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Preservice Secondary Teachers Perceptions of College-Level Mathematics Content Connections With the Common Core State Standards for Mathematics

Travis A. Olson
University of Nevada, Las Vegas
travis.olson@unlv.edu

Abstract

Preservice Secondary Mathematics Teachers (PSMTs) were surveyed to identify if they could connect early-secondary mathematics content (Grades 7-9) in the Common Core State Standards for Mathematics (CCSSM) with mathematics content studied in content courses for certification in secondary teacher preparation programs. Respondents were asked to identify college-level mathematics courses they had taken in their preparation programs that specifically addressed connections to early-secondary school mathematics content in the CCSSM. The analysis of the results specifically focuses on PSMTs' inability to self-identify connections between this CCSSM content and college-level mathematics coursework. In this study, the goal that content coursework for teachers emphasize visible connections between the content PSMTs will teach and mathematics coursework in preservice preparation programs was seemingly not achieved.

There is a significant body of work on the importance of mathematical and pedagogical content knowledge of elementary and secondary mathematics teachers (Ball, Thames, & Phelps, 2008; Hill, Rowan, & Ball, 2005). Knowledge that teachers possess related to the mathematics content they teach has an impact on the choices made when selecting appropriate tasks, and generating questions when students explore the tasks chosen (Stein, Grover, & Henningsen, 1996; Stein, Remillard, & Smith, 2007). Although it is known content knowledge impacts teachers' abilities to provide students opportunities to learn mathematics, preservice teachers' abilities to recognize connections in their mathematics coursework preparation to what they will teach remains an important area of investigation and discussion. The authors of the recent Institutes for Educational Advancement's Teacher Education and Development Study in Mathematics report [TEDS-M report] (Tatto et al., 2012) compare aspects of the preparation programs of primary and secondary teachers in 17 countries. The authors provide discussion of general connections between overall coursework taken in teacher preparation programs and general mathematical topics in K-12. However, they report no data on preservice secondary mathematics teachers' (PSMTs') abilities to recognize content connections between specific college-level mathematics coursework taken and specific grades 7-12 mathematics content.

There exist, however, reports regarding guidelines for specific college-level mathematics coursework that PSMTs should encounter in teacher preparation programs in the United States. In particular, the report by the Conference Board of Mathematical Sciences (CBMS) on the Mathematical Education of Teachers (2001), and the CBMS report on the Mathematical Education of Teachers II [MET2 report] (2012) significantly outlined the mathematical foundations that elementary and secondary mathematics teachers should be expected to attain in their preservice preparation programs.

Furthermore, the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices, Council of Chief State School Officers [CCSSM], 2010) has arguably provided a de facto delineation of basic mathematical concepts teachers are expected to understand in order to provide meaningful learning experiences and connections for their students. This recognition of the CCSSM as a referent point for the development of PSMTs' mathematical knowledge needed for teaching mathematics is an important consideration in recommendations for preservice programs outlined by CBMS (2012). However, a gap persists in what is known about PSMTs' mathematical preparation with specific regard to the content of the CCSSM.

Heck et al. (2011) suggest a set of priorities for research on the influence of the CCSSM. One area of interest identified is understanding ways in which certification programs and institutions of higher education are responding to the CCSSM. Consequently, a critical component in understanding the PSMT's ability to implement the CCSSM lies in the identification of what PSMTs report being exposed to mathematically, specifically with regard to the content outlined in the standards.

The research reported in this paper addresses the research question: Do preservice secondary mathematics teachers report connections between mathematics concepts studied in their college mathematics preparation, and the mathematical content delineated in the CCSSM early-secondary standards (Grades 7-9)?

Theoretical Perspectives

Beyond the MET and MET2 reports, Papick, Olson, and Regis (2010), Papick (2011), and Wu (2011a, 2011b) have discussed various issues on preservice secondary mathematics teachers' needs for specific coursework related to the content they will be expected to teach. Much of this research and discussion focuses on the goal for PSMTs preparation to provide a "mathematics preparation that develops a deep understanding of the mathematics [PSMTs] will teach" (Papick, Olson, & Regis, 2010a, p. 32). The mathematics PSMTs will likely teach is currently, in large part, defined in the CCSSM, at least within the 45 states, the District of Columbia, four territories, and the Department of Defense Education Activity that have adopted the standards (CCSSM, 2013).

