Learning to measure length

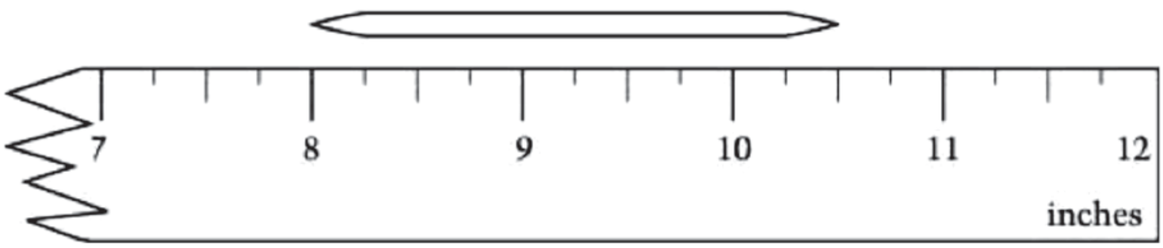
The problem with the school ruler

Michael Drake
Victoria University of Wellington
<michael.drake@vuw.ac.nz>

Ever wondered why children have difficulty using a ruler? In this article Michael Drake investigates some of the difficulties students encounter and provides some ideas for teaching about and learning to use rulers.

The desire to quantify our world and experience, that is to measure, is a uniquely human trait. Within this desire, the measurement of length holds a special place, not only as one of the simplest attributes to measure but also as one of the most interesting. How tall am I? How high have I climbed? How far have we walked? Do I have enough material to make this dress? However, international assessments and research over several decades have indicated that, worldwide, some students have difficulty understanding the measurement of length — regardless of which system of measurement is being used. Yet, understanding linear measurement is important, not just because it is a common measurement activity but because it is used in many branches of mathematics and the sciences, where scales based on linear measurement are commonly found on measurement instruments and in graphs. Therefore, it is important to develop our understanding of why students might find linear measurement difficult to understand.

One task that frequently has been used to test student understanding of linear measurement is called the broken ruler problem. Figure 1 (National Center for Education Statistics, 2012) shows one version of the problem. Other versions may show an object that is offset so the left end is not aligned with zero, or may involve physical measurement with a broken ruler. So why do some students find the broken ruler problem difficult? What can we do differently that might help? This article seeks to answer the first question by closely examining some rulers to see exactly what it is that students are struggling to use. To answer the second question, the insights gained from examining rulers are combined with insights from research to create suggestions for teaching and learning.



What is the length of the toothpick in the figure above?

Figure 1. The Grade 4 toothpick problem from NAEP (National Center for Education Statistics, 2012).

Figure 1 shows an American version of the broken ruler problem that was used in the main *National Assessment of Educational Progress* (NAEP) from 1996 to 2003. Results for this item are shown in Table 1. They indicate that even by Grade 8, roughly a third of students were not able to answer the question correctly. Other versions of the broken ruler problem have been used in the NAEP long-term trend assessments roughly every four years since the 1980s. They show similar results (Kloosterman, Rutledge & Kenney, 2009). Thus we have useful data against which the effectiveness of regularly-voiced teaching suggestions for improving student understanding of rulers can be measured.

Table 1. Percentage of Grade 4 and 8 students who correctly answered the NAEP toothpick problem.

Year	1996	2000	2003
Grade 4	24	25	20
Grade 8	64	64	58

Source: Males, Sweeny, Gilbertson, & Gonulates (2011).

Common errors

One common error for this problem is that some students simply read the end-point that aligns with the toothpick, producing the answer $10\frac{1}{2}$ inches. In NAEP 2003, 14% of Grade 4 and 7% of Grade 8 students gave this answer (US Department of Education, 2003). Hiebert (1984) suggests that this is because some students learn a procedure for reading a ruler without really understanding that the answer represents the distance between the beginning point and the end point of the object. One of his suggestions to overcome this was to show how individual unit-long pieces could be lined up to make a measure, then introduce students to a ruler as a way to abbreviate this tedious process. The other was to place zero a short distance from the end of the ruler because on such a ruler students must directly deal with the idea that the length of the object is the number of units between the endpoints. Early teaching approaches built around the construction of rulers have been echoed in research for some years now, and New Zealand and Australian rulers have their start point a short distance from the end of the ruler (e.g., Figure 2). However, my experience is that some students still have trouble understanding linear measurement and all that the extra space has succeeded in doing is introducing a new error. Bragg and Outhred (2000) report similar results. When measuring with such a ruler, a common error among younger children is to align the end of the ruler (rather than zero) with the edge of the object, as they would line up their hands if using them to measure the width of their desk. Older students more commonly make the mistake of aligning the ruler at 1, probably because counting starts at one.

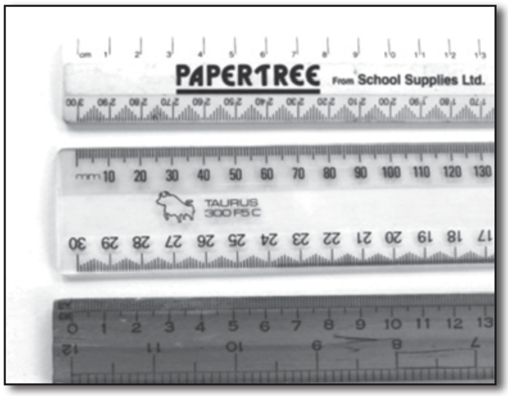


Figure 2. A selection of New Zealand school rulers.

The second common error with the broken ruler problem is to produce an answer which is one more than the correct answer. For the toothpick in Figure 1, this answer would be $3\frac{1}{2}$ inches; in NAEP 2003, 23% of Grade 4 and 20% of Grade 8 students gave this answer (US Department of Education, 2003). One common way to obtain this answer is to count the mark that aligns with the start of the toothpick as 1, then continue to count the marks. Traditional wisdom to remedy this error is to get students to focus on counting the units (the spaces) rather than the marks, a seemingly simple solution. Unfortunately, NAEP results suggest that students' success rates with the broken ruler problem have changed little since the 1980s (Kloosterman et al., 2009).

Problematic issues

What are we missing? Let us look more closely at our rulers. Go to Figure 1 and observe the length of the marks on the ruler. You should notice that the unit marks, half marks, and quarters are all different sizes, which suggests that the size of the marks provide important scaffolding information for users of this ruler. Also note that to use this ruler to correctly obtain a half unit, a student must identify that the end of the object aligns with the mark that indicates halfway. It seems that with this ruler, students must be able to work effectively with the marks, not