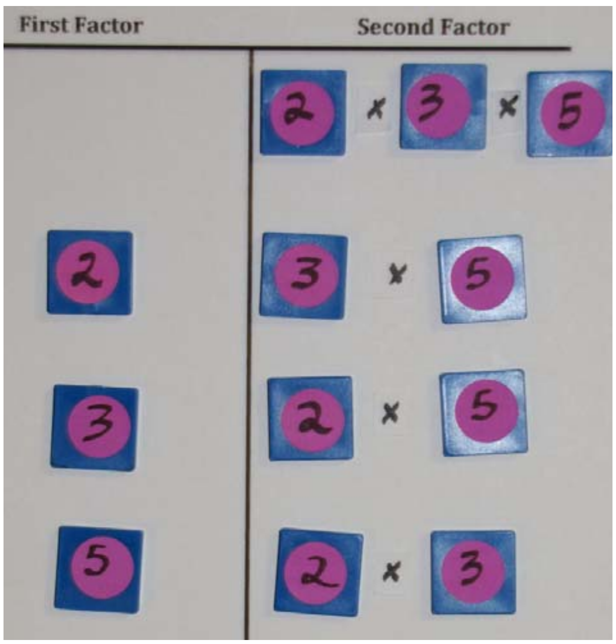


*Figure 4.* The factor groups that yield all the factors of 30 are shown.

***Recognizing squares, cubes and roots***

The student is given a perfect square (1,764) but not told the number is a perfect square (see Figure 5). The student finds the prime decomposition using the prime factor tiles. The student is asked to arrange all the prime factor tiles into two identical groups. Is this possible? What does this mean? If the student multiplies the tiles of the first group and records this number, the student will then discover that the number is a perfect square. What is the relationship between the recorded number and the original given number?

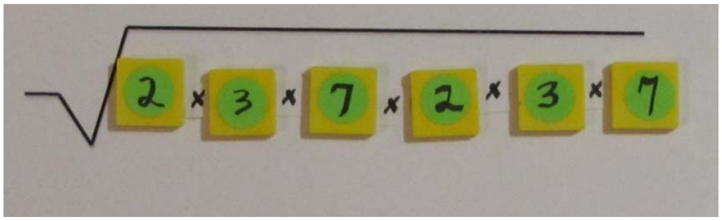


*Figure 5.* The square root representation of 1,764 is shown.

The instructor gives provides another number (not a perfect square) and asks the student to repeat the activity. Why couldn't the student separate the tiles into two identical groups? Based on your findings, when is a number a perfect square? When the number is a perfect square, how can the square root be determined using the prime factor tiles? This activity can be generalized for perfect cubes. How can a perfect cube be identified from the prime decomposition? How do we find the cubic root?

The same idea can carry over to finding simplified roots of numbers that are not perfect squares or cubes. For example the $\sqrt{180}$ can be investigated using prime factor tiles. Students can build the prime decomposition of 180 (2x2x3x3x5). How can $\sqrt{180}$ be simplified? What would the rewritten root look like? It can be rewritten as $6\sqrt{5}$.

**Methodology**

*Participants*

The prime factor tiles were used with preservice teachers in a *Mathematics for Elementary School Teachers* course. The preservice teachers had not yet entered the pedagogical portion of their program; they had not been exposed to educational theory. Data were gathered from ten preservice teachers; nine were female.

*Procedure*

Preservice teachers engaged in several activities over two class meetings (1 hour 15 minutes each) to help guide their understanding of prime decomposition supported by the use of prime factor tiles. Four distinct topics were examined: defining prime and composite numbers, explorations involving factors, using Venn diagrams to find the GCF and LCM, and application questions (discussed whole class). Observational notes were taken during and after all classroom interactions. In addition, data were collected from the written responses of the participants. During the third meeting, they completed an individual assessment encompassing prime decomposition concepts.

*Defining prime and composite numbers*

They were asked to develop the meaning of the terms prime and composite after prime decomposing fifteen different numbers using prime factor tiles. They put the numbers that were decomposed into two distinct categories and provided a description. They were then asked to describe where the number 1 fit in terms of their generated definitions. (See Kurz and Garcia (2010) for the specific activity.)

*Investigative Explorations*

Next, they investigated various explorations using prime factor tiles. For example, find (Kurz & Garcia, 2010, p. 259):

- Two numbers that share the factors 2, 3, 4 and 5. Find the smallest number that has all four factors
- A number that is even and has factors of 17 and 41

- The smallest number that has 3 and 11 as factors; to find it, multiply the two numbers. To find the smallest number that has 3 and 12 as factors, you cannot multiply the two numbers (it would not result in the smallest number possible). Why not?

