Australian Curriculum linked lessons

Reasoning in Number and Algebra

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In the *Australian Curriculum: Mathematics* "Number and Algebra are developed together as each enriches the study of the other" (ACARA, 2014, p. 2). The content strands indicate what students should be learning and the proficiency strands indicate the types of learning experiences students should be exposed to in order to learn the content. Most of the content descriptors in Years F–6 algebra refer to a progression of patterning and finding unknowns in number sentences. We could see from this that there are two major themes running through the curriculum; that of patterns and relationships, and that of equations and expressions. These two aspects of algebra are closely linked and, as such, should be developed together (Ministry of Education, 2014).

The Reasoning Proficiency is about children making sense of the mathematics by explaining their thinking, giving reasons for their decisions and describing mathematical situations and concepts. Children need to be able to speak, read and write the language of mathematics. Important mathematical reasoning language includes the language of thinking, the language of justification and the language of proof.

In order to link the Reasoning Proficiency to algebra means that students need to be provided with opportunities to identify patterns, explain their thinking, justify their decisions and to generalise (ACARA, 2014) while investigating pattern and relationships alongside equations and expressions. Students need to access learning experiences that allow them to make and describe patterns, make rules and find general rules to predict the results of patterns. Implicit in this is the notion of relationships. At a Primary level these relationships are generally described in words and/or diagrams with the symbolic representation coming later on. At the same time children need opportunities to build on their understanding of the number system and equivalence to formulate generalisations and test conjectures (ACARA, 2014). So how do we provide these types of opportunities at a level that is suitable for primary aged children?

Line Up!

One of my favourite lesson starters is a problem called *Line Up!* This is a problem that students as young as Year 2 (see Figure 1) can access by utilising problem solving strategies. For students of this age searching for a method to solve the problem is a challenge in itself, but the real power of the problem is finding out how the students thought about the problem and what that means for algebraic reasoning.

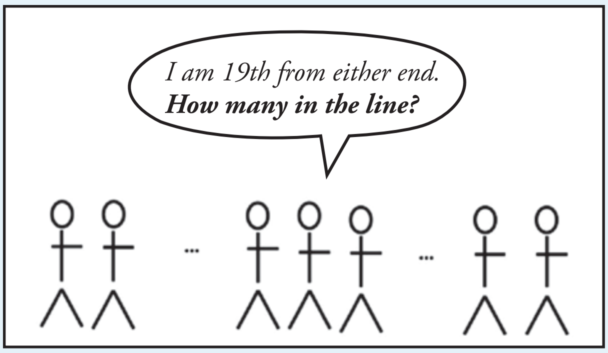

Figure 1. Line Up! problem.

By prompting students to think carefully about how they visualised the problem when they were standing in the line, generally three scenarios emerge. I find it useful to try to get students to draw a picture to explain their reasoning (see Figure 2), tell me a story about their reasoning and to then write a number sentence to represent their reasoning. For example for the pictures shown below the stories might be something like:

- I look to the left and there are 18 people, I look to the right and there are 18 people and I haven't counted myself so I add me in.
- I look to the left and there are 19 people including me. I look to the right and there are 18 people. So I add 19 and 18 together to get 37 people in the line.
- I look to the left and there are 19 people including me. I look to the right and there are 19 people including me. Now I have counted myself twice so I have to take one away.

In this way the children are able to see two very important ideas emerge: that different students 'see' problems in different ways and that there are several ways to arrive at a correct answer, and that there are several ways to write equivalent expressions for the same problem.

Four Arm Shapes

Another favourite lesson which assists students to begin the process of generalising from a young age (and moving from additive to multiplicative thinking) is a lesson documented in one of my most valuable resources *Maths300* (Education Services Australia [ESA], 2010). This lesson is set in the context of building four paths to an attraction in a park.

