Using challenging tasks for **formative assessment on quadratic functions** with senior secondary students

Karina J. Wilkie
Monash University, Victoria, Australia
<karina.wilkie@monash.edu>

Senior secondary mathematics students who develop conceptual understanding that moves them beyond "rules without reasons" to connections between related concepts (Skemp,1976, 2002, p. 2) are in a strong place to tackle the more difficult mathematics application problems. Current research is examining how the use of challenging tasks at different levels of schooling might help students develop conceptual knowledge and proficiencies in mathematics as promoted in the Australian curriculum—understanding, fluency, problem solving, and reasoning (ACARA, 2009). Challenging tasks require students to devise solutions to more complex problems that they have not been previously shown how to solve, and for which they might develop their own solution methods (Sullivan et al., 2014). Another key area of research is on formative assessment which has been found to be effective for increasing student motivation and achievement under certain conditions (for example, Brookhart, 2007; Karpinski & D'Agostino, 2013).

This article describes one study within a larger project on challenging tasks. It explored 87 Year 10 students' responses to a quadratics task, and their views on learning with challenging tasks and with multiple solution methods. Some ideas are shared on the potential for using challenging tasks, not only for conceptual learning, but also for formative assessment. This increases the benefit to students by not only providing opportunities for them to grapple with mathematics concepts relationally, but also giving them timely feedback that motivates them to address gaps between their knowledge and learning goals. It also provides teachers with valuable information on their students' current levels of understanding to help them make adjustments in their teaching approaches during the learning process.

Using challenging tasks with secondary students

Although challenging tasks have been found in numerous studies to promote effective conceptual learning (for example, Hiebert & Grouws, 2007; Stein & Lane, 1996) some issues that might constrain teachers' implementation of them have been highlighted. In some studies, secondary students were found to "resist task engagement or negotiate the task demands downwards" (Anthony, 1996, p. 42) when the task was difficult, involved higher-level thinking, or did not produce readily available answers. Some students have been found to exert pressure on teachers to explain the task or provide simpler ones and teachers find it difficult to manage this (Sullivan, Clarke, Clarke, & O'Shea, 2009).

Research on students' learning goals in mathematics has found that helping students to develop a growth mindset rather than a fixed mindset increases the likelihood of their persisting with more challenging work. A growth mindset views intelligence as something that is not fixed but can be improved through effort (Dweck, 2000; 2007). Finding ways to encourage students to engage in productive struggle with challenging tasks because of the benefit to their conceptual learning is a focus of the overall research project of which this article describes one part. In the study with Year 10 students, the mathematics teachers included alongside their use of challenging tasks, explicit explanations to the students about the value of putting effort into the tasks and that they were deliberately chosen to be more difficult than standard exercises or class work.

After the quadratics task from this study was trialled (Appendix 1), the students were invited to complete an anonymous reflective questionnaire, sharing their views on challenging tasks and on learning to tackle problems using different solution methods. Nearly 60% indicated that they did not mind challenging tasks and 30% indicated that they liked them. Nearly two-thirds liked to learn new ways to approach tasks from peers and another two-thirds liked to learn multiple strategies from the teacher. It appears that, over time, given repeated experiences with challenging tasks and explicit encouragement from teachers, students perceive such tasks as beneficial to their learning and are more likely to engage with them. (For further details on this aspect of the study, please see Wilkie, in press). Yet there is much more to learn about engaging and motivating students to move beyond instrumental or procedural learning and grapple with concepts relationally (Skemp, 1976; 2002). The following section describes some specific findings from the literature on helping students to develop their conceptual understanding of functions.

Developing students' conceptual knowledge of functions

Students develop their algebraic thinking through understanding, connecting, and moving between different representations of functions. They learn to move flexibly and fluently between representations rather than simply knowing isolated procedures in each. "A mathematical representation cannot be understood in isolation… The representational systems in mathematics and its learning have structure, so that different representations within the system are richly related to one another" (Goldin & Shteingold, 2001, p. 2). Functions can be viewed in two ways: as process and as object (Moschkovich, Schoenfeld, & Arcavi, 1993). A process view focuses on the relationship between two variables: "for each value of *x*, the function has a corresponding *y* value" (p. 71). It also considers sets of individual points on the Cartesian plane. An object view sees functions as objects that can be worked with 'as a whole', such as parameterised classes of functions, transformations of whole graphs on the Cartesian plane, and operations on functions. Figure 1 presents a framework that demonstrates how students develop their conceptual understanding of functions by moving between different types of representations (horizontally) and between the two views, process and object. An additional dimension relates to students either having to construct or interpret a particular representation (Romberg, Carpenter, & Fennema, 1993).