*Using Venn diagrams for finding the GCF and LCM*

Preservice teachers used Venn diagrams to develop a method of finding the GCF and LCM of two numbers. Using the prime factor tiles, they built the prime decomposition of 48 and 72. Then, they were supplied with overlapping circles and asked how the tiles would be properly placed in the circles. After the tiles were placed, they were asked to find a method using Venn diagrams that would lead to the GCF and LCM. They worked through several examples to test their conjectures.

*Application questions*

The next set of questions was designed specifically to observe whether the preservice teachers could apply some of the knowledge they gained in the investigations/lessons. These problems generally give preservice teachers difficulty (Zazkis & Campbell 1996). They examined the following questions (Zazkis & Campbell, 1996, p. 542):

- Consider the number $M=3^3 \times 5 \times 7$
- Is M divisible by 7? Explain
- Is M divisible by 5, 2, 9, 63, 11, 15? Explain.

The questions were analyzed whole class. Algorithmic methods were discussed (dividing) along with reasoning techniques.

*Individual assessments*

During the next course meeting, the preservice teachers completed an individual assessment. Using a calculator and/or tools, these questions were designed to provide an opportunity to apply prime decompositions techniques to new and previously explored examples. They answered the following questions:

1. Tell whether 97 is prime, composite or neither.
2. Tell whether 1 is prime, composite or neither.
3. Find the prime factorization of 1002 and $1002^2$.
4. True or False. “If a number is divisible by 6, then it is divisible by 2 and 3.” (Billstein, Libeskind & Lott, 2010, p. 297)
5. True or False. “If a number is divisible by 2 and 4 then it is divisible by 8.” (Billstein, Libeskind & Lott, 2010, p. 297)
6. “Find the least divisible number by each natural number less than or equal to 12.” (Billstein, Libeskind & Lott, 2010, p. 312)
7. Find the GCF and LCM of 400 and 75.
8. Find the GCF and LCM of *abc* and *bcd*.
9. True or False. 5 is a multiple of 25.

Because these questions were not specifically discussed in class (other than problem 2), the researchers wanted to determine whether or not the preservice teachers could apply the reasoning techniques explored in class to different questions addressing very similar mathematical ideas.

## Results and Discussion

### *Initial Classroom Interactions*

When first using the tiles, they preservice teachers were perplexed and could not immediately understand how to use them. They could not grasp how the tiles connected to prime decomposition. A few of the preservice teachers were very resistant to the tiles stating, “These are hard to use, I just want to build factor trees” and “I don’t like these tiles.” While the preservice teachers were permitted to use alternative techniques, all but one used the tiles. When they did use the tiles, some first built factor trees (either on paper or in their heads) and aligned the tiles with the factor tree. It took the preservice teachers some time to connect the tools to their algorithmically based procedures. In the beginning, some preservice teachers were not able to distinguish the tools as mathematically helpful. As time progressed however, they relied less on factor trees and more on the tiles.

Table 1 displays the overall accuracy of the individual assessment questions asked at the completion of the unit. These findings will be used to support the analysis of preservice teachers' development in the four prime decomposition activities explored.

Table 1. Accuracy of Individual Assessment

| Question | Answer | Accuracy |
|---|---|---|
| 1. Tell whether 97 is prime, composite or neither. | Prime | 100% |
| 2. Tell whether 1 is prime, composite or neither. | Neither | 90% |
| 3. Find the prime factorization of (a) 1002 and (b) $1002^2$. | (a) $2 \cdot 3 \cdot 167$<br>(b) $2^2 \cdot 3^2 \cdot 167^2$ | (a) 80%<br>(b) 60% |
| 4. True or False. "If a number is divisible by 6, then it is divisible by 2 and 3." (Billstein, Libeskind & Lott, 2010, p. 297) | True | 100% |
| 5. True or False. "If a number is divisible by 2 and 4 then it is divisible by 8." (Billstein, Libeskind & Lott, 2010, p. 297) | False | 50% |
| 6. "Find the least divisible number by each natural number less than or equal to 12." (Billstein, Libeskind & Lott, 2010, p. 312) | 27,720 | 10% |
| 7. Find the (a) GCF and (b) LCM of 400 and 75. | (a) 25<br>(b) 1,200 | (a) 60%<br>(b) 70% |
| 8. Find the (a) GCF and (b) LCM of *abc* and *bcd*. | (a) *bc*<br>(b) *abcd* | (a) 80%<br>(b) 70% |
| 9. True or False. 5 is a multiple of 25. | False | 60% |