The students begin with tiles to build a growing pattern of four paths to a central tile which

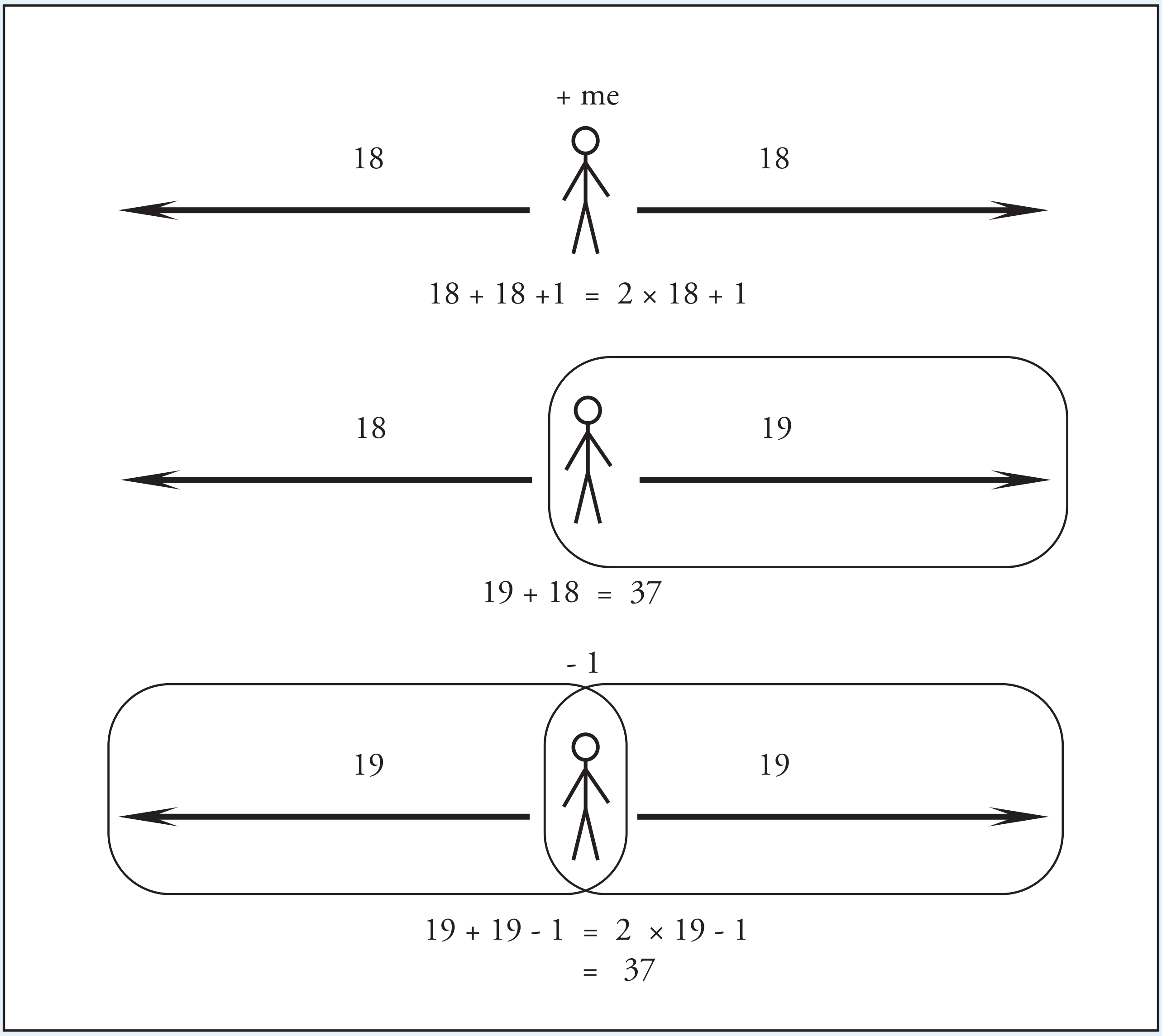

Figure 2. Three Line Up! picture scenarios.

represents the attraction (see Figure 3). As each set of four tiles is added students can calculate how many paving tiles have been used. Rather than just adding four tiles each time children can be encouraged to look at 'four lots of' tiles to encourage the multiplicative thinking rather than the additive thinking.

