Examining the Impact of Writing and Literacy Connections on Mathematics Learning

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Abstract

In this study, we examine how literacy connections with multiple step mathematics problems affected mathematics learning for 4th grade students. Three fourth grade teachers incorporated writing activities in their mathematics classroom for two weeks. The level of teacher scaffolding decreased as students progressed through the problems. The analysis of the students’ writing revealed several findings: (a) their understanding was expressed in accurate and coherent written responses, (b) their challenges became more apparent, and (c) the depth of their understanding was evident. The factors of teacher support, scaffolding, and problem difficulty are examined in the context of students’ problem solving. Implications for teacher practice and future research are shared.

Overview

The Common Core State Standards Initiative (2012) defines the CCSS as a state-led effort coordinated by the National Governors Association Center for Best Practices and the Council of Chief State School Officers (NGA/CCSSO). The CCSS signify a movement toward valuing writing across the curriculum and students’ communication in various content areas. In mathematics, the CCSS include Standards for Mathematical Practice that emphasize the National Council of Teachers of Mathematics’ (NCTM) process standards of problem solving, reasoning, proof, communication, representation, and
connections (NGA/CCSSO, 2011). The CCSS Initiative (2012) highlights the writing standards for the English language arts (ELA), which stress the ability to write logical arguments based on substantive claims, sound reasoning, and relevant evidence. This overlaps with the aims of the standards for mathematics. The initiative states that mathematically proficient students should understand and use stated assumptions, definitions, and previously established results in constructing arguments. The initiative further defines proficient students as being able to justify their conclusions, communicate them to others, and respond to the arguments of others. The NCTM (2000) called for teachers to provide students with opportunities to communicate about mathematical concepts in a clear and coherent manner. The CCSS for mathematics (CCSS-M) echo those remarks by calling for students to "construct viable arguments" and "attend to precision" in the Standards for Mathematical Practices. The CCSS and NCTM focus on writing as a tool for mathematics learning, formative assessment, and a way for students to further clarify their thinking.

Mathematics Learning and Literacy Connections

Researchers have examined the role of writing in advancing metacognitive behaviors. These behaviors are important for students to effectively acquire problem-solving skills and maintain conceptual knowledge and understanding (Bangert-Drowns, Hurley, & Wilkinson, 2004; Muth, 1997; Ediger, 2006; Garofalo & Lester, 1985, 1987; Liedtke & Sales, 2001; Ntenza, 2006; Puagalee, 2001). By using content-area literacy strategies, students enhance their ability to internalize course content and develop conceptual understanding of a particular subject (Stephens & Brown, 2000). Brown and Palincsar (1982) suggest flexibility in planning and monitoring components of writing as skills that enhance problem solving. Preplanning and monitoring are an integral part of the writing process. Students engaged in the writing process are involved in one of the most disciplined ways of creating meaning and an effective method for examining one’s thinking (Murray, 2004). Metacognitive behaviors can be exhibited by statements made about the problem or the problem-solving process (Artz & Armour-Thomas, 1992).

The research supports combining writing instruction and mathematics education to strengthen students’ understanding of the content. Studies pertaining to teaching and learning mathematics identify reflection and communication as vital parts for constructing understanding (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, & Human, 1996; MacGregor & Price, 1999; Manouchehri & Enderson, 1999; Monroe, 1996). Students engage in active learning when writing in the mathematics classroom. This literacy connection has shown to be useful in the development of mathematical concepts, vocabulary, and skills (Bangert-Drowns et al., 2004; Baxter, Woodward & Olson, 2005; Muth, 1997; Draper, 2002; Ediger, 2006; Lester & Garofalo, 1987; Kline &
Ishii, 2008; Liedtke & Sales, 2001; NCTM, 2000; Neil, 1996; Ntenza, 2006; Pugalee, 2001; Thompson & Chappell, 2007). Writing about mathematical ideas provides teachers a way to capture and examine mathematics thinking that is both inexpensive and nonintrusive (Powell, 1997). Mathematics journals seem to offer a specific place to capture ideas and engage in many different forms of writing.

**Mathematics Journals**

Some research connects personal writing with a possible change or improvement in students’ beliefs about mathematics and has linked a person’s beliefs and their achievement (Thompson & Chappell, 2007). Goldsby and Cozza (2002) posit that mathematics journals provide an opportunity to see into the mind of students as they engage in mathematical activities. Through writing, students begin to develop deeper understanding of mathematics and use mathematical vocabulary accurately (Draper, 2002; NCTM, 2000; Ntenza, 2006; Thompson & Chappell, 2007; Tuttle, 2005). Journals become a communication conduit between teacher and student. This creates space for individualized instruction to occur (Pugalee, 1997).