		Type of representations				
		Real-world context	Verbal representation	Tabular representation	Algebraic representation	Graphical representation
Perspective of functions	Process	Constructing / Interpreting				
	Object	Constructing / Interpreting				

Figure 1. A functions framework for developing students' knowledge conceptually with multiple representations and perspectives (Moschkovich, et al., 1993; Romberg, et al., 1993).

The challenging quadratics task used in this study was deliberately open-ended to enable students to make their own decisions about which representations they could use to explore the position of different parabolas that match each criterion—crossing the x-axis once, twice, or not at all. Although the students in this cohort appeared to have some prior instrumental knowledge of the quadratic formula, they were encouraged by the wording of Part A in the task to use and connect their conceptual knowledge of parabolas by constructing multiple examples that met each criterion and by using algebraic and graphical representations. This could be achieved using either view of functions (process or object) but it appeared that those who used an object view seemed to be more able to develop effective generalisations in Part 2, which asked them to notice patterns in their different examples. The task was also designed to encourage students to think about and even connect the different algebraic representations of parabolas—turning point form (which connects well to graphical representations and their transformations), general form (with $\Delta = b^2 - 4ac$), or factorised form—but this may be more likely with older students studying Mathematical Methods in Years 11 and 12. The third part of the task sought students' exploration of horizontal lines and their transformations to meet each criterion—crossing a particular parabola once, twice, or not at all. It provided another opportunity to use different representations (algebraic and graphical) and to make connections between their knowledge of lines and parabolas, which can lead to generalisations about their intersection.

Challenging tasks that focus on generalisation and connections between different perspectives of functions, like the one shared in this article, provide students with the opportunity to develop their conceptual knowledge of functions. Emphasising to students the features of such tasks that make them productive for their learning helps them focus on meaningful aspects of learning and encourages their effort (Black & Wiliam, 1998a). In addition to the types of tasks and activities chosen by teachers, the ways they are used by teachers is another avenue for considering how to improve the effectiveness of student learning. The next section discusses research on formative assessment and how it might increase student motivation and achievement.

Using formative assessment to motivate students and improve achievement

Formative assessment involves a teacher's appraisal of student work for the dual purpose of providing constructive feedback to students and of guiding their teaching decisions during the learning process (Karpinski & D'Agostino, 2013). It contrasts with summative feedback, which is given at the conclusion of the learning process. In recent years educational policy and research initiatives have been advocating a greater focus on formative assessment. Reviews of research have shown that formative assessment can promote student motivation and achievement when it has the following characteristics:

- students understand the nature of the task, its objectives, and its purpose for formative assessment;
- the criteria for achievement are identified;
- teachers and students communicate about students' current level of knowledge and future directions;
- feedback avoids comparison with other students;
- students are actively involved in their own learning; and
- teachers respond to the appraisal of student work by modifying or adjusting their teaching approaches during the learning process (Black & Wiliam, 1998a, b).

The key aspect of formative assessment that increases student motivation is descriptive feedback from teachers which focuses on the learning in the task itself. A grade or affective comment emphasises ability, competition, and comparison with others, which draws attention away from the task and towards self-aspects, which can decrease motivation. Students benefit from feedback on what they completed correctly, and also specific guidance on how to improve (Kluger & DeNisi, 1996; Wiliam, Lee, Harrison, & Black, 2004). Task-related feedback, which focuses on understanding the task and supports self-regulated learning, has been found to be more effective than general praise (Timperley, 2013). Giving students a grade with no comments, or simply giving praise on their performance as a type of formative assessment during the learning process, has been found to de-motivate some students. Descriptive information on their completion of a task with suggestions for overcoming difficulties was found to be more effective for improving their achievement than simply informing them about how well they did (Lipnevich & Smith, 2009).

Students who learn to focus on how to improve their solutions to tasks and to evaluate their own progress through self-assessment have been found to have increased both motivation and achievement (Schunk, 1996; Fontana & Fernandes, 1994). Student self-assessment is an essential component of formative assessment but students need both a clear overview of the desired learning goals and to be trained to assess their current position effectively so that they can use the information to close the gap in future efforts (Black & Wiliam, 1998a, b).

Ideas for giving students feedback on challenging tasks as part of formative assessment

Giving students feedback on their written responses to a challenging task may create an additional impetus for encouraging them to persist with the task. It provides them with an opportunity to use their current knowledge, to engage in productive struggle, and to put in effort knowing that they will receive specific feedback to guide their future learning. To be useful to students, feedback from the teacher needs to:

- provide meaningful assessment tasks linked to key learning objectives;
- detect current performance level;
- communicate the gap between current performance and reference levels; and
- provide follow-up activities that help students monitor their progress in closing the gap (Brookhart, 2007).