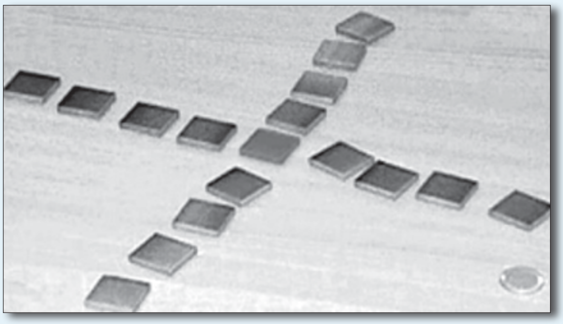

Figure 3. Four Arm Shapes.

Then questions such as "If each path was 10 tiles long, how many tiles would we have in total?" may be asked, and more importantly "How do you know? Can you show me on your model?" Then you can move on to asking about how many tiles would be used if each path was 100 units long. I find this use of 100 very powerful, as it is too big to physically build the model but the number is easy to work with arithmetically. If students can explain their reasoning with the paths 100 units long, then they will be able to answer the next question "If I told you how long each path was, explain how would you work out how many paving tiles I would need to build the path? Can you show me on your model how you worked it out". If students can articulate their reasoning in this way, they have found a general rule. Whether you decide to write this rule in words or symbols will depend on the age and experience of your students.

This general rule can then be tested by substituting numbers into the rule to check whether the rule always works. The checking can be done by making the paths and counting the tiles. Some students could even look at the problem in reverse to answer questions such as "If the gardeners used 48 paving tiles, how long would each of the paths be?" By working backwards in this way, the children are effectively solving equations, thus linking the pattern and generalisation to the equations and expressions. Another extension to this problem is seeing how the general rule might change if the central tile (the red tile shown in Figure 3) has to be included in the total number of tiles. Older students might even plot the graph of the length of each path and the total number of tiles so that the idea of a linear function may be introduced. The possibilities within this simple task are almost endless.

Game of 22

The *Game of 22* is a strategy game that involves students using their addition skills to be the person to turn over the card that makes the total exactly 22. Firstly the cards from one to six in each suit need to be set out as shown in Figure 4. The game is played in pairs with each student taking a turn at turning over a card and maintaining a running total of the sum. The student who turns over the card that makes the total exactly 22 is the winner. After playing several games students are able to begin to devise a strategy which allows them to always win the game.

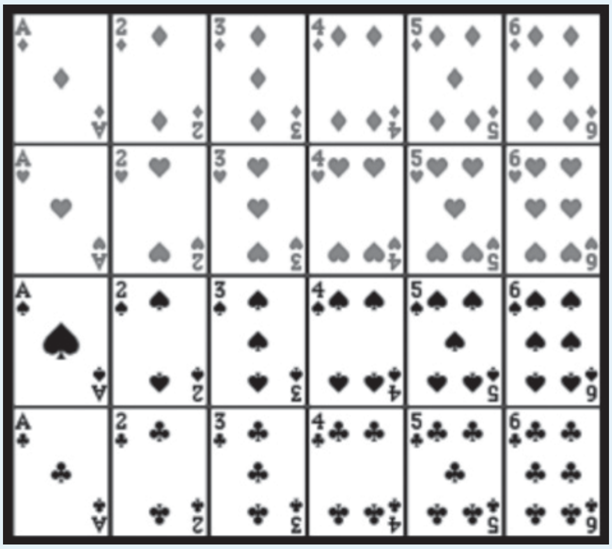

Figure 4. Game of 22 set up.