*Defining prime and composite numbers*

Classroom Interaction Results

During the class activity, 90% preservice teachers stated that 1 was a prime number. When questioned about their generated definitions, most of the preservice teachers provided a memorized definition "a prime is divisible by 1 and itself." The professor asked how does the number 1 fit into that definition. Responses included "One can only go into itself and by 1." When further questioned, the preservice teaches continued to insist 1 was prime number because it is divisible by 1 and itself. One preservice teacher objected, "No, because a prime number is divisible by 1 and itself but 1 is only divisible by itself [which is also 1]." After a brief whole class analysis of her statement, the group still felt 1 was prime. The professor defined the number 1 as neither prime nor composite. Some of the preservice teachers continued to insist 1 was a prime number in their other mathematics classes. The case that 1 was neither prime nor composite was reiterated. They asked, "Are you sure?" The professor assured them that once they walked out of the classroom, 1 would continue to be neither prime nor composite forever; they laughed.

Other than one participant, the preservice teachers were not able to understand that 1 was neither prime nor composite. They just took the professor's word on its categorization. While this discourse shows the professor's failure at helping preservice teachers understand why 1 is neither prime nor composite, it is important to recognize that this was a very difficult concept for these preservice teachers to conceptually understand. Perhaps the difficulty stems from being taught incorrectly in the past, as nearly all of the preservice teachers stated that they were told that 1 was a prime number. Because the trajectory of learning and quality of instruction cannot be mapped at this point, there is no way to know. This finding shows the difficulty of this content and the need to address these misconceptions through curriculum development.

Assessment Results

In terms of the individual assessment, all of the preservice teachers recognized 97 as prime. Nine out of the ten said 1 was neither prime nor composite. The preservice teacher who answered incorrectly stated 1 was a prime number. Zazkis and Liljedahl (2004) found similar misconceptions. Preservice teaches were able to define what prime meant and could recognize prime numbers. However, the application of this knowledge was much more difficult.

*Investigative explorations*

Classroom Interaction Results

The explorations were the most difficult component of the prime decomposition lessons. The preservice teachers struggled throughout the investigations. They commented, "These problems are hard." Some of the preservice teachers were struggling because they randomly selected a number and then tested the divisibility (guess and check unsupported by conjectures).

They were not working from the factors. The professor guided their investigations by encouraging the preservice teachers to work with the factors. Once the preservice teachers understood that it was easier to start from the factors rather than a randomly generated number, they became more successful. They were able to work through the rest of the explorations. Working with one another, they correctly solved most of the explorations. The final exploration gave the preservice teachers the most difficulty: Find “the smallest number that has 3 and 11 as factors; to find it, multiply the two numbers. To find the smallest number that has 3 and 12 as factors, you cannot multiply the two numbers (it would not result in the smallest number possible). Why not?” (Kurz & Garcia, 2010, p. 259). Some explanations were not mathematically clear: “Because 12 is not prime. 12 is smallest.” “Because it is divisible by 3 and 11” “Because 12 is made up of several twos.” Others had a clearer understanding: “With 3 and 11 both are only divisible by itself. 12 and 3 can be divisible by 4 and 3.” “Because 12 has a factor of 3” “Because 12 has multiple factors but 3 and 11 are both prime.”

Assessment Results

The difficulties with the explorations were also demonstrated in problem 6 of the individual assessment; “Find the least divisible number by each natural number less than or equal to 12” (Billstein, Libeskind & Lott, 2010, p. 312). Only one preservice teacher correctly answered this question. Even though this question somewhat aligned with the second problem “What is the smallest number that has [2, 3, 4 and 5 as] factors?” the preservice teachers were unable to find a solution. Perhaps the problem was in the decoding the question (for example “natural number” or “least divisible number”). However, it seems to go deeper than vocabulary. They were perplexed by the explorations; the factor tree model was taken out of its standard context and the preservice teachers had to think in a more complex manner. The algorithmic procedure did not work; other methods had to be developed and evaluated. They had to work backward from the factors.