Research regarding the importance of PSMTs' conceptual understandings further underscores the issue of PSMTs' lack of development of deep understanding of early-secondary mathematics (Olson & Olson, 2013; Papick, Olson, & Regis, 2010; Papick, 2011; Wu, 2011a, 2011b). In light of such research and the recommendations outlined by Heck et al. (2011), it is critical to identify a baseline of what, if any, mathematical connections PSMTs identify between what they have learned in college-level mathematics courses and what they will teach in early-secondary classrooms.

Importance of PSMTS' Conceptual Understandings

Papick, Olson, and Regis (2010) discuss the importance of PSMTs' understanding of mathematical structures underlying middle and early-secondary grades mathematics content related to patterning concepts. In their analysis of classic patterning problems involving three given initial terms of a sequence, the authors note that, "the only polynomial functions g that have the values $g(1) = 3$, $g(2) = 5$, and $g(3) = 7$ are induced by polynomials of all degrees other than two" (p. 31). The authors present this statement as a consequence of identifying $g(x) = (2x + 1) + (x - 1)(x - 2)(x - 3)h(x)$, where $h(x)$ is some polynomial with rational coefficients. As such, Papick, Olson, and Regis argue it is important for teachers to understand there are infinitely many polynomial functions that could satisfy simple patterning statements, such as "3, 5, 7, …. What's next?" In other words, "there are infinitely many functions g defined from the positive integers into the integers having the values, $g(1) = 3$, $g(2) = 5$, and $g(3) = 7$, and moreover there are infinitely many polynomial functions with these given rules" (p. 31). As such, these authors argue understanding mathematical structures is critical for early-secondary teachers who may want to engage their students by posing the following question: "Given three distinct points on a line, does there exist a quadratic polynomial function whose graph passes through those points? Justify your answer" (Papick, Olson, & Regis, p. 31). That is, a teacher versed in these mathematical constructs is better prepared to engage students in significant mathematical conversations and tasks.

Wu (2011a, 2011b) and Papick (2011) echo similar sentiments regarding the need to develop PSMTs' conceptual understandings related to mathematical structures. Wu (2011b) provides three examples for mathematicians to consider including in college curricula for PSMTs. He states such examples included in the curriculum that expose PSMTs to mathematical knowledge must satisfy two conditions: "It is relevant to teaching, i.e., does not stray far from the material they teach in school. It is consistent with the fundamental principles of mathematics" (p. 373). Wu (2011a) also acknowledges a potential source for the lack of alignment between PSMTs' development of mathematical knowledge development and conceptual understandings of school mathematics they will teach.

Papick (2011) expands on the importance of teachers' conceptual understandings that reflect understanding of mathematical structures, as well as developmentally appropriate ways to engage students with such mathematics. Papick notes that teachers should know the following:

> …how to represent and connect mathematical ideas so that students may comprehend them and appreciate the power, utility, and diversity of these ideas, and they should be able to understand student thinking (questions, solution strategies, misconceptions, etc.) and address it in a manner that supports student learning. (p. 389)

However, the question of how PSMTs currently view the alignment between the content in their mathematical coursework and the content they will teach (i.e., the CCSSM) persists.

In addition to positing what PSMTs should experience, recent studies on PSMTs' conceptual understandings of fraction concepts within worded problems indicate a need for deep conceptual understanding among PSMTs (Izsák, Jacobson, de Araujo, & Orrill, 2012). Specifically, Izsák, Jacobson, de Araujo, and Orrill (2012) discuss fraction tasks used in a professional development course with middle school teachers and the strategies they used in drawing models. These authors note a need for teachers to identify appropriate referent units and possess deep understandings of structures in order to use drawn models for a variety of concepts, including qualitative comparisons and multiplicative relationships. That is, the deeper understandings that middle (and early-secondary) teachers have, the more they can foster conceptually driven mathematical conversations that include drawn models. Moreover, the authors note they identified, "…fundamental challenges that need to be met if we are to better prepare teachers for standards-based curricula that use drawn models as a basis for developing general numeric methods with students" (p. 423). Such concerns over teacher preparedness are timely not only with respect to standards-based curricula, but also with regard to the implementation of the CCSSM. Given the importance of content coursework taken by PSMTs and high school student achievement (Monk, 1994), an investigation into studies and reports on PSMTs' content preparation was initiated to better understand the recommendations by Wu and Papick, as well as the discrepancies noted by Izsák, Jacobson, de Araujo, and Orrill (2012).

