UNIQUELY PRECISE: IMPORTANCE OF CONCEPTUAL KNOWLEDGE AND MATHEMATICAL LANGUAGE

By

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ABSTRACT

The importance of mathematical concept development and language is recognized early in children's schooling as they mature through shape and counting experiences. The reader may recall instances of a youngster referring to a "corner" of a shape before the reader has the language of vertex. This language precision needs to continue to grow as the learner moves through arithmetic into algebra, geometry, and further mathematics. This precision is essential and is reinforced in the common core standards for mathematics (2010). If the primary goal is to facilitate proficiency in math for all students (including students with disabilities), there needs to be an emphasis on the deeper conceptual development and the uniquely precise nature of mathematics language both at the pre-service and in-service levels. This is essential as literature suggests that there is a significant relationship between teachers' mathematical knowledge and student achievement. The lack of teachers' mathematical knowledge prevents explicit instruction in the area of math concepts and/or a lack of focus on the mathematical language. This in turn causes barriers for k-12 students as they advance in the math curriculum. In this paper, the authors will discuss (a) the importance of mathematical concept development and language; (b) provide an example of a lack of precise conceptual understanding of prime number among pre-service teacher math educators; and (c) list explicit strategies that can be used to facilitate both the conceptual and language development at the pre-service level.

Keywords: Mathematics Language, Pre-service Teachers, Common Core Math Standards, Conceptual Knowledge, Prime Numbers.

INTRODUCTION

The results of the recent National Assessment of Educational Progress indicate that a large achievement gap exists between the students with and without disabilities in the area of math in the United States. The 2013 results indicate that only 18% percent of students with disabilities were performing at proficient or advanced levels compared to 45% of students without disabilities at the 4th grade level (CEC Policy Insider, 2013). Similarly 8% of the students with disabilities were performing at proficient or advanced levels compared to 39% of students with disabilities (CEC Policy Insider, 2013). This 27% point gap at the 4th grade level and 31% point gap at the 8th grade level impelled the Council for Exceptional Children, the largest International Professional organization advocating for the education of students with disabilities, for an examination of current teaching practices so that required improvements can be undertaken to meet the diverse needs of students with disabilities. Similarly, the low performance of U.S. students on National and International tests has renewed calls for reform of math education with an emphasis on deeper conceptual understanding (Carney, Brendefur, Thiede, Hughes, & Sutton, 2014) and led to the development of Common Core Math Standards.

Recent federal mandates in the field of education (NCLB, 2001 and IDEIA 2004) have led to more students with disabilities being included in the general education classes to access the general education curriculum. For example, in fall 2010, 95% of students identified with disabilities were educated in the regular schools and 60.5% of the students spent 80% or more time in the general education class (U.S. Department of Education, National Center for Education

Statistics, 2013).This has led to co-teaching and other teaching arrangements between the general and special education teachers (Cramer & Nevin, 2006). However, most general education math teachers have extensive content expertise and limited knowledge and skills needed to design instruction for students with special needs. Given that higher percentage of students with special needs are being educated in the regular classrooms, it is necessary to examine the teaching practices of general education math teachers to make necessary teaching improvements to facilitate the learning of all students.

Importance of Mathematical Language

Children do not innately learn the language of mathematics and mathematics is a foreign language mostly learned at school (Cimbricz, 2013). Furthermore, mathematics has its own definitions that are contrary to definitions students bring to the classroom. For example, a table where the child sat for breakfast is certainly different than the table of numerical values that they explore later in the morning during their mathematics class. In school, students explore squares and its respective properties in two dimensions. This is a very different, potentially a contrary experience, to the experience of sitting in their town's square. The low level of student achievement in mathematics, including students with disabilities, is due to the misconception among teacher educators that mathematics is cultural and language free due to the use of mathematic symbols. In reality, success in mathematics is dependent upon good conceptual understanding of concepts for which language and concrete experiences are quintessential (Garrison & Mora, 1999). Thus, it is essential that teachers facilitate the acquisition and use of academic language skills to deepen students' conceptual knowledge of the discipline (Cimbricz, 2013). Furthermore, teachers' mathematical knowledge was found to be significantly related to student achievement gains (Hill, Rowan, & Ball, 2005). Hence, both pre-service teachers and in-service teachers need to understand the importance of mathematics language, focus on explicit teaching the mathematical understanding of the concepts and the language associated with it.

