Mathematics Self-Realted Beliefs and Online Learning

Cherie Ichinose and Martin Bonsangue
California State University, Fullerton

Abstract

This study examined students' mathematical self-related beliefs in an online mathematics course. Mathematical self-related beliefs of a sample of high school students learning mathematics online were compared with student response data from the 2012 Programme for International Student Assessment (PISA). The treatment group reported higher levels of both mathematical anxiety and mathematical self-efficacy than did the PISA group, suggesting that online learning can be either an effective or ineffective mode for learning mathematics depending on the experience and expectations of the individual student. This information may help inform online teaching and learning in mathematics at both the secondary and post-secondary levels.

Introduction

The emergence and rapid expansion of online learning is now a reality. The International Association for K-12 Online Learning (iNACOL) reported that more than three million K-12 students were engaged in some kind of online learning for the 2012-2013 school year (Watson, Murin & Pape, 2014), the most recent year in which such data was reported (Rose, 2014). Moreover, the document states that:

> Beginning in the 2013-14 school year, the Civil rights Data Collection will require all public school districts to report whether they have any students enrolled in distance learning programs. In addition, school districts may voluntarily report for 2013-14 the number of students

> enrolled in distance learning programs by race, ethnicity, sex, ELL, and disability; but in the 2015-16 school year, all school districts will be required to report such information to the U.S. Department of Education's Office for Civil rights in the CrDC. (p. 14)

Both the growing number of students participating in online learning, as well as the data-reporting protocol now required by the federal government, indicate that increasingly, students and their parents/caregivers are choosing to do part or all of the students' education in an online setting.

The presence of online learning is impacting post-secondary education as well as K-12 schools. This may be especially true for local two-year and four-year colleges that have traditionally been closely associated with the high schools in their geographic area. In his book, *Management Challenges for the 21st Centur*y, Peter Drucker stated that, with technology, one must redefine what education means: "Long-distance [online] learning may well make obsolete within twenty-five years the freestanding undergraduate college (1999, p.101)." Drucker's now fifteen-year-old comment has a prophetic ring to it: students can now complete all degree requirements without ever setting foot on a campus. Clayton Christensen predicts that by 2019, more than 50% of the instruction will be delivered online (Christensen, Horn, & Johnson, 2011). As a result, post-secondary institutions must meet the demands of the up and coming 21st century learner.

There is a considerable amount of research connecting student self-related mathematical beliefs in traditional face-to-face learning environments. Self-related mathematical beliefs are students' beliefs in their own mathematics abilities. The three constructs, self-efficacy (the extent to which students believe in their own ability to solve mathematics problems), self-concept (students' beliefs in their own mathematics abilities) and anxiety (feelings of helplessness and stress when dealing with mathematics), have been identified as being the key predictors of mathematics achievement and behavior (Bandura, 2002; Barkatsas, Kasimatis, & Gialamas, 2009; Hoffman, 2010; OECD, 2013; Pajares & Schunk, 2001; Reed, Drijvers, & Kirschner, 2010; Wadswarth et al., 2007). The three student beliefs and attitude constructs are interrelated. A higher level of mathematics self-efficacy has been shown to be related to mathematical performance, persistence, reduced math anxiety, and greater future interest in mathematics (Haskett, 1985; Hodges, 2008; Hoffman, 2010; Taylor & Mohr, 2001).

Most of this research is based on traditional classroom settings; there is limited research on student mathematical motivation and beliefs in an online learning environment. The study presented in this article was conducted with the purpose of extending the body of knowledge of students learning mathematics online. While the data used in this study was based upon a sample of high school aged students, there is evidence that the findings may readily apply to the college level as well, particularly in developmental mathematics courses.

Self-Related Mathematical Beliefs

With online instruction, the goal of understanding how student beliefs motivate mathematical learning is no different than with traditional face-to-face instruction; however, the modality may dictate motivation. Wadsworth et al (2007) state that the online learning environment "May support students' motivation for learning; it also may create a situation in which motivation is ever more important to students' success than in a traditional classroom face-to-face setting." (p. 7) Due to the autonomous nature of online learning, there is an important link between success in an online learning environment and students' beliefs in and motivation to achieve in online learning environment (OECD, 2013; Spence & Usher, 2007).