Reports on PSMTs' Preparation

Two recent reports primarily contributed to the frameworks utilized in the development of the survey presented in this paper, and in the interpretation of the results in the broader context of informing the mathematics education system. The first, TEDS-M report, provides a very broad picture of the current status of teacher preparation programs across 17 countries. The second, MET2 report, provides professional suggestions for the structure and content of teacher preparation and professional development programs within the United States.

TEDS-M report

As part of their report, Tatto, et al. (2012) examined academic mathematics courses that aim to provide the following:

> …mathematics knowledge to a population of university students that may or may not include future teachers, and are designed to treat content beyond the mathematics learned at the secondary school level, that is mathematics at the university level (e.g., abstract algebra, functional analysis, differential equations, etc.). (p. 106)

As part of investigating teachers' experiences with regard to the academic mathematics courses (hereafter identified as mathematics *content* classes), as well as more pedagogically oriented classes (hereafter identified as mathematics *methods* classes), the authors looked at responses to a prompt that asked respondents to identify general mathematics topics (often taught at the primary or secondary level) they have studied in their current preparation program. However, in the report it is not clear as to whether or not the identification of college coursework related specifically to topics studied in academic mathematics courses or in pedagogically oriented courses. Additionally, the general mathematics topics surveyed at the K-12 level were not in any way associated with the CCSSM. This gap was a focal point of the development of this study; that is, the lack of connections identified by PSMTs specifically between their *content* preparation coursework (prior to *methods* coursework) and the content they will teach as defined by the CCSSM.

In the development of the survey reported in this paper, it was imperative to specifically survey PSMTs on the connection between precisely the two elements, college-level mathematics coursework and CCSSM, rather than investigate more broad connections between preparation *program*s and school mathematics that is not specifically grounded in the CCSSM document. Such connections between college-level mathematics coursework and the CCSSM are posited, but not necessarily studied in the MET2 report.

MET2 report

In discussing the "mathematics of high school" the authors of the MET2 report indicate that such mathematics "does not mean simply the syllabus of high school mathematics, the list of topics in a typical high school text. Rather it is the structure of mathematical ideas from which that syllabus is derived" (p. 54). Such a distinction was pertinent in the development of the survey reported in this paper. Rather than a list of topics "often taught" (i.e., in typical text, perhaps) as was reported in the TEDS-M report, for the study reported in this paper, it is timely to investigate connections to specific CCSSM content clusters. Consequently, the focus of the survey reported in this paper became examining the connections that PSMTs' self-report between college-level mathematics coursework and the early-secondary CCSSM content.

Methods

A survey was developed and went through several iterations; feedback was obtained from several mathematics education faculty members at institutions of higher education with PSMTs' preparation programs. When completed, the survey was distributed to instructors who are known to the author to teach courses in the methods of teaching secondary mathematics, as it is assumed that all, or nearly all mathematics content coursework is taken prior to *methods* coursework. In this regard, the participants in this study were sampled by convenience. The instructors of these courses were asked to provide the survey to the students he or she felt would be classified as preservice secondary mathematics teachers.

In developing the survey, 22 content clusters were selected[1]: all 9 content clusters in Grade 7, all 10 content clusters in Grade 8, and 3 Number and Quantity content clusters from High School (the two clusters under N-RN, and the cluster under N-Q)[2]. Table 1 provides the list of content clusters surveyed.

[1] The term "content cluster" is from the CCSSM (CCSSI, p. 5), as it denotes a cluster under which there may be one or multiple standards. For the purposes of brevity of the survey, the focus was on the "cluster" rather than expanding to each individual standard.

[2] The justification for the early-secondary grade-level focus stems from work with PSMTs (Olson & Olson, 2013) and difficulties displayed regarding mathematical structures that occur, in the author's opinion, more predominantly in early-secondary grades, or within number and quantity discussion in early-secondary courses. Furthermore, it is this author's institutional experience that PSMTs often teach at the early-secondary level because of the availability of job, even though the PSMTs are often State-certified to teach Grades 7-12.