Although the use of more frequent formative assessment has been advocated, constant testing can overshadow the process of learning (Bangert-Drowns et al., 1991). In addition, the use of recall or rote activities and simply giving students a grade or score, may not necessarily promote deeper conceptual learning or lead to increased motivation or achievement (Black & Wiliam, 1998a, b). The challenging task itself needs to be designed to elicit students' explanations and reasoning (ACARA, 2009) so that the teacher can find evidence of student thinking in the written responses, which will help give task-specific feedback to the students and support their decision-making about further teaching (Harlen, 2007). It is important to ask students to write their responses comprehensively and to explain their thinking clearly in writing as this is important for both later assessment and for their actual learning.

One possible way to provide feedback on a challenging task is to use a rubric communicating the different levels of responses that could be made to the task. A teacher might choose to assess each student's response to a task and provide them with feedback on the rubric by highlighting their current level, or they might guide students through self-assessing their work using the rubric. Since the rubric is levelled, it can communicate to each student both his or her current level of understanding and future directions to improve.

A sample rubric for the quadratics challenging task

A sample rubric for each part of the quadratics task is provided in Table 1. The first column contains illustrative examples of responses to each of the three parts of the task (A–C). The second column explains the type of response according to levels (a higher number indicates a more sophisticated response). The third column provides further levelling within a particular type to highlight for a student their current level of understanding, and information about higher levels of response.

Table 1. Formative assessment rubric for Year 10 quadratics task Part 1 with illustrative examples.

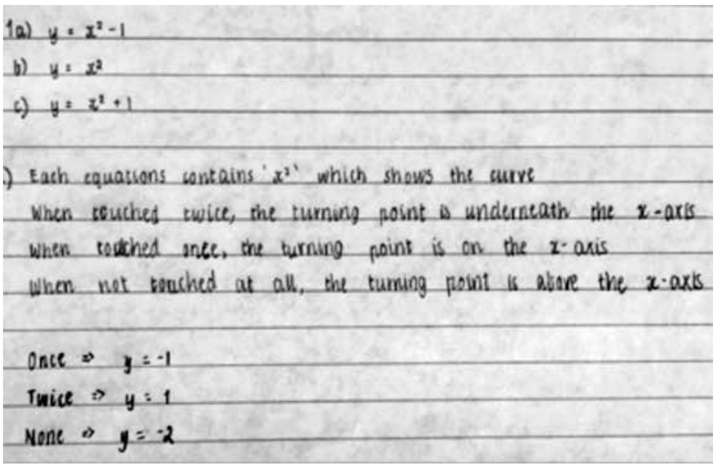
PART A. Can you find some equations of parabolas that: **a) Cut across the x-axis twice?** **b) Cut across the x-axis once?** **c) Don't cross the x-axis at all?**		
Illustrative response	Type of response	Level of response
a) $y = x^2 - 1$ b) $y = x^2$ c) $y = x^2 + 3$ (A1.1)	A1. One example for each	A1.1 Variations on x^2 A1.2 Turning point/multiple forms—ad hoc A1.3 Turning point/multiple forms—systematic
a) $y = (x - 1)^2 - 3y = -(x - 1)^2 + 3$ $y = (x + 1)^2 - 3y = -(x + 1)^2 + 3$ b) $y = (x - 1)^2 y = (x + 1)^2$ $y = -(x - 1)^2 y = -(x + 1)^2$ c) $y = (x - 1)^2 + 3y = (x + 1)^2 + 3$ $y = -(x - 1)^2 - 3y = -(x + 1)^2 - 3$ (A2.5)	A2. Multiple examples for each	A2.1 Variations on x^2 A2.2 Turning point/multiple forms—ad hoc; positive parabolas only A2.3 Turning point/multiple forms—systematic; positive parabolas only A2.4 Turning point/multiple forms—ad hoc; positive and negative parabolas A2.5 Turning point/multiple forms—systematic; positive and negative parabolas
PART B. What do you notice about each of the different groups of parabolas that you have found?		
Illustrative response	Type of response	Level of response
"All of the equations have an x^2 in them" (B1.1) $b^2 - 4ac > 0 : 2$ solutions $b^2 - 4ac = 0 : 1$ solution $b^2 - 4ac < 0 : 0$ solutions (B1.2)	B1. Comment related procedurally/instrumentally to parabolas but not clearly linked to conceptual understanding	B1.1 Noticing the term x^2 B1.2 Stating formula details $\Delta = b^2 - 4ac$
"For the parabola to cross the x-axis twice, the turning point must be below the x-axis and the parabola must be positive (smiley)." (B2.1)	B2. Partial generalisation with some conceptual understanding.	B2.1 Noticing some feature/s of parabolas B2.2 Reference to one form of parabolic equation (general or turning point or factorised) B2.3 Reference to more than one form
"In the turning point formula, if k is negative, the parabola will cross twice; if k is zero, it will cross once; if $k > 0$ it won't cross" (B3.1)	B3. Generalisation with conceptual understanding: 3 types of positive parabolas	B3.1 Reference to one form of parabolic equation (general or turning point or factorised) B3.2 Reference to more than one form
"In the t. p. formula, if k is negative, the parabola will cross twice; if $k = 0$, it will cross once; if the parabola is upside down, it won't cross at all if k is negative" (B4.1)	B4. Partial generalisation with conceptual understanding: mixture of positive and negative parabolas	B4.1 Reference to one form of parabolic equation (general or turning point or factorised) B4.2 Reference to more than one form
"In the t.p. formula, a and k must have opposite signs to each other to cross twice; if $k = 0$, a can be + or – but the parabola will cross once; if a and k have the same sign, the parabola will not cross the x-axis; if the equation can be factorised + or $-(x + d)(x + e)$ then the parabola will cross twice; if $d = e$, it will cross once. If it can't be factorised, it won't cross" (B5.3)	B5. Full generalisation with conceptual understanding: all 6 types	B5.1 Description based on visualisation of graphs B5.2 Reference to one form of parabolic equation (general or turning point or factorised) B5.3 Reference to more than one form