The difficulties the preservice teachers had with the explorations were not predicted; the questions were written for the upper elementary student and not perceived by the researchers as too complex. They started with a random number rather than using factors to determine the number. What these findings indicated was that preservice teachers need activities to analyze the factor tree procedure and work toward the application and decoding of prime decomposition. They could create factor trees and could decompose numbers using tiles. But when they had to apply reasoning beyond the procedure, difficulties resulted. Zazkis and Campbell (1996) found similar results in that “checking whether or not an object has a certain property appears to be easier than constructing an object that has such a property” (p. 550).

*Using Venn diagrams for finding the GCF and LCM*

Classroom Interaction Results

Using a Venn diagram, the preservice teachers were provided with an opportunity to develop a method for finding the GCF and LCM. After building the prime decomposition of 48 and 72, they placed tiles in the Venn diagram. The preservice teachers then conjectured that the center numbers multiplied together yielded the GCF while all of the numbers multiplied together yielded the LCM. Other numbers were then tested to investigate whether the preservice teachers discovered a method that always works.

This was the last activity that integrated prime factor tiles. The preservice teachers were more at ease using the tiles; there were no complaints. Perhaps their comfort with the tiles was a result of exposure. They had already explored using the tiles and became more aware of the tiles’ features. When asked to describe the process, a preservice teacher responded:

To find the GCF between the two numbers, you multiply all the numbers that are in the intersecting section, which are the common factors between the numbers. Once you have multiplied them, you have found the GCF between the two numbers. To find the LCM is also very simple, you multiply all the numbers in both circles, including the intersecting section, and once you have done so, you have found the LCM.

Assessment Results

This process seemed to transfer to the individual assessment but there were some issues. The preservice teachers were more successful when finding the GCF using variables rather than numbers (problem 8). When finding the GCF of 400 and 75, the overlapping portion of the Venn diagram contained $5^2$. The preservice teachers who missed this problem failed to place both fives in the center of the Venn diagram or they placed both fives but failed to multiple them. This was the case for 3 of the incorrect preservice teachers. Another preservice teacher wrote a 5 for the LCM. The preservice teachers were able to find the LCM (for the most part) but had difficulty recognizing that 5 is not a multiple of 25. They could find the LCM (70% for numbers and 80% for variables) but could not accurately identify a multiple for problem 9 (60%). When Zazkis (2000) researched preservice teachers' understanding of factors, divisors and multiples, she found that multiple was the most difficult concept. The majority of her participants did not have a thorough understanding of the meaning of multiple. Multiple implied multiplication according to Zazkis' participants; they perceived a multiple as either a product or a factor as these are the components of a multiplication problem.

*Application questions*

Classroom Interaction Results

The next questions were analyzed whole class. The first two parts were written on the whiteboard (Zazkis & Campbell, 1996, p. 542):

- Consider the number $M=3^3 \times 5 \times 7$

- Is M divisible by 7? Explain.

Some preservice teachers began to multiply M then divide. The professor queried, “Is there a quicker way to solve this?” A preservice teacher stated, “Because 7 is a factor then that means that 7 will go into the number with no leftovers.” The preservice teachers agreed that the logic was true. The next component was presented (Zazkis & Campbell, 1996, p. 542):

- Is M divisible by 5, 2, 9, 63, 11, 15? Explain.

The preservice teachers went through the numbers and stated M was (or was not) divisible by the listed numbers based on its factors. One preservice teacher said, “In the case of 9, we have 3 to the third power which nine is dividable by that number. On the other hand, we have 2 and 11 which are not divisible of the numbers in the equation.” Another preservice teacher commented that while she understands the logic, “I’d still have to check by multiplying it out and then dividing.” The professor asked why. She said, “Because then, I’d know for sure. I don’t know for sure using the shortcut.” She was asked, “Why don’t you know for sure?” She said that the calculator would tell her for sure because she could see whether or not the number divided into it without a remainder. This aligns with Zazkis and Liljedahl’s (2004) findings; preservice teachers divide to check a number’s divisibility to be on the safe side. There is a reliance on procedures to justify reasoning and feel certain (Zazkis & Campbell, 1996).

Assessment Results

Three questions from the individual assessment were designed to observe whether preservice teachers were able to apply these techniques. Find the prime factorization of 1002 and $1002^2$ were asked with the hope that the preservice teachers would square the prime factorization of 1002. Eighty percent accurately factored 1002. Sixty percent accurately factored $1002^2$. Out of those who accurately factored $1002^2$ all but 1 worked from the factorization of 1002. Of the two who accurately factored 1002 and then inaccurately factored $1002^2$, the preservice teachers squared 1002 and then factored.