According to Bandura's social cognitive theory, self-efficacy refers to beliefs in one's capabilities to organize and execute a task, especially when facing difficult circumstances (1997). Mathematics self-efficacy is described as one's confidence in the ability to perform mathematical tasks well and the ability to overcome difficulties (Haskett, 1985; OECD, 2013). The level of mathematics self-efficacy is dependent on a student-perceived ability to successfully complete a given, and often difficult, task. For example, students with lower mathematics self-efficacy may find solving a system of linear equations out of their grasp. They cannot visualize the solution and, believing that the task is too hard, create road-blocks to prevent them from solving the problem. Conversely, students with higher levels of mathematics self-efficacy believe that they can solve, visualize and successfully execute the solution. Bandura celebrates the usefulness of online learning if students possess self-efficacy for regulating their own learning, and proposes that this will result in positive [mathematics] self-efficacy (Hodges, 2008). However, two studies revealed mixed findings. Spence and Usher (2007) reported that online students had significantly lower self-efficacy than their face-to-face counterparts, resulting in lower mathematics performance. Another study, focusing on a developmental mathematics course, showed no significant difference in mathematics self-efficacy between the online and face-to-face course (Wadsworth et al., 2007).

In addition to mathematics self-efficacy, student beliefs can be measured by the students' reported attitudes of their own abilities. Self-concept, according to Bandura, is essentially a view of oneself based on direct experience (Bandura, 1997). Mathematics self-concept is best described simply as students' beliefs in their own mathematical ability. Mathematics self-concept has been found to drive one's motivation to study mathematics. That is, students believe that they are "good at mathematics," this has a significant influence on what and how they study. Students with higher levels of mathematics self-concept tend to see themselves as smart and competent in mathematics, and usually believe mathematics has important implications for future study (Artino, 2010; Bong & Skaalvik, 2003). Ireson and Hallam (2009) reported that students with low mathematics self-concept did not want to pursue future mathematical studies, whereas students with more positive self-concept specified that they would like to learn more mathematics in the future. Studies have shown that mathematics self-concept is also linked to gender. In a number of studies, males showed to have a higher perception of their mathematics abilities than females in a traditional face-to-face class setting (Ireson & Hallam, 2009; OECD, 2013) and online setting (Barkatsas, Kasimatis, & Gialamas, 2009; Vale & Leder, 2004). Measurable differences between males' and females' beliefs suggest that males may be more motivated to learn because they believe that mathematics will help them in their later careers.

Mathematics can evoke a variety of emotions, both positive and negative. For some, this can be a negative emotion that has a direct impact on decision-making and learning outcomes. Students with high levels of anxiety may have adverse physiological reactions to mathematics, going to extreme lengths to avoid mathematics altogether (Reed et al., 2010). Thus, math-anxious students may deeply feel that for them, learning mathematics is not possible.

Instruction in a traditional face-to-face classroom can cause fear, anxiety, and avoidance of mathematics by some students (Tobias, 1981). Therefore, students with higher levels of mathematical anxiety may choose to learn mathematics online. At the 19th International Conference on Technology in Collegiate Mathematics, Spence (2007) reported that students chose an online course in order to avoid the physical face-to-face mathematical interactions that they find less favorable and uncomfortable. Although the sample size of Spence's study was relatively small, the implications of the results are worth noting and again suggest that future research is warranted.

Methodology

The purpose of this study was to examine students' self-related mathematics beliefs taking an online mathematics course. Comparisons were made between samples of high school students participating in traditional face-to-face and online learning environments. Two research questions were derived from a previous study (Ichinose, 2011):

RQ1. Are there differences in mathematics self-related beliefs between students in an online setting compared with students in a face-to-face setting?

RQ2. Are there differences between males' and females' mathematics self-related beliefs in either the online group or for the face-to-face group?

Sample

Participants in the study came from the larger group of 2,051 high school students taking an online mathematics course offered by California Virtual Academies (CAVA), a virtual high school in California. Each of the students was sent an online survey to assess their experience in an online mathematics course, with 458 students responding (22.3%). This response rate was consistent with the published research of online surveys ranging from 15-29% (Comley, 2000). Of the 458 respondents, 156 (34.1%) self-reported as male, 263 (57.4%) as female, with 39 (8.5%) not answering. This sample was representative of the overall CAVA population.