Research in teaching and learning mathematics proposes that reflection and communication are important components for increasing mathematical understanding (MacGregor & Price, 1999; Monroe, 1996). Reflection is defined as examining and evaluating one’s thoughts and actions. Schuster and Anderson (2005) suggest writing in mathematics should entail students showing how they came to understand a concept along with the foundation of that concept. This suggestion necessitates a certain level of reflection from the writer (Shuster & Anderson, 2005).

Kostos and Shin (2010) employ a mixed-method action research design with second graders from a large suburb of a northeast city to investigate the effect of mathematics journals on mathematical thinking and communication. Sixteen students were part of the study. The data include students’ mathematics journals, pre- and post-math assessments, and interviews. The results indicate an increased use of mathematical vocabulary. Post-tests showed statistically significant improvement over pre-test scores. McIntosh and Draper (2001) utilize action research within their own classrooms to illustrate how writing can be used in mathematics. Their research supported journal writing as valuable for both mathematical learning and assessment. They coined the term “learning log” to describe a continuous commentary that students used to reflect on what they were learning and to learn while they were reflecting. These studies indicate writing positively impacted student learning, increased accurate use of vocabulary, and assisted students with reflection.

A review of the literature indicated studies support writing in mathematics and note the positive effects for student learning; however, the need for more
current work in this area is evident from the limited published work over the last decade. This study intends to add to this body of work.

Research Questions

Literacy connections, in the form of writing in mathematics, have potential for positively affecting mathematics learning. Research needs to further examine the progression and growth of students’ writing, the types of problems that are posed, and the context of writing. We grounded our research questions in these areas:

1. How do students begin to use writing in their problem solving process?
2. How do students’ mathematical writing responses evolve as they continue to use writing in their problem solving process?
3. What differences exist when comparing students’ earlier problem solving written responses compared to later written problem solving responses?

Methods

Setting and Participants

This study took place in three fourth grade classrooms in a high-need elementary school in the southeastern United States. The school received Title I funding from the federal government due to a high percentage of students who qualify for free and/or reduced lunch. Three fourth grade teachers posed the writing prompts to their students and allowed them to work on them during the school day. The study was completed in one month’s time. The packet of writing prompts was kept in the classroom, ensuring that only students were working on their packets. Each teacher was licensed to teach elementary school (Grades K-6) in the state where the study took place. Each teacher also taught mathematics for 80 minutes a day.

Fifty-one students participated in the study, including 42% Hispanic students, 27% African American students, and 31% Caucasian students. When comparing earlier problem solving with later problems, only students that that completed those problems were analyzed. Students’ academic ability varied among the group. On the state end of grade tests the year before for these fourth grade students, 61.2% of the students scored at or above the proficiency mark, and 81.2% of the students scored at or below the proficiency mark. The students were ability grouped into a high, middle, and low classrooms based on both their performance on the third grade end of grade assessment, as well as a placement test given at the beginning of the school year.

This study was the first time that students had been asked to write about mathematics concepts in their school career. While students completed tasks and problems from the standards-based curriculum *Investigations in Number, Data, and Space* (TERC, 2008), they had no previous experience writing extensively about mathematical ideas.
Data Sources and Data Analysis

The sole data sources in this study were students’ responses to writing prompts about mathematics problems. An inductive thematic data analysis (Coffey & Atkinson, 1996) was employed. Each author independently coded an equal amount of writing prompts. Students were randomly assigned to one of the two researchers. Prior to independent coding, both researchers independently coded 3 prompts, compared their findings, and had complete agreement on each of the codes and categories.

The data analysis was kept in a Google Docs Spreadsheet. Each prompt was labeled as whether it was correct, partially correct, or not correct. Open coding was used to circle phrases, words, and computation in students’ work and these codes were then condensed into themes (Ezzy, 2002; Patton, 2005). Further notes were recorded about what strategies students used while responding to the prompts, as well as salient quotes from students.

Findings

Question One: Early Journal Entry Trends

The first problem listed below included in this study asked students to evaluate addition and subtraction problems, find and describe the error, explain the reasons for the miscalculation, and to offer additional strategies.