Figure 5 shows the total of 10 after each student has had two turns. I find that students are quite quick to work out that 15 is a special number, that is if they are able to turn over the card that gives a total of 15 they can always win the game. On prompting the students can tell me that this is because 15 is seven less than the target of 22 and seven is one more than the highest card available to turn over. The reasoning is that if they have turned over the card that gives a total of 15 then their partner has to turn over a card that puts the target of 22 in reach.

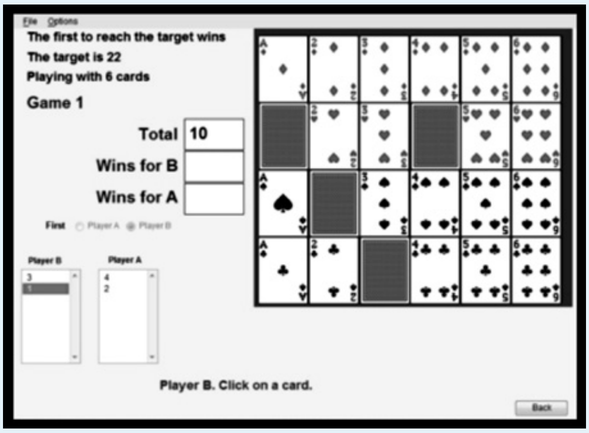

Figure 5. A total of 10 after four cards have been turned.

This prompts the next question: "If 15 is a special number are there other special numbers that you should be aiming for?" After much thinking most students are able to reason that eight and one are also special numbers. So, if you go first you should turn over a one and if you go second you should try to make a sum of eight. So far this is a number task that assists students with addition fluency and uses reasoning to devise a strategy. It is translated into an algebraic reasoning task by the investigation of several other games using the same structure. To assist with this, there is a piece of software from the *Maths300* (ESA, 2010) collection called *Game of 31*. This allows students to investigate changing both the target number and the number of cards used to come up with a general rule for winning the games.

Figure 6 shows a *Game of 53* using nine cards, for example. After investigating several different games, students are able to see that there is always a set of special numbers that are linked to the target total and the number of cards being used. For the *Game of 53*, the special numbers are 43, 33, 23, 13 and 3. So the person going first should turn over a three and the person going second should endeavour to make the total 13. This is now a very rich algebraic reasoning task where students have generalised the winning strategy for any of the games.

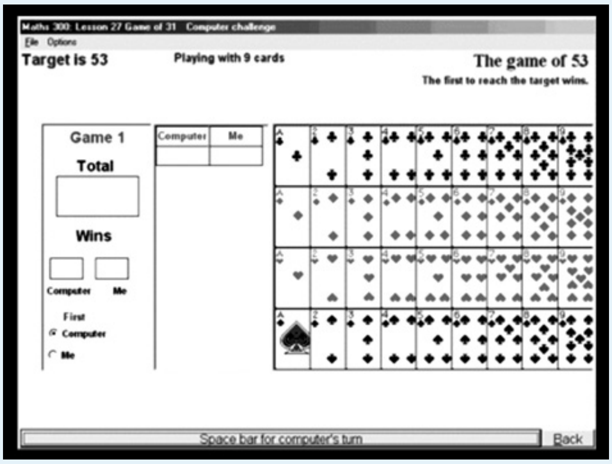

Figure 6. Game of 53 using nine cards.

The mysterious world of algebra is where we often lose students, as they see it as a mass of rules to be followed without any genuine understandings attached. By introducing algebraic reasoning through rich investigative tasks set within meaningful contexts, we provide students with the opportunity to understand some of the big ideas of algebra before they are overwhelmed by the symbolism and rules. Rich investigative tasks allow all students to enjoy some level of success while being challenged to think, reason and communicate mathematically. By providing students with opportunities to find patterns, explain their thinking, justify their decisions and generalise we are able to turn simple tasks that often begin as number tasks into rich, investigative algebraic reasoning tasks.

References

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