PART C. Can you use your previous answers to find the equations of horizontal lines that cut across the parabola $y = x^2 - 2x$: a) twice b) once c) not at all?		
Illustrative response	**Type of response**	**Level of response**
a) $y = 1$ b) $y = -1$ c) $y = -2$ (C1.1)	C1. One example for each	C1.0 Some incorrect equations for horizontal lines C1.1 Correct equations for horizontal lines
a) $y = 1$ $y = 2$ $y = 3$ b) $y = -1$ (only one possibility) c) $y = -2$ $y = -3$ $y = -10$ (C2.1)	C2. Multiple examples for each	C2.0 Some incorrect equations for horizontal lines C2.1 Correct equations for horizontal lines
a) y = B where B > –1 b) $y = -1$ (only one line possible) c) y = B; where B < –1 (C3.2) "For any positive parabola, y = B; B > k from t.p. formula to cross twice; $y = k$ to cross once; y = B; B <k to not cross at all" (C3.3)	C3. Generalisation	C3.1 Reference to horizontal lines C3.2 Partial—to given example only C3.3 Partial—to all positive parabolas C3.4 Full—to all positive and negative parabolas

Examples of student responses to the quadratics task with suggested rubric feedback

In this section, seven samples of Year 10 student responses are discussed to demonstrate the use of the rubric for providing formative feedback. It is suggested that students receive a full copy of the rubric with their current level of response highlighted so that they are additionally provided with information about what a higher level of response would involve.

Figure 2 shows a minimal response to each of the three tasks, with only one example provided for each type of parabola in Part A, partial generalisation using features of positive parabolas, and one example horizontal line for each type in Part C. A student who receives feedback about each of these responses would learn task-specific information by being able to see on the rubric the additional levels above their highlighted level.


1a) $y = x^2 - 1$
b) $y = x^2$
c) $y = x^2 + 1$

) Each equations contains 'x^2' which shows the curve
When touched twice, the turning point is underneath the x-axis
When touched once, the turning point is on the x-axis
When not touched at all, the turning point is above the x-axis

Once ⇒ $y = -1$
Twice ⇒ $y = 1$
None ⇒ $y = -2$

Rubric score:

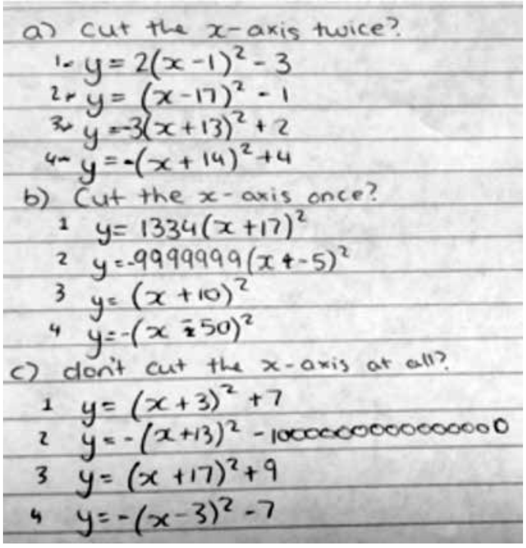
A1.1

B2.1

C1.1

Figure 2. Sample student response—minimal details for Parts A to C.