Problem 1. 126 + 56 = 172       130 – 19 = 129   142 + 9 = 141

Look carefully at each of these problems. There is an error in each one that is making the solution incorrect. Write about the mistake that is being made in each problem and describe how to correctly solve the problem. Have you made this miscalculation while you were learning? Explain why you think these mistakes can easily happen. What strategies have you used to help you add and subtract larger numbers?

The teachers reported using a significant amount of scaffolding and modeling to introduce problem solving and writing. It was reported that this problem was done more as a whole group and a full example was produced as a model. The data in Table 1 shows the percentage of students that answer correctly, incorrectly, and partially correct.

<table>
<thead>
<tr>
<th>Question</th>
<th>Correct</th>
<th>Incorrect</th>
<th>Partial</th>
<th>Total</th>
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</thead>
<tbody>
<tr>
<td>Problem 1</td>
<td>34</td>
<td>0</td>
<td>16</td>
<td>50</td>
</tr>
<tr>
<td>Percentage</td>
<td>68%</td>
<td>0%</td>
<td>32%</td>
<td>100%</td>
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</table>
The table indicates that the majority of students were able to identify the mistake, explain the error and reasons, and provide an additional strategy. The 32% of the students that responded partially correct identified and corrected the error with the traditional algorithm; however, they did not provide additional strategies that could be used to solve the problem. The correct responses also relied heavily on the traditional algorithm and primarily offered the number line as a strategy. The teachers, modeling, and scaffolding were evident in the similarities of student responses. The question asked students to evaluate the errors and reflect on personal experiences. The written responses of students were limited in their depth and reflection. Figure 1 shows correct and Figure 2 shows partially correct responses from students.

Figure 1. Problem 1 early writing examples.
All three correct responses explain the algorithm mistake almost identically and offer the number line strategy with some differences in the use of the number line. The partially correct responses identifies and corrects using the traditional algorithm; however, they provided little explanation on how the error occurred and offer no additional strategies.

All of the responses seemed to follow a similar pattern for organizing work and written responses. The correct problem responses included a number list and transitional phrases to explain which problem the student was evaluating. The similarities across responses may be an indicator of the teachers’ modeling for the first problem.
Question Two: Later Journal Entry Trends

The next problem, listed below, focused on area and perimeter and identifying the relationship that occurs when the length of the sides are doubled.

Problem 2. If a square has sides that are 5 inches long, explain how to find the area and perimeter. If the sides of the square double in length, what is the new measurement for area and perimeter? What is the relationship between the old area and new area when the sides were doubled? What is the relationship between the old perimeter and new perimeter when the sides were doubled?

Teachers began to scale back modeling and scaffolding for this problems as the students were more familiar with the activity and expectations. Table 2 shows the percentages of students that answered correctly, incorrectly, and partially correct.

Table 2. Problem 2 Responses

<table>
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<tr>
<th>Question</th>
<th>Correct</th>
<th>Partial</th>
<th>Incorrect</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 2</td>
<td>24</td>
<td>21</td>
<td>2</td>
<td>47</td>
</tr>
<tr>
<td>Percentage</td>
<td>51%</td>
<td>45%</td>
<td>4%</td>
<td>100%</td>
</tr>
</tbody>
</table>

The data indicated a large percentage of students were able to complete correct calculations and identify the relationship for the new area and perimeter. For this problem, however, there were a number of students that were unable to solve the task correctly.

The correct responders consistently drew pictorial representations of the square with measurements listed in the problem and one with the side lengths doubled. Their writing expressed a consistent recognition of the double and quadruple relationship. The descriptions were well written, included how area and perimeter was used, and reflected real life examples. The partially correct responders also included pictures of the squares with different lengths and solved the old and new perimeter. In some cases the picture included the measurements for area and perimeter, but the written responses were unclear and did not address how these were calculated or the relationship between the new and old measurements. In other examples the pictures, written responses, and calculations were accurate and clear; however, the change in relationship between old and new measurements were not addressed.

Figure 3 shows examples of the types of responses categorized as correct and partially correct. The two students with incorrect responses included incorrect answers for both parts of the task with little work shown to support their answers.

The correct responders provided a detailed description of what perimeter and area measure, the equation used to solve for each of these measurements,
a real life example of their use, the new perimeter and area calculations, and a paragraph describing the relationship between the old and new measurements. The written response showed a clear organization of thinking and transitioning from the procedures to the conceptual. The partial response indicated the student was familiar with the procedures required for solving perimeter and area; however, the responses appeared to be limited to just the procedure. The written response lacked clear articulation of how to solve for perimeter and area and did not address the questions on relationship. The incorrect response showed some calculations; however the process was unclear, the relationship between the new perimeter and area measurements was not explained, and there was no written explanation.
The next problem, included below, was a multi-step problem that could be solved with several different combinations of multiplication, addition, and subtraction. Similar to the first problem it also required students to examine a wrong response and offer their thinking on the mistake, and a way to correct.