Objectives

The purpose of this paper is to (a) define mathematical language; (b) provide an example of a lack of precise conceptual understanding among pre-service math educators using the concept of prime number; and (c) list strategies to facilitate the mathematic language and conceptual development of pre-service math teachers. This is essential as teachers' mathematical knowledge is related to students' math achievement.

What is Mathematical Language?

Mathematics differs from language arts in that it requires the use of precise language and formal models to develop conceptual knowledge (Bernstein, 1996). Academic language in the area of mathematics is wide-ranging and includes the use of (a) mathematically specific meanings that are different from the meaning implied during conversational language (i.e., ruler, right angle, tan, etc.), (b) specific words that define various mathematics concepts (fraction, rational number, equilateral, square root, etc.), (c) symbols/syntax (including equations), (d) visual displays, and (e) precision (Merino & Zozakiewicz, 2015). In order to develop and deepen students' mathematical understanding, it is necessary to identify and support their conceptual development and underlying academic language in mathematics.

The importance of mathematics language is highlighted both in the Common Core Math Standards (National Governors Association Center for Best Practices & Council of State School Officers, 2010) and the recent pre-service teacher assessments such as the edTPA. The Common Core Standards for Mathematics (CCSM, 2010) puts forth eight mathematical practices. Included in this list is a strong emphasis on "Attend to Precision" that is defined as · "mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions" (p. 7).

Precision in mathematics, called for in CCSM, includes appropriate units of measurement (for example, area is not measured in gallons), inclusion of mathematical conventions (i.e. m represents slope), labeling axes correctly (ordered pairs listed as (x, y) pairs), and being able to calculate accurately and efficiently (i.e. arithmetic automaticity). According to CCSM, precise communication has to be responded by "others." Others can include peers as well as the teacher (Imm & Stylianon 2012).This precision in language is dependent on students' conceptual understanding of the mathematical concepts and procedures.

Example: Concept of Prime Number

As detailed above, conceptual knowledge and precise mathematical language are essential to improve the achievement of all students. the authors will highlight the importance of conceptual understanding and precise language using an example of their work with pre-service teachers majoring in math. Pre-service teachers at our institution are required to video classroom episodes during their field experiences and compose reflections on their teaching. One of the authors regularly records a class session in an effort to support the students as they prepare their electronic teaching portfolios. The authors will provide a vignette (with pseudonyms) to highlight issues with precise conceptual understanding and associated mathematics language among pre-service teachers using the concept of prime number. The lack of reciprocal precise communication and conceptual understanding became apparent in this recorded session when the question, "Is one a prime number?" was posed by the instructor.

Marco: arguably.

Scott: I've been told no.

Marta: yeah I've been told most cases it's not.

Scott: every prime number has to be divisible by one and itself and one is itself.

When Katherine was asked if she wanted to get in on this discussion she politely responded "no." This led to the instructor asking additional questions to elicit student understanding with two examples and a non-example before returning back to the question if one was a prime number.

Instructor: Is two a prime number?

Students: Two is a prime number because of the factor pair of (1, 2)

Instructor: Is three a prime number?

Students: Three is a prime number as its factors are (1, 3).

Instructor: Is six a prime number?

Students: No, It is composite as the factor pairs were (1, 6), (2, 3).

Instructor: So… what are the factors of one?

Students: One only has one factor namely itself.

Instructor: So… is one a prime number?

There was no consensus about whether the number one was prime number, and the notion that one was not a prime number in "most cases" left everyone in a moment of wonder. This sense of wondering continued when students reexamined the frequently incomplete understanding that prime numbers are divisible by itself and one…i.e., "only have two factors."

This above communication highlights the importance of examining teachers' mathematical knowledge (i.e., understanding and making explicit use of definitions) as it is the essential pre-requisite for facilitating their students "explicit use of definitions" (p. 9) that CCSM (2010) standards of practice calls for. Attempting to develop the concept of prime numbers includes the need for the precision offered in the definition. If any of the pre-service teachers had read Rooney (2008), their response is understandable. Rooney states: "Primes are a special class of integers: they are numbers which have no factors (cannot be divided by anything) except themselves and 1. The primes under 20 are 2, 3, 5, 7, 11, 13, 17 and 19 (1 is usually not included)" (p. 48). By Rooney's standard, Marco was correct. This necessitates the need for explicit instruction in which students (pre-service teachers in our case) conceptual understanding is facilitated using carefully designed examples and non-examples.