Table 1. *CCSSM Content Standards Surveyed, and Associated Domains and Abbreviations*

Content Cluster	Domain (Abbreviation)
Analyze proportional relationships and use them to solve real-world and mathematical problems.	Ratios and Proportional Relationships (7.RP)
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	The Number System (7.NS)
Use properties of operations to generate equivalent expressions.	Expressions and Equations (7.EE1)
Solve real-live and mathematical problems using numerical and algebraic expressions and equations.	Expressions and Equations (7.EE2)
Draw, construct, and describe geometrical figures and describe the relationships between them.	Geometry (7.G1)
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	Geometry (7.G2)
Use random sampling to draw inferences about a population.	Statistics and Probability (7.SP1)
Draw informal comparative inferences about two populations.	Statistics and Probability (7.SP2)
Investigate chance processes and develop, use, and evaluate probability models.	Statistics and Probability (7.SP3)
Know that there are numbers that are not rational, and approximate them by rational numbers.	The Number System (8.NS)
Work with radicals and integer exponents.	Expressions and Equations (8.EE1)
Understand the connections between proportional relationships, lines, and linear equations.	Expressions and Equations (8.EE2)
Analyze and solve linear equations and pairs of simultaneous linear equations.	Expressions and Equations (8.EE3)
Define, evaluate, and compare functions.	Functions (8.F1)
Use functions to model relationships between quantities.	Functions (8.F2)
Understand congruence and similarity using physical models, transparencies, or geometry software.	Geometry (8.G1)
Understand and apply the Pythagorean Theorem.	Geometry (8.G2)
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	Geometry (8.G3)
Investigate patterns of association in bivariate data.	Statistics and Probability (8.SP)
Extend the properties of exponents to rational exponents.	The Real Number System (NRN1)
Use properties of rational and irrational numbers.	The Real Number System (NRN2)
Reason quantitatively and use units to solve problems.	Quantities (NQ)

For each content cluster, PSMTs were asked to identify courses in which they were taught how the content of the course related to their future as a mathematics teacher. That is, did PSMTs see the instruction in a course connecting college-level mathematics to the teaching and learning of early-secondary mathematics implied by the wording of each CCSSM content cluster?

The list of college-level mathematics courses was formulated by examining coursework in PSMT preparation programs at four institutions of higher education across four different States. Coursework included in the programs of study for PSMTs' at these four institutions was compared with the suggestions for the preparation of PSMTs in the MET report. Similar course names were identified, and a list of courses was generated that reflected the required content coursework for PSMTs across these four institutions. For purposes of brevity in the survey, where possible, courses with common characteristics were combined. Table 2 contains the finalized list of courses.

Table 2. *Coursework Options Provided in the Survey for Connections to CCSSM Content Clusters*

Course Name Used for Survey
Calculus or Advanced Calculus
Discrete Math or Computer Science
Linear or Matrix Algebra
Abstract Algebra or Group Theory
Euclidean, Non-Euclidean, or Projective Geometry
Probability, Combinatorics, or Statistics
Sets and Logic
Number Theory
Theory of Equations
History of Mathematics
Numerical Analysis
Real or Complex Analysis
None of the Listed Courses
Other

For each of the content clusters identified in Table 1, PSMTs were prompted in the survey with the following typical question:

> In which of the mathematics courses were you taught how the content of the course related to your future as a mathematics teacher who would teach children about *analyzing proportional relationships and using them to solve real-world and mathematical problems*[3]? Check any that apply.

As noted as options in Table 2, PSMTs were allowed to also choose "other" or "none." However, if they chose "other," they were prompted for a written response to specify the courses they considered not to be in the provided list. Any number and combination of the courses in Table 2 could be selected for each content cluster. However, if "none" was chosen, then no other coursework could be selected. That is, a PSMT *could not* choose "none" and also "abstract algebra" for a given content cluster, but *could* choose "Calculus; Abstract Algebra; Sets and Logic; and Real or Complex Analysis" for a single given content cluster.