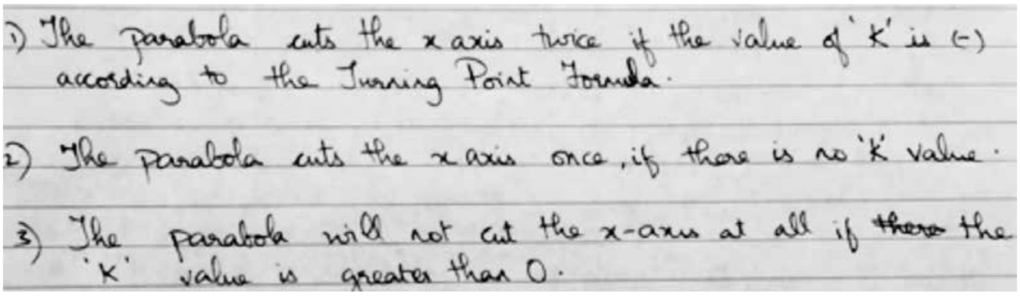
Figure 3 shows a more comprehensive response to Part A with multiple examples of parabolas being given for each type. The examples are all in turning point form and there are both positive and negative parabolas. Such a response is more likely to support students' generalisations in Part B since multiple examples highlight the different types of parabolas that meet the conditions of crossing twice, once or not at all. Having experience of open-ended challenging tasks which lead to generalisation will help students in future to follow the same process, which is an important application of algebraic thinking across all domains.


a) Cut the x-axis twice?
1- $y=2(x-1)^2-3$
2- $y=(x-17)^2-1$
3- $y=-3(x+13)^2+2$
4- $y=-(x+14)^2+4$
b) Cut the x-axis once?
1 $y=1334(x+17)^2$
2 $y=-9999999(x+-5)^2$
3 $y=(x+10)^2$
4 $y=-(x\mp 50)^2$
c) don't cut the x-axis at all?
1 $y=(x+3)^2+7$
2 $y=-(x+13)^2-10000000000000000$
3 $y=(x+17)^2+9$
4 $y=-(x-3)^2-7$

Rubric score: A2.5

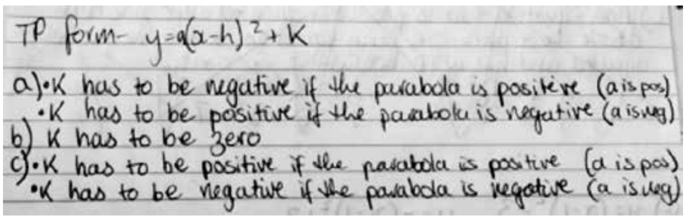
Figure 3. Sample student response to Part A—multiple examples,turning point form, systematic, positive and negative parabolas.

Part B of the challenging task aims to elicit students' use of Part A to make generalisations about all of the different types of parabolas but without directing them to do so in a particular way or with a particular formula. Figure 4 shows a response that generalises for positive parabolas using the turning point general equation. Interestingly, although there is mention of k needing to be greater than zero for parabolas that do not cross the x-axis, the student refers to there not being a k value for crossing once, rather than to k equalling zero. This misconception was noticeable across a number of responses and provides useful insight into how students view the turning point formula. It might be useful to put the equation $y = x^2$ into turning point form and discuss the values of each parameter to help students with this issue: $y = 1(x - 0)^2 + 0$.


1) The parabola cuts the x axis twice if the value of 'K' is (-) according to the Turning Point Formula.
2) The parabola cuts the x axis once, if there is no 'K' value.
3) The parabola will not cut the x-axis at all if ~~there~~ the 'K' value is greater than 0.

Rubric score: B3.1

Figure 4. Sample student response to Part B—generalisation for 3 types of positive parabolas (but with misconception about *k*).

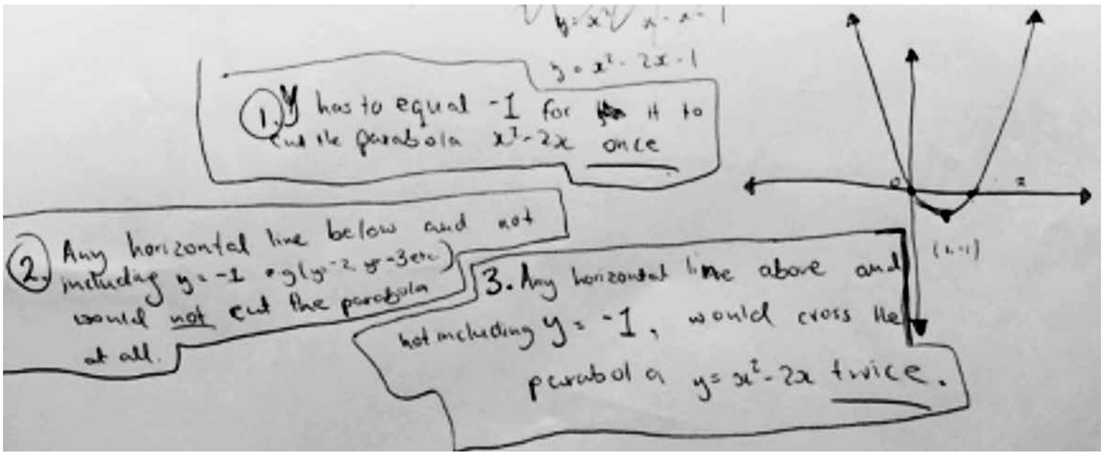

TP form- $y=a(x-h)^2+K$
a)•K has to be negative if the parabola is positive (a is pos)
•K has to be positive if the parabola is negative (a is neg)
b) K has to be zero
c)•K has to be positive if the parabola is positive (a is pos)
•K has to be negative if the parabola is negative (a is neg)

Rubric score: B5.2

Figure 5. Sample student response to Part B—full generalisation with referenceto turning point equation.