To examine students' self-related mathematical beliefs, data was also obtained from the OECD 2012 database to compare the online sample with students who completed the 2012 PISA Student Questionnaire. The Programme for International Student Assessment data included 271,323 students in grades 7 through 12. Of the PISA respondents, 49.9% were male and 50.1% were female. As with the online sample, most (88.1%) of the PISA students were in 9th or 10th grade at the time of the study.

Survey Instrument

The current study included, with permission, six 4-point Likert scale mathematical beliefs related questions, from the 2012 Programme for International Student Assessment (PISA) student questionnaire, cited with high degree of validity and reliability (OECD, 2013). The 2012 theme of the student questionnaire focused on students' self-related mathematical beliefs.

Each construct (self-efficacy, self-concept and math-anxiety) consisted of three questions such that each is scored on a 4-point Likert scale. For instance, a participant answering questions pertaining to self-concept and math-anxiety will respond with the range of: Strongly Disagree (1), Disagree (2), Agree (3) or Strongly Agree (4). Questions related to self-efficacy were answered with the range of: Not Confident at all (1), Not Very Confident (2), Confident (3), and Very Confident (4).

A participant is evaluated upon his/her response to each of the questions within a given construct. For example, a participant's score associated with self-efficacy was the sum of his or her responses. Thus, a student with the scores of 4, 3, and 4 was assigned a score of 11. Since 8 was the median score of this construct, this participant's score (11) was classified as "High Self-Efficacy."

Data Analysis

To explore the mathematical beliefs of online and face-to-face learners this study used statistical analyses that included frequencies, chi-square (χ^2) test of independence, and odds ratios. Analysis of data involved the use of 2 x 2 contingency tables. One dimension reflected the two learning environments, PISA (face-to-face) and online learning, while the second dimension reflected the level of self-related mathematical beliefs: self-efficacy, self-concept, and anxiety. The χ^2 table was entered with one degree of freedom. The 0.05 level of significance was used in judging the association between the two learning environments and self- related mathematical beliefs. Odds ratios were used to determine the intensity of the occurrence of each group (online or PISA) to report higher levels of self-related mathematical beliefs.

Results

Data from both the online and PISA groups were analyzed. The results comparing the groups (online vs PISA) are reported in the sequence based on the three constructs: mathematics self-efficacy, mathematics self-concept, and mathematics anxiety. Attention was made to examine the subgroup gender within each group (online vs PISA).

Research Question 1

Recall: self-related mathematical beliefs are students' beliefs in their own mathematics abilities. Research Question 1 states, "Are there differences in mathematics self-related beliefs between students in an online setting compared with students in a face-to-face setting?" The results that follow here compares the mathematics self-efficacy, mathematics self-concept, and mathematics anxiety between the online and PISA groups.

Students were asked to what extent they believe in their own ability to handle difficult learning situations in mathematics. There were mixed findings with each question with regard to the level of confidence in solving difficult mathematics situations when comparing the online and PISA groups. The majority (92.4%) of the online group reported they were confident or extremely confident in their ability to solve a multi-step linear equation, compared with

89% of the PISA students. The online group (69.9%) reported being confident or extremely confident in their ability to solve an area problem, compared to 68.6% of the PISA students. When asked how confident students felt in calculating a 30% discount, the PISA group reported a slightly higher level of confidence (79.6%) compared to 75.9% for the online group.

Results taken from the contingency table (Table 1) indicate an association between mathematical self-efficacy and the learning modality, χ^2 (1, N=50632) = 4.828, p = .03. Examination of cell frequencies showed that students with higher levels of mathematics self-efficacy are more likely to be an online learner (76.0%) than the PISA group (71.1%) (odds ratio = .777, 95% CI [.621, .974]). That is, students in an online setting had statistically higher levels of mathematics self-efficacy than did their PISA counterparts.