Lastly, four prompts were given to PSMTs to identify general characteristics of their experiences. These prompts included asking PSMTs to self-identify their gender, current year in their certification program (i.e., freshman through graduate), their type of certification program (i.e., undergraduate, graduate, alternative route), and the institution in which they are currently enrolled.

[3] This italicized statement represents, with slight modification, the general placement of the CCSSM content clusters (Table 1) within the survey format.

Findings

In total, in the Fall of 2011, 33 PSMTs at three institutions of higher education responded fully to all the prompts in the survey[4]. Of these 33 respondents, 19 were Female, 13 Male, and one not identified. The data in Table 3 indicate the year in the program of study self-identified by the PSMTs; similarly, the data in Table 4 indicate the nature of the certification program self-identified by the PSMTs.

Table 3. *Current Year of PSMT in Their Certification Program*

Year in Certification Program	Number of Respondents
Freshman	1
Sophomore	0
Junior	1
Senior	26
First Year Alternative	3
Second Year Alternative	1
Third Year Alternative	0
No Answer	1

Table 4. *Type of PSMT Certification Program*

Type of Certification Program	Number of Respondents
Undergraduate	28
Graduate leading to certification and masters degree	2
Alternative Route (non-degree leading)	2
No response	1

Notably, the vast majority of respondents reported they were in an undergraduate certification program in their senior year of studies. This self-report aligns well to the request for instructors to present the survey to students in a course on the methods of teaching secondary mathematics. Thus, the demographics of the respondents aligned with the goals of the research in that the PSMTs should have experienced most, if not all, of their mathematics coursework.

Tallies were made for the number of times a mathematics course was identified by a PSMT to be one in which connections were drawn between the teaching of the mathematics represented by a particular CCSSM content cluster and the content of said mathematics course. The data in Table 5 represent four tally counts: the total counts for each mathematics course identified per CCSSM content cluster, the total counts for each content cluster across all courses (last column in the table), the total counts of the number of times a particular course was chosen per CCSSM grade level (last row

[4] Due to the nature of the convenience sample, it is unclear how many PSMTs received the survey but did not attempt or complete it.

within each grade level), and the total counts per course across the three grade levels (last row in the table). Given that PSMTs were allowed to select more than one mathematics course per content cluster, the totals per content cluster (i.e., the last column, "Tl") could be more than 33.

Table 5. *CCSSM Coverage by College Mathematics Courses (Completed Surveys; n = 33)*

Grade	Abbreviated Course Names														
Strd	Clc	DM	LA	AA	Gm	PS	Lg	NT	Eqn	Hst	NA	RA	No	O	Tl
7															
RP	12	4	11	6	6	10	2	1	0	5	2	0	11	4	59
NS	5	5	6	6	1	5	0	2	1	1	2	0	16	6	34
EE1	11	6	9	9	3	5	3	1	0	2	1	1	12	4	51
EE2	12	9	10	5	2	14	1	0	0	2	1	0	9	5	56
G1	8	2	5	4	15	1	1	0	0	3	0	1	9	4	40
G2	13	1	2	3	10	3	1	0	0	4	0	0	11	4	37
SP1	1	0	1	1	2	16	1	0	0	0	0	0	13	2	22
SP2	1	2	1	3	0	13	1	0	0	0	1	0	13	2	22
SP3	1	4	0	1	1	17	1	0	0	1	1	0	12	3	27
Tot 7	64	33	45	38	40	84	11	4	1	18	8	2	106	34	348
8															
NS	4	3	3	6	1	0	0	1	0	0	1	3	20	1	22
EE1	11	4	6	7	2	3	2	3	1	3	1	2	15	4	45
EE2	5	3	10	2	5	2	1	1	2	2	1	0	16	2	34
EE3	5	3	16	6	0	0	1	1	1	0	0	0	12	3	33
F1	18	7	8	8	1	3	2	1	0	1	1	3	11	5	53
F2	12	6	4	7	1	1	0	1	1	0	0	1	13	4	34
G1	2	5	4	2	10	0	0	1	0	1	1	0	15	4	26
G2	8	3	3	1	8	0	0	0	1	5	0	1	13	3	30
G3	14	2	0	2	8	0	0	0	0	1	0	1	13	4	28
SP	0	3	0	1	0	8	0	0	0	0	1	0	18	2	13
Tot 8	79	39	54	42	36	17	6	9	6	13	6	11	146	32	318
9															
NRN1	7	3	1	5	0	0	0	2	0	2	0	1	18	2	21
NRN2	5	3	3	8	1	0	1	2	0	1	0	3	15	3	27
NQ	10	7	4	3	6	6	2	0	0	1	1	1	13	4	41
Tot 9	22	13	8	16	7	6	3	4	0	4	1	5	46	9	89
Totals	165	85	107	96	83	107	20	17	7	35	15	18	298	75	755