Explicit Strategies to Facilitate Conceptual and Language Development

When authors consider Scott's remark, "I've been told no," the authors immediately relate this to students "being told" or given the information as a "fact" without understanding. The authors conjecture that supporting the language of mathematics and its concepts (meanings) should not occur by students "receiving" the information as "fact" (i.e., memorizing a definition). For example, frequently students are told "prime numbers" only have two factors, one and itself. A long list of examples of the prime numbers is then provided. However, without explicit instruction, students associate prime numbers mostly with odd numbers. This is especially true with students with special needs. This method of instruction where prime number is taught as a "fact" inhibits the understanding of the concept. So when a learner is asked if "one" is a prime number, we experience many different responses as discussed above. We suggest that students wrangle with this notion of prime number, argue about whether one should be included, after all it only has one factor, by relating the example and non-examples with the precise definition that mathematics offers. In this case, one has only one factor, itself, so it serves as a non-example, an important experience for students. The use of carefully designed examples and non-examples is essential and is commonly used to facilitate the conceptual understanding of students with disabilities. We extend this need for all students. The lack of conceptual understanding inhibits the ability of the pre-service teachers to effectively teach their pupils.

Implications

It is also important to understand that a lack of conceptual understanding and precise definitions also hinder students understanding of subsequent mathematical knowledge. For example, consider Gallian's (1994) statement of the Fundamental Theorem of Arithmetic, which states that "every integer greater than 1 is prime or a product of prime numbers and this product is "unique," except for the order in which the factors appear" (p. 7). Schumer (1996) shares an example about the importance of "unique." "For example, 91 is represented as 7 times 13 or $91=7\bullet13$, and there is no other way to represent 91 as a product of prime numbers save for $13\bullet7$" (p. 38). For students to understand this they need to have the conceptual and precise understanding that 1 is not considered a prime number. If it were, then the prime factorization of an integer would no longer be unique. For example, if one was not a prime number, then students can represent "91 as follows: $91=7\bullet13=1\bullet7\bullet13=1\bullet1\bullet7\bullet13$ and so on".

Elementary children spend time learning about odd, even, prime, and composite numbers. The authors frequently see pre-service teachers creating factor trees, rewriting a composite number as a product of primes. But if the elementary children, who we see many years later as pre-service teachers, don't have exposure to the precise conceptual understanding of prime number, confusion is a likely outcome. If, as the CCSM (2010) Mathematical Practices suggests that students are to critique the reasoning of their peers, attention to this precision could support students to embrace language precision. We cannot expect students to critique the precision of others, if elements of the precision are lacking. Hence it is necessary to develop students' conceptual understanding through the use of careful language along with explicit designed examples and non-examples. National Council of Teachers of Mathematics (NCTM) standards call for teaching of new concepts using concrete examples and experiences and then scaffolding using semi-concrete graphical and abstract symbolic, verbal, and written representations to facilitate a thorough understanding of the concept (Garrison & Mora, 1999).

Conclusion

If Schleppegrell (2010) is correct in that "attempts to make mathematics related directly to students' experience and to eliminate or downplay the technicality of mathematics may also be problematic, since the technicality is functional for making the kinds of meanings that are relevant to constructing knowledge in mathematics" (p. 86) then the researchers must work with pre-service (and in-service educators) to support them to "help students adopt the mathematical discourse that will enable them to participate in mathematics in the formal context of schooling" (Schleppegrell, 2010, p. 87). We posit, from our experience, that examples and non-examples play a critical role in the students' conceptual development and associated language precision. Furthermore, the researchers have found many textbooks lacking in precision. The researchers know that many educators are textbook dependent, this all the more increases the demands on the teachers to make their instruction explicit. Students also need repeated exposure to use the vocabulary and be able to examine and share their understanding. Kranda (2008) states: "Students resist using precise mathematical language in their solutions because it is not natural to them; therefore, specific vocabulary instruction (with examples and non-examples) and repetition in using the vocabulary during instruction is necessary to make using the language more natural for the students" (p. 17).

Teachers are habituated into the traditional ways of instructional delivery, which is the transmission of knowledge to the student (Carney, Brendefur, Thiede, Hughes, & Sutton, 2014). Most of this traditional route constitutes memorizing facts and knowing the procedures. To enact meaningful changes at the classroom level, the researchers need to make the teachers and their pupils engage in participatory dialogue to develop deeper conceptual understanding without which learning remains outside the person.

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