Problem 3. Marley ran 5 miles a day for 5 days. On the sixth day she ran 4 miles and on the seventh day she ran 6 miles. How many total miles did she run in seven days? Marley needs to complete 30 miles a week for her training, did she complete the needed miles? If so, did she go over and by how much? If not, how many miles did she miss in her training?

Explain how you solved this problem and what operations you used to get your answers. What was challenging about solving this problem? What advice would you give about solving multi-step word problems?

Bert answered this problem in the following sentence:
Marley completed 20 miles of her training in the last seven days. She needed to complete 30 miles for the week, therefore she missed 10 miles of her training this week.

Write a response to Bert’s answer explaining what error he made in his calculation.

This problem was administered with the least amount of modeling and scaffolding from the teacher. By this point the teachers reported students were familiar with what was being asked and started working with minimal questions. This problem included the most questions for students to address. Table 3 presents the percentages of students that answer correctly, incorrectly, and partially correct.

Table 3. Problem 3 Responses

<table>
<thead>
<tr>
<th>Question</th>
<th>Correct</th>
<th>Incorrect</th>
<th>Partial</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 3</td>
<td>34</td>
<td>12</td>
<td>5</td>
<td>51</td>
</tr>
<tr>
<td>Percentage</td>
<td>67%</td>
<td>24%</td>
<td>10%</td>
<td>100%</td>
</tr>
</tbody>
</table>

The data indicated most of the students were able to correctly solve and respond to the second part of the problem. The incorrect responders were those that miscalculated and the partial responders had difficulty addressing the second part of the problem.

The students’ writing for this problem clearly described each of the steps the student did to solve. Students exhibited a higher level of specificity in what they did to solve the problem; however, there was greater variation in their specificity for what “Bert” did incorrectly. In some cases, students
identified the mathematical miscalculation that he made by interpreting the problem incorrectly, others noted that he needed to read more carefully and offered their way of solving as advice. The partially correct responders showed difficulty in evaluating Bert’s work and identifying the error. Figure 3 shows an example of a correct, incorrect, and partially correct response.

All three responders used writing to show their thinking and the places in which they were having difficulty in the problem-solving process became evident. The organization of the work also varied between the students. In some cases, the calculations were embedded in the writing and in others students decided separating their mathematical work from their writing was helpful. There were many student examples, not included in Figure 2, which showed students underlining and circling parts of the question that appeared to be important for solving. The mathematics, written responses, and dissecting of the actual problem displayed more individuality.

**Question Three – Mathematical Writing Progression**

The students’ writing with problem solving evolved from the earlier problems to those offered toward the end. The level of teacher scaffolding decreased as students became more familiar with the process and expectations. The first question included in this study indicated that teacher scaffolding encouraged students to write about how they solve the problem, organize their work, and offer the number line strategy as another way to solve. For this problem there were no incorrect responses. There were a few students that did not respond to the second part of the question and were labeled as partially correct. The traditional algorithm was used by every student and those that provided another strategy only suggested a number line. There was almost no variation in responses. The scaffolding appeared to assist students in establishing a process; however, it seemed to limit individual thought.

The response for the next problem presented variation in students’ ability to solve the problem and explain the area and perimeter relationship with a new length measurement. The problem asked students to “explain how to find area and perimeter.” Some students interpreted this question as a place to explain area and perimeter including a formula and reasons for using these calculations; whereas other students used their calculations of the actual problem as an explanation on how to solve. The responses that were limited to the drawing with a value for area and perimeter suggested these students may be more comfortable using the formula with mental math and may have found explaining and evaluating the relationship between the two problems more challenging. The variation in the amount of writing, the parts of the problem addressed or not addressed, and the ways in which the students addressed those questions provided insight into their thinking.

The last problem analyzed in this study offered the most variation in responses and included the least amount of teacher scaffolding. In many cases, the responses were longer and more detailed than previous problems.
The evaluation question presented the most challenge for students. Students highlighted the mathematical error and suggested reading slowly and carefully as a strategy, while others presented just the strategy and did not address the miscalculation, and some did not address this part. Figure 4 shows the progression of the three problems for one student.

