Classroom Ideas

Dividing fractions:
a pedagogical technique

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Dividing fractions, as easy as pie,
Flip the second fraction, then multiply.
And don't forget to simplify,
Before it's time to say goodbye.

We have all heard the rhyme, and perhaps taught it ourselves to our students. When dividing one fraction by a second fraction, invert, that is, flip the second fraction, then multiply it by the first fraction. To multiply fractions, simply multiply across the denominators, and multiply across the numerators to get the resultant fraction. So by inverting the division of fractions we turn it into an easy multiplication of fractions problem.

I received a phone call from a primary school teacher who was teaching this method to her Year 6 class. She had been asked a question, one that she had never before been asked. An inquisitive 12 year old was not happy to just accept the methodology taught; he wanted to know why you 'flip' the second fraction over. I teach a bridging mathematics course at university and hardly ever have I had anyone ask "why"—why would you would invert a fraction and then multiply?—so it is not surprising that this young teacher has not encountered the question before. Knowing why we perform a certain mathematical action rather than just rote learning will lead to deep understanding (Cheng, 2013). In practice, I find that explaining 'why' cements that deep understanding rather than remembering a rhyme we were taught years ago.

I went over two reasons with my colleague and the answers may be of interest to you as well. The first explanation would be suitable for younger students learning fractions and presents the explanation in an intuitive way. The advantage of this method is that it shows the student that the methodology does work, and also explains why it works.

Reason 1

Start off by solving a simple fraction. For example, how many times does $\frac{1}{2}$ go into 4?

$$\text{Solve: } \frac{4}{\frac{1}{2}}$$

Most students can visualise how many halves go into four, as there are two halves in one, and there are four 1's in four, so therefore there must be 2×4 or 8 halves in four. In essence, we are multiplying the numerator with the denominator of the bottom fraction. To make this clearer, let's display this problem in fraction format. Remember that 4 can be represented as a fraction by placing it over 1.

$$\text{Solve: } \frac{4}{1} \div \frac{1}{2}$$

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Flip the second fraction: $\frac{4}{1} \times \frac{2}{1}$

Then multiply: $\frac{4 \times 2}{1 \times 1} = \frac{8}{1} = 8$

Answer: 8

Since multiply is the opposite to divide, by flipping the fraction over and reversing the division we have kept the equality in the equation while solving it. So we have shown that flipping over the fraction does give the correct answer. We can also show how this works algebraically. This solution would be suitable for those students who have been exposed to basic algebraic calculations.

Now, let us look at a more complicated division of fractions to illustrate the concept effectively.

Solve: $\frac{3}{8} \div \frac{2}{3}$

Using rote learning the student would invert the second fraction and multiply.

Solution: $\frac{3}{8} \times \frac{3}{2} = \frac{9}{16}$

Reason 2

Proving the solution algebraically:

$$\frac{3}{8} \div \frac{2}{3} \text{ can be re-written as } \frac{\frac{3}{8}}{\frac{2}{3}}$$

We can eliminate the bottom fraction by multiplying both top and bottom fractions by $\frac{3}{2}$.

$$\frac{\frac{3}{8}}{\frac{2}{3}} \times \frac{\frac{3}{2}}{\frac{3}{2}}$$

(we are effectively multiplying the fraction by 1)

Cross multiplying both top and bottom fractions and we end up with:

$$\frac{\frac{9}{16}}{\frac{1}{1}}$$

The bottom fraction is 1, and so we are left with:

$$\frac{9}{16}$$

This solution does require some basic algebra but I have found that this solution is also elegant in its simplicity, and contributes to the deep understanding that students require without resorting to rhymes or rote learning.

References

Cheng, An-Chih, Jordan, Michelle E, & Schallert, Diane L. (2013). Reconsidering assessment in online/hybrid courses: Knowing versus learning, *Computers & Education, 68*, 51-59.