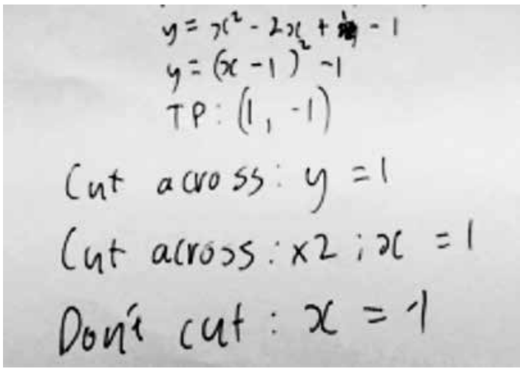
Figure 5 shows a sophisticated response in which the relationship between a and k has been described in order to generalise for any parabola (quadratic functions, not sideways parabolas!) to meet the given conditions. A step beyond this example would see students looking at connections between the turning point form of an equation, the general equation, the use of $b^2 - 4ac$ and also the factorised form of an equation. Making these connections between different expressions of parabolas supports conceptual learning rather than isolated procedural knowledge about different formulae.

Part C of the challenging task aimed to elicit further generalisation about horizontal lines crossing a parabola. Again, there is not a prescriptive direction to approach the question in a particular way, but the intent is to lead to using a particular example to generalise about it, with the possibility of generalising for any positive and negative parabolas. A number of responses included a graph of the given parabola and also the use of the turning point form of its equation. Figure 6 shows the finding of the turning point but then some issues with expressing the equations of horizontal lines correctly (in the form $y = B$). These responses would signal to the teacher the need for further conceptual development about sets of lines in subsequent teaching.



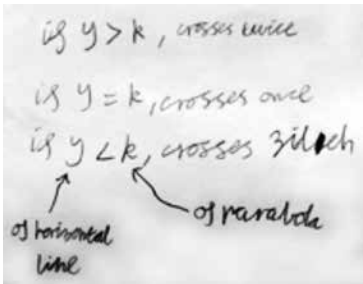
Rubric score: C3.2

Figure 7. Sample student response to Part C— partial generalisation for given example.



Rubric score: C1.0

Figure 6. Sample student response to Part C —one example with some incorrect equations for horizontal lines.



Rubric score: C2.0

Figure 8. Sample student response to Part C—attempt at generalisation for positive parabolas (but with misconception about expressing a family of lines rather than a region).

Figure 7 shows the use of words to describe generalisations for the particular parabola given as a starting point in Part C. This response demonstrates a conceptual understanding of the different horizontal lines that meet the given conditions. A step beyond this would involve expressing these ideas mathematically to capture all possible lines, before then using these to generalise for any positive or negative parabola.

Figure 8 shows an attempt to generalise for all positive parabolas by connecting the k value in the turning point equation to the value of y for a horizontal line. Unfortunately, this has led to an inaccurate expression mathematically (a region rather than a set of horizontal lines) which means that the response can't be scored as $C3$ on the suggested rubric.

Quite a few similar responses occurred with this particular cohort; using this task for formative rather than summative assessment flags to the teacher the need for further discussions about how generalisations are expressed mathematically.

Using an example of students' own attempts to do this in Part C may give them additional conceptual insight into the general equations of parabolas they have learnt and also the different purposes for pronumerals, such as representing variables like x and y or representing parameters like a, h and k, in the general turning point equation. Kieran (2007) argued that teachers help students make connections between symbolic and graphical representations of functions through the use of visualisation and transformations. This also relates to the previously presented framework (Figure 1) and the importance of students learning to select and move fluently between cells horizontally and vertically to develop proficiency in understanding functions conceptually—an important prerequisite for success in higher level mathematics such as Calculus.

As emphasised in the research literature on formative assessment, it is important for teachers to use the information they ascertain from their assessment of their students' current level of understanding to modify the direction or approach of their subsequent teaching. The misconceptions demonstrated in a few of the examples (which were noticeable in this cohort's responses) could then be effectively addressed by the teacher and lead to improved conceptual understanding.