The scaffolding appeared to have encouraged writing that was detailed and descriptive. This example showed depth and richness in their description and organization. The last problem presented some additional organizational strategies. The student used lines and arrows to separate the parts of the problem along with individual questions within the problem. This student demonstrated his/her own strategy as the teacher stepped away from leading the problems and the actual problem contained more steps.

Figure 4. Progression of writing – One student
Discussion

The research questions guided this study through the progression of using writing as a tool for mathematical problem solving. The findings have implications for student learning as well as the role of teachers in mathematics classrooms that want to encourage writing. The areas of modeling and scaffolding played a role in student responses and set students up with expectations that later revealed more individual differences.

Modeling and Scaffolding

The first research question investigated how students begin to use writing in their problem-solving process. As this study was conducted with students that were unaccustomed to writing in mathematics, the teachers played an important role in modeling and scaffolding. The first problem was also based in addition and subtraction of three-digit numbers. Addition and subtraction was, at this point, a strong skill set for most students in these classes. The familiarity with the concept and teacher modeling led to a majority of the students having correct responses, a small number of partially correct and no incorrect responses. Students’ work had similar organization and teacher modeling was evident in the written work.

Pugalee (1997) suggests that student writing becomes a communication that reveals places where additional support becomes apparent and can be used to individualize instruction. In this study, the teacher modeling and scaffolding was important for students to become acclimated to writing in mathematics. The scaffolding and modeling for problem one however, limited the differences seen in student work. Even with organization and writing appearing similar for the first problem, the partially correct responses revealed a trend in student thinking. The partially correct responses were largely due to the lack of providing additional strategies to solve addition and subtraction problems. The writing was affected by teacher modeling, but still remained a tool to assess learning and allowed the teacher to reinforce different strategies (McIntosh & Draper, 2001).

Differences in Depth of Student Responses

Our second research question followed the evolution of students’ writing as the combination of writing and problem solving became more comfortable, expectations were understood, and teacher modeling was reduced. The third question compared the students’ writing from earlier problems to those offered later in the study. The findings show an increase in variation in writing, organization, number of correct, partially correct, and incorrect. The differences in depth of thinking became evident in student responses. The correct responses (67%) provided a clear explanation of perimeter and area, gave examples of how it is used, showed calculation, and accurately
evaluated and identified the changes that occurred between the smaller and larger shape. The writing combined with problem solving promoted a deeper understanding of the material (Draper, 2002; NCTM, 2000; Ntenza, 2006; Thompson & Chappell, 2007; Tuttle, 2005). In some cases, the partial responses showed pictures and answers that appear to be calculated mentally. The descriptions were minimal and the question of relationship between the two problems went unaddressed. This provided insight into areas of needed instruction (Puglaee, 1997). The last problem continued along this pattern of greater variation in responses and depth of response. Since there were several steps to problem 3 embedded in a story problem, the ways in which students begin to dissect and organize their strategies became more evident in their writing. The teacher modeling for the first problem and increased familiarity with writing to support problem solving and thinking seemed to foster growth in the students’ ability to communicate in a coherent, clear, and detailed manner. The students that exhibited difficulty in this area appeared to be challenged by the mathematical content and the writing process highlighted those areas of struggle.

**Implications for Research**

This study shows the results of introducing writing into mathematical problem solving. The students were inexperienced with using this literacy tool in mathematics. The teachers’ increased modeling and scaffolding were highly evident in students’ first responses. This role appeared to be helpful for students to understand how to use writing in their problem solving; however, it limited individuality in the first responses. The role of the teacher in bringing writing and mathematical problem solving together seems to be an area that needs further research. The evolution of teacher support in the writing process and embedded instruction where needed may be ways to further the benefits of using writing in mathematics. Writing can support changes in beliefs about mathematical ability (Thompson & Chappell, 2007), increase understanding of material (Kostos & Shin, 2010), and be a place to reteach (Puglaee, 1997). Examining the teachers’ role in this process is an important step to making writing during mathematics an effective and reflective activity for both students and teachers.

The problem solving and writing in this study occurred at the individual level; however scaffolding, modeling, and support can also be provided by peers. Another area of research should examine the role of peer interactions to increase growth in written communication. Several studies identify the active learning that takes place as students engage in mathematical writing (Bangert-Drowns et al., 2004; Baxter et al., 2005, Muth, 1997; Draper, 2002). There appears to be a need to examine how a community of student writers could support one another in their learning and enhancing each other’s ability to communicate their thinking in writing.
References


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