Note. The first column is organized first by grade, then by standard abbreviation (i.e., "Strd"), with a final total count for each grade level provided (i.e., "Tot 7"). See abbreviations in Table 1 for "Strd" reference.
Note. The abbreviations for the Abbreviated Course Names are as follows – CLc = Calculus; DM = Discrete Mathematics; LA = Linear Algebra; AA = Abstract Algebra; Gm = Geometry; PS = Probability or Statistics; Lg = Logic; NT = Number Theory; Eqn = Theory of Equations; Hst = History of Mathematics; NA = Numerical Analysis; RA = Real or Complex Analysis
Note. The final three columns are related to the non-course options in the survey. In particular, No = None of the Listed Courses, and O = Other. The final column, T1, represents the Total Number of Courses Identified per Standard (which excludes "No" and "O" columns).

In Table 5, the numbers under Abbreviated Course Names represent the number of times a course was selected as pertaining to PSMTs experiencing a particular content cluster within academic mathematics coursework. That is, the shaded cell indicate that 6 PSMTs identified Abstract Algebra or Group Theory as a course in which they recall engaging in studying connections between the college-level coursework and the content cluster, 7.RP: *Analyze proportional relationships and use them to solve real-world and mathematical* *problems*. Furthermore, the cell directly to the left of the shaded cell indicates that 11 PSMTs identified Linear Algebra as pertaining to the same content cluster. Within the same row, the "Tl" column indicates that in total the content cluster, 7.RP, was identified as pertaining to college-level coursework 59 times among 33 PSMTs. That is, college-level mathematical connections were identified by the PSMTs with respect to the ratio and proportion content cluster in Grade 7 over twice as many times as were connections to each of the statistics and probability content clusters (7.SP1 and 7.SP2) in Grade 7, and over four-times as many times as the statistics and probability content cluster (8.SP) in Grade 8. Given the myriad connections between statistics, probability, and proportional reasoning, this disparity between PSMTs selection of a 7th grade ratio and proportion standard and the lack of selection of the 7th and 8th grade probability and statistics standards is particularly noteworthy.

In total, PSMTs identified connections between college-level mathematics and CCSSM content clusters most for None of the Listed Courses (298), Calculus or Advanced Calculus (165), Probability and Statistics (118), Linear Algebra (100), and Discrete Mathematics or Computer Science (96). Although coursework in calculus or algebra was selected most frequently among specific courses, "none" was selected more than one-and-one-half times Calculus or Advanced Calculus. The "none" choice was also the most selected option in 14 of the 22 content clusters surveyed. That is, for 63% of the surveyed content clusters pertaining to early-secondary CCSSM mathematics instruction, 33 PSMTs could not identify a mathematics course in which they reported being exposed to connections between college-level mathematics coursework and mathematics instruction related to early-secondary CCSSM content.

Other findings were evident when shifting the lens of analysis to the *least chosen* coursework. In particular, coursework related to Logic was only identified 20 times among all the surveyed content clusters. In six of the Grade 8 content clusters, Logic was not identified as a course where connections were made between the college-level and CCSSM mathematics content. Importantly, this lack of connection related to Logic included a lack of connection to the three content clusters involving Geometry (i.e., 8.G1, 8.G2, and 8.G3); arguably a key component of the study of geometry is a heavy foundation of logic.

Coursework related to Real or Complex Analysis was identified as being less connected than Logic to the early-secondary CCSSM content clusters. In particular, the two high school standards related to the real number system (i.e., NRN1 and NRN2) were only identified by one and three PSMTs respectively, as being connected to coursework involving Real or Complex Analysis. In other words, less than 10% of the PSMTs identified any connection between the CCSSM content cluster: *Use properties of rational* *and irrational numbers*, and a Real or Complex Analysis course, a course arguably largely focused on the axiomatic study of the real number system.