As suggested before, rather than assess each response for themselves, teachers might alternatively show students how to self-assess their own level of response for each part; this might be a viable option for giving timely feedback, as individual analysis by the teacher can be time-consuming. Students can each be given a copy of the rubric and be shown how to use it to work out their level for each part (Wylie & Lyon, 2015). Discussing worked examples of responses at different levels may also help students understand how to improve the quality of their own work (Stiggins, 2007; Wylie & Lyon, 2015). Teachers may want to ask students to reflect specifically on what they think they now need to do to improve their knowledge, or to discuss their work with each other and encourage collaborative peer-assessment (Harlen, 2007). To be effective, peer–assessment does need to occur in the context of a classroom environment in which there are explicit structures, guidance and routines for providing appropriate types and formats of feedback to a peer (Wylie & Lyon, 2015). With experience and guidance, students can become proficient at and motivated by collaboration with each other. Follow-up tasks related to similar concepts from the challenging task are an effective way to help students use the self-assessment information to improve their conceptual understanding (Brookhart, 2007). Some suggestions for follow-up tasks on quadratics are given in the next section.

Other ideas for challenging quadratics tasks

The research literature emphasises the use of follow-up tasks that enable students to act on the information they have received from formative assessment and improve their learning (Brookhart, 2007). Some other examples of tasks that teachers might like to trial with their students are provided in Appendix 2. As with the actual task trialled with the Year 10 students, the tasks are deliberately open-ended and elicit examples of equations that are intended to lead the student to making generalisations about parabolas and/or their intersections with straight lines. There are no prescribed solution methods and differing levels of sophistication are possible in the strategies that could be used. Teachers could construct an initial rubric based on their own attempt at each task and their students' year level, and then refine it after examining the range of their students' responses.

Concluding comments

There is still much to understand about students' mathematics learning and the roles that types of learning tasks and activities, motivation, and interactions with teachers, play in promoting effective learning and achievement. Challenging tasks are a promising avenue for considering how to move students past instrumental learning—rules without reasons—to relational learning that develops students' conceptual knowledge, needed for tackling successfully the more difficult mathematics problems at and beyond school. Formative assessment has been found to promote motivation and increased learning when it provides students with feedback that helps them improve in reaching task-related objectives, and also guides teachers' approaches during the learning process. Finding sustainable and practical ways for teachers to utilise both challenging tasks and formative assessment with secondary mathematics students is a worthwhile and ongoing endeavour. This article has suggested some possible ideas for teachers to explore with their students. Feedback to the author on their experiences would be gladly received.

Acknowledgements

The author would like to acknowledge with appreciation the teacher and student participants from the Encouraging Persistence, Maintaining Challenge project, funded by an Australian Research Council Discovery Project Grant (DP110101027), who contributed to the study on which this article is based.

References

Anthony, G. (1996). *Classroom instructional factors affecting mathematics students' strategic learning behaviours.* Paper presented at the 19th Annual Conference of the Mathematics education research group of Australasia (MERGA), University of Melbourne.

Australian curriculum assessment and reporting authority (ACARA). (2009, January, 2011). *The Australian curriculum: Mathematics.* Retrieved January 3, 2014, from http://www.australiancurriculum.edu.au/Mathematics/Curriculum/F–10

Black, P., & Wiliam, D. (1998a). Assessment and classroom learning. *Assessment In Education: Principles, Policy & Practice, 5*(1), 7–74. doi: 10.1080/0969595980050102

Black, P., & Wiliam, D. (1998b). Inside the black box: Raising standards through classroom assessment. *Phi Delta Kappan, 80*(2), 139–148.

Brookhart, S. M. (2007). Expanding views about formative classroom assessment: A review of the literature. In J. H. McMillan (Ed.), *Formative classroom assessment: Theory into practice* (pp. 43–62). New York: Teachers College Press.

Dweck, C. S. (2007). Boosting achievement with messages that motivate. *Education Canada, 47*(2), 6–10. Dweck, C. S. (2010). Mind-sets and equitable education. *Principal Leadership, 10(*5), 26–29.

Fontana, D. ,& Fernandes, M. (1994). Improvements in mathematics performance as a consequence of self-assessment in Portuguese primary school pupils. *British Journal Of Educational Psychology, 64(*3), 407–417.

Goldin, G., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. In A. A. Cuoco & F. Curcio (Eds.), *The roles of representation in school mathematics* (pp. 1–23). Reston, VA: National Council of Teachers of Mathematics.

Harlen, W. (2007). Formative classroom assessment in science and mathematics. In J. H. McMillan (Ed.), *Formative classroom assessment: Theory into practice* (pp. 116–135). New York: Teachers College Press.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 371–404). Charlotte, NC: National Council of Teachers of Mathematics, Information Age Publishing.

Karpinski, A. C., & D'Agostino, J. V. (2013). The role of formative assessment in student achievement. In J. Hattie & E. M. Anderman (Eds.), *International guide to student achievement* (pp. 202–204). New York, NY: Routledge.

Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 2, pp. 707–762). Charlotte, NC: National Council of Teachers of Mathematics, Information Age Publishing

Kluger, A. N., & DeNisi, A. (1996). The effects of feedback interventions on performance: a historical review, a meta-analysis, and a preliminary feedback intervention theory. *Psychological Bulletin, 119*(2), 254–284.

Lipnevich, A. A., & Smith, J. K. (2009). Effects of differential feedback on students' examination performance. *Journal of Experimental Psychology, 15*(4), 319–333.

Moschkovich, J., Schoenfeld, A. H., & Arcavi, A. (1993). Aspects of understanding: On multiple perspectives and representations of linear functions and connections among them. In T. A. Romberg, T. P. Carpenter & E. Fennema (Eds.), *Integrating research on the graphical representation of functions* (pp. 69–100). Hillsdale, NJ: Lawrence Erlbaum Associates.

Romberg, T. A., Carpenter, T. P., & Fennema, E. (1993). Toward a common research perspective. In T. A. Romberg, T. P. Carpenter & E. Fennema (Eds.), *Integrating research on the graphical representation of functions* (pp. 1–9). Hillsdale, NJ: Lawrence Erlbaum Associates.

Schunk, D. H. (1996). Goal and self-evaluative influences during children's cognitive skill learning. *American Educational Research Journal, 33*(2), 359–382. doi: 10.3102/00028312033002359

Skemp, R. (1976, 2002). Instrumental and relational understanding. In D. Tall & M. Thomas (Eds.), *Intelligence, learning and understanding in mathematics: A tribute to Richard Skemp.* Flaxton, QLD: Post Pressed.

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation, 2*(1), 50–80. doi: 10.1080/1380361960020103

Stiggins, R. J. (2007). Conquering the formative assessment frontier. In J. H. McMillan (Ed.), *Formative classroom assessment: Theory into practice* (pp. 8–28). New York: Teachers College Press.

Sullivan, P., Clarke, D., Clarke, B., & O'Shea, H. (2009). Exploring the relationship between teacher actions and student learning. In M. Tzekaki, M. Kaldrimidou & H. Sakonidis (Eds.), *Proceedings of the 33rd conference of the International Group For The Psychology Of Mathematics Education* (Vol. 5, pp. 185–193). Thessaloniki.

Sullivan, P., Askew, M., Cheeseman, J., Clarke, D., Mornane, A., Roche, A., & Walker, N. (2014). Supporting teachers in structuring mathematics lessons involving challenging tasks. *Journal of Mathematics Teacher Education,* Online first 1–18. doi: 10.1007/s10857-014-9279-2

Timperley, H. (2013). Feedback. In J. Hattie & E. M. Anderman (Eds.), *International guide to student achievement* (pp. 402–404). New York, NY: Routledge.

Wiliam, D., Lee, C., Harrison, C., & Black, P. (2004). Teachers developing assessment for learning: Impact on student achievement. *Assessment in Education: Principles, Policy & Practice, 11*(1), 49–65. doi: 10.1080/0969594042000208994

Wilkie, K. J. (2015). Rise or resist: Exploring senior secondary students' reactions to challenging mathematics tasks incorporating multiple strategies. *Eurasia Journal Of Mathematics, Science & Technology Education,* in press.

Wylie, E. C., & Lyon, C. J. (2015). The fidelity of formative assessment implementation: issues of breadth and quality. *Assessment in Education: Principles, Policy & Practice, 22*(1), 140–160. doi: 10.1080/0969594X.2014.990416, in press.

Appendix 1: Quadratics task from the study

A. Can you find some equations of parabolas that:

- Cut across the x-axis twice?
- Cut across the x-axis once?
- Don't cross the x-axis at all?

B. What do you notice about each of the different groups of parabolas you have found?

C. Can you use your previous answers to find the equations of horizontal lines that cut across the parabola $y = x^2 - 2x$ once, twice, or not at all?

Appendix 2: Sample follow-up challenging tasks

Example 1

A. For the parabola $y = x^2$ can you find some equations of straight lines that:

- Cut across the parabola twice?
- Cut across the parabola once?
- Don't cross the parabola at all?

B. What do you notice about each of the different groups of lines you have found?

C. If you were given the equation of a straight line, e.g., $y = x$ can you find a way to tell how many times it crosses the parabola $y = x^2$ (or not at all) and at which point/s?

D. Can you use your previous answers to explore which straight lines cut across the parabola $y = x^2 - 2x$ twice, once or not at all?

Example 2

A. Two friends stand 20 metres apart and kick a soccer ball to each other. The ball leaves the foot of one friend, travels in the air and touches the ground right at the feet of the other friend. Can you find some realistic equations for describing the path of the ball? Show all working and explain your reasoning to justify your answers.

B. What can you say in general about the family of parabolas that you have found to describe the path of the ball? What are their similarities and differences?