All the preservice teachers recognized that if a number is divisible by 6, then it is divisible by 2 and 3. But once the statement was altered, if a number is divisible by 2 and 4 then it is divisible by 8, they were generally unable to observe a difference in the rule. A number divisible by 8 would need $2^3$ as a factor, not $2^2$ as indicated. Only half recognized the statement as false.

### *Student Feedback*

At the conclusion of the course investigations, preservice teachers were asked to comment on what they thought of the tools. Many preservice teachers felt that the tools should be used as a supplement to factor trees. They felt that the tools would only help certain kinds of learners and that the factor trees were better because they were a specific, prescribed method. When asked if students would understand conceptually the mathematics using factor trees, the preservice teachers felt that students would not develop conceptual understanding but did not seem concerned with their observation. One commented, “The factor trees are easier because there is a process; the tiles make you think and they are harder.” This response emulates what would be expected from preservice teachers who have yet to be exposed to educational theories or approaches to learning. They focused more on the procedures of mathematics rather than the sense making often encouraged (Cooney, 1999).

## Conclusion

The brief exposure to prime factor tiles did not solve the issues that preservice teachers have in relation to factoring and prime factorization. However, that does not mean that the tools were not valuable. The preservice teachers did use the methods they discovered for finding the

GCF and LCM. They also became more comfortable using the tiles as time went on. Because they did not like the tools does not mean that they were ineffective. On the contrary, the prime factor tiles made them think harder, perhaps bringing about richer experiences. Lubinski and Otto (2004) discuss the importance of mathematics to go beyond memorizing procedures. Instead, mathematics should encompass sense making; this is more beneficial. The preservice teachers were also able to understand that the tools provided a different method for investigating prime factorization above and beyond what can be done with factor trees alone. But most importantly, the preservice teachers were able to multiplicatively decompose numbers in more than one way; they experienced an alternative method.

The real issue is the amount of exposure. If the course structure provided the time to focus on prime decomposition for more than several hours, perhaps the preservice teachers would have developed a deeper understanding. In Griffiths' (2010) framework, mathematical investigations focusing on the single theme of prime factorization lasted over many weeks; this depth was helpful in developing students' greater understanding of the theme. Szydlik, Szydlik and Benson (2003) provide investigations that delve deeper into the concepts of prime decomposition. This may be what is necessary for preservice teachers as well. However if this depth is provided, other curriculum topics must then be slashed and how does one decide what to slash? What makes a mathematical topic less important than another?

If more time is not an option, the explorations seem to be the most needed curriculum in relation to prime decomposition. Because preservice teachers had such trouble, it is imperative that these difficulties are addressed. It may be helpful to have preservice teachers solve the explorations and then create their own explorations. This provides the opportunity to move beyond the creation of factor trees while focusing more deeply on prime factorization.

Additionally, Szydlik et al. (2003) provide a rich question for investigating prime decomposition: find the number less than 1,000 with the most factors. Robbins and Adams (2007), Kurz and Garcia (2010) and Kurz and Garcia (2012) provide readymade handouts that can be used to delve deeper into prime decomposition. Deeper and more meaningful investigations with explorations may be more advantageous than the curriculum implemented in this inquiry. Our focus was on several topics scratched just below the surface. Perhaps fewer topics that go deeper may be more fruitful if time is an issue (Lubinski & Otto, 2004).

Our findings concur with Zazkis and Campbell (1996) that there needs to be an increased focus on basic arithmetic ideas. In addition, number theory should be an integral part of preservice teachers' program of study (Brown, 1968). Future studies should measure impact after additional time was provided for preservice teachers to mathematically explore and evolve. These results give further support for the need to go deeper (Lubinski & Otto, 2004) and more analytical when teaching prime decomposition. A richer environment that has a greater emphasis on the prime factor tiles should be investigated. While tiles were encouraged during the investigations, they were not required. Preservice teachers who were challenged by the tools tended to disregard them. It would be interesting to research whether a deeper emphasis on prime factor tiles with labs, class activities and creation of explorations would improve understanding of prime decomposition. In addition, concept maps can be used to guide growth and understanding of terms (Bolte, 1999). Because the findings presented were based on an exploratory inquiry, there needs to be a larger study with a control and experimental group. This kind of study will help shed light on exactly what mathematical concepts the tiles can support beyond a lecture-based, factor tree approach.

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