Discussion and Implications

The data presented in this paper provide an examination of whether PSMTs report receiving instruction in college-level mathematics coursework connected in some way to the CCSSM content they will teach. This study serves as a critical baseline regarding content connections and the CCSSM within preservice secondary mathematics teacher preparation, particularly given the timeliness of the data to the wide-adoption of the CCSSM. The data indicate that although the PSMTs were able to identify *some* connections, in general there are gaps in their ability to connect college-level mathematics coursework they experienced with early-secondary CCSSM content. However, given that courses such as Calculus I and II often include mathematics developed at a secondary level (i.e., ratios, rates of change, fractions, and so forth), it is unclear as to whether PSMTs drew connections between that college coursework and CCSSM content due to familiarity with recently studied high school content, or because the college-level instruction specifically highlighted connections to early-secondary CCSSM mathematics.

In examining these data, questions arise as to when in their college-level mathematics coursework PSMTs should be provided experiences to connect specific theoretical and axiomatic mathematical foundations to the CCSSM content. For example, a stark question driven by the data from this survey is when in their college-level mathematics coursework should PSMTs be provided experiences in which they examine the axiomatic structures of the real numbers with regard to the study of the *properties of rational and irrational numbers* in secondary school mathematics? Although coursework exists outside of Real Analysis in which such foundational discussion can take place, a course focused on the axiomatic study of the real numbers appears to be an optimal point in PSMTs' content development that understandings of number systems could be developed. In particular, in discussing recommendations for the study of the real number system, the authors of the MET2 report note the following:

> Teachers need to know how to prove what is unstated in high school in order to avoid false simplifications and be able to answer questions from students seeking further understanding. Thus, a construction of the real numbers, a proof that they satisfy the properties of operations (the CCSS[M] term for the field axioms), and a proof that they satisfy the Completeness Axiom are necessary for teachers. (CBMS, 2012, p. 60)

That is, the finding that only three PSMTs could connect NRN2 to Real or Complex Analysis coursework, and only one PSMT connected this content cluster to Logic course work indicates that there is a critical gap between their experiences and the MET2 report recommendations.

Furthermore, the authors of the MET2 report note a recommendation by Wu for preparing PSMTs. "In contrast with the normal courses which are relentlessly "forward-looking" (i.e., the far-better-things-to-come in graduate courses), considerable time should be devoted to looking back" (CBMS, 2012, p. 26). In other words, connections between mathematical structures and early-secondary mathematics teacher should not be the sole prevue of courses on methods in teaching secondary mathematics. Rather such mathematics coursework should significantly complement methods courses by utilizing the approximately 36 semester credit hours[5] in college-level mathematics prior to such methods courses to "looking back" (i.e., connecting to secondary school mathematics) as well as "looking forward" (i.e., connecting to future graduate studies).

Conclusions

The data presented in this paper provide a basis for understanding PSMTs' mathematical experiences with regard to CCSSM. That is, this paper serves as a snapshot, "of the indicators of the health and quality of the mathematics education system at a given point in time, and for examining variations over time" (Heck, et al., 2011, p. 6). Moreover, these data illuminate what PSMTs identify as the, "mathematics classes in preservice and alternative teacher certification programs aligned with the content…specified in the CCSSM" (Heck, et al., 2011, p. B-1).

The responses by the PSMTs arguably support concerns expressed by Wu (2011a) that, "because of the teacher preparation programs' failure to teach content knowledge relevant to K-12 classrooms, the vast majority of preservice teacher do not acquire a correct understanding of K-12 mathematics while in college" (p. 9). Given that the survey was completed shortly after the publication and adoption of the CCSSM, the responses offer compelling evidence that if the coursework experienced by PSMTs is not more clearly and purposely aligned to the CCSSM content, the concern articulated by Wu will continue to be perennially voiced.

References

Ball, D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching: What makes it so special? *Journal of Teacher Education*, *59*(5), 389-407.

[5] The approximation of 36 credit hours was chosen as it is the number of credit hours required by the State in which the author resides for certification in teaching mathematics, Grades 7-12.

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