Table 1

Mathematics Self-Efficacy Between Online and PISA (Including Expected Values)

	Lower Levels	Higher Levels
Online	100 (123.4) 24.0%	316 (292.6) 76.0%
PISA	50532 (50511.7) 28.9%	124146 (124166.3) 71.1%

Students were asked about their beliefs in their own mathematical competence. More than three-fifths (63.9%) of the online group reported they agreed or strongly agreed that "I get good grades in mathematics," compared to the PISA (57. 5%) group. The PISA group answered with higher levels of agreement (39.1%) that mathematics was their best subject as compared to the online group (34.6%).

Examination of cell frequencies (Table 2) indicates that students with higher levels of mathematics self-concept were online learners (50.6%), compared to that of the PISA group (47%). However, chi-square and odds ratio analyses revealed no association between the learning modality and mathematics self-concept, χ^2 (1, N=173072) = .569, p =.450 (odds ratio = .926, 95% CI [0.759, 1.130]).

Table 2

Mathematics Self-Concept Between Online and PISA (Including Expected Values)

	Lower Levels	Higher Levels
Online	151 (158.5) 35.9%	270 (262.5) 64.1%
PISA	65005 (64997.5) 37.7%	107646 (107653.5) 62.3%

Students were asked to what extent they feel helpless, tense, nervous, or under emotional stress when dealing with mathematics. Overall, students from the online group reported higher levels of anxiety. Specifically, more online students reported feeling tense when learning mathematics (46.8%) than the PISA (34.8%) sample. Further, 44.6% of the online group compared to the 33.9% of the PISA group felt nervous while doing mathematics.

Results taken from the contingency table (Table 3) indicates students with higher levels of mathematical anxiety were more likely to be an online learner (55.2%) than the PISA group (46.0%), χ^2 (1, N=173187) = 10.169, p < 0. 001 (odds ratio = .731, 95% CI [.602 , 887]). That is, students in an online setting had statistically significant higher levels of mathematical anxiety than their PISA counterparts.

Table 3

Mathematics Anxiety Between Online and PISA (Including Expected Values)

	Lower Levels	Higher Levels
Online	186 (218.4) 44.8%	229 (196.9) 55.2%
PISA	90955(90922.6) 52.6%	81814 (81849.4) 47.4%

Research Question 2

Research Question 2 states “Are there differences between males’ and females’ mathematics self-related beliefs in either the online group or the face-to-face group?” The results that follow here compares the mathematics self-efficacy, mathematics self-concept, and mathematics anxiety between gender within each the online and PISA groups.

Table 4 shows that male students from the PISA group had higher levels of mathematics self-efficacy χ^2 (1, N=174678) = 1436.10, $p < 0.001$ and self-concept χ^2 (1, N=172651) = 2632.68, $p < 0.001$ when compared to that of female students. Conversely, female students from the PISA group reported higher levels of mathematical anxiety than did males χ^2 (1, N=172772) = 1197.741, $p < 0.001$.

Table 4

Self-Related Beliefs with Gender and PISA (Including Expected Values)

		Lower Levels	Higher Levels	χ^2
Self-Efficacy	Male	30775(35988.9) 25.4%	90567(857353.1) 74.6%	2099.32*
	Female	42902(37688.1) 33.8%	84169(89382.9) 66.2%	
Self-Concept	Male	38324(44730) 31.8%	82172(75766) 68.2%	2981.43*
	Female	53260(46854) 42.2%	72958(79364) 57.8%	
Anxiety	Male	70872(65083.4) 58.8%	49611(55399.4) 41.2%	2185.51*
	Female	62584(68372.6) 49.4%	63988(58199.4) 50.6%	

* Significant with $p < .001$

The intensity of these occurrences was confirmed by the odds ratio: self-efficacy (odds ratio = 0.669, 95% CI [.655, .683]), self-concept (odds ratio = .599, 95% CI [.587, .610]), and mathematical anxiety (odds ratio = 1.397, 95% CI [1.371, 1.424]). Thus, students in the PISA group with higher levels of mathematics self-efficacy were more likely to be male.

The same analyses (Table 5) were performed with the online group. Male students reported higher levels of self-concept χ^2 (1, N=407) = 8.32, $p < 0.001$ (odds ratio = 1.817, 95% CI [1.209, 2.730]). With mathematical anxiety, female students from the online group reported higher levels of anxiety than did males χ^2 (1, N=401) = 11.96, p < 0. 001 (odds ratio = 0.487, 95% CI [0.323, 0.734]). However, chi square analysis revealed no significant association between mathematical efficacy and gender χ^2 (1, N=403) = 2.835, p = 0.092. Further, males were no more likely to have higher levels of mathematical self-efficacy than females as confirmed by the odd ratio analysis, (odds ratio = 1.522, 95% CI [0.932, 2.487]).

Table 5

Self-Related Beliefs with Gender and Online Sample (Including Expected Values)

		Lower Levels	Higher Levels	χ^2
Self-Efficacy	Male	29(36) 19.6%	119(112) 80.4%	2.835
	Female	69(62) 27.1%	186(193) 72.9%	
Self-Concept	Male	62(76.1) 41.1%	89(74.9) 58.9%	8.322*
	Female	143(128.9) 55.9%	113(127.1) 44.1%	
Anxiety	Male	84(67.3) 56%	66(82.7) 44%	11.962**
	Female	96(112.7) 38.2%	155(138.3) 61.8%	

* Significant with p <.005
** Significant with p <.001

Discussion

This study revealed a relationship between learning modality (online or face-to-face) and student self-related mathematical belief of mathematics self-efficacy, mathematics self-concept and mathematical anxiety. Students in an online course reported higher levels of mathematics self-efficacy when compared to a face-to-face group. With the online group, higher levels of mathematics self-efficacy were not associated with gender. This implies that the alternative online learning modality may create a less biased learning environment between genders.

There was no significant relationship between mathematics self-concept and learning modality. Results from this study support the current mathematics education literature that in both learning environments, mathematics self-concept remained linked to gender (Barkatsas, Kasimatis, & Gialamas, 2009; Ireson & Hallam, 2009; OECD, 2013; Vale & Leder, 2004). Finally, students in an online class had higher levels of mathematical anxiety than those from PISA, perhaps indicating that online students were avoiding the more personal face-to-face mathematical environment.

As stated earlier, the three student beliefs and attitude constructs are interrelated. Traditionally a higher level of mathematics self-efficacy has been shown reduced math anxiety, and greater future interest in mathematics (Haskett, 1985; Hodges, 2008; Hoffman, 2010; Taylor & Mohr, 2001). However this study showed students in the online group with high math anxiety to also have high mathematics self-efficacy with compared to the PISA group. Future research must examine the relationship between these constructs.

Limitations

There were several limitations in this study. The sample, while reflective of the population from which it was drawn, was limited by its response rate and perhaps by some students' willingness to participate in the study. Since the study was based in part on self-reported information, the validity of some of the findings depended on the respondent's choices in representing their behaviors and opinions accurately. Also the large sample size of the PISA group resulted in comparison data that were essentially parameters rather than sample statistics.

Conclusion

The emergence and rapid expansion of online learning at both the secondary and post-secondary levels is present in virtually every such institution in the country. The flexibility and convenience of entertainment technologies can provide students live on-demand learning resources, without the restriction of time and geographical limitations.

The present study confirmed the link between success in an online learning environment and students' beliefs in and motivation to achieve in an online environment for some, but not all, students (OECD, 2013; Spence & Usher, 2007). Through instruction and content support, online teachers and instructional designers must continue to create and implement experiences that will foster student beliefs and motivation that can accommodate students' collective as well as individual experiences. With the use of 21st century learning technologies, college instructors can create settings that promote challenging mathematics in a safe online learning environment. In light of the relative newness of research on online learning, additional studies will be necessary to help further inform future educational choices.

While the current study was based on a sample of high school students' experience compared to that of an international set of data, the findings may help inform post-secondary learning environments as well. In particular, the experience reported by students in this study, and especially female students, are consonant with that reported by women enrolled in college mathematics courses, especially for non-traditional aged students (Haskett 1985; Tobias 1981). There is power in the anonymity that the online learning environment provides, thereby relieving any pressure either way to participate in a discussion based on gender. The online environment may lend itself to an equality between males and females where, historically, male students have out-performed female students in mathematics.

Since many students begin or re-start their academic trajectories at the community college, the issues that surfaced in this study may be very useful for shaping college level mathematics courses for the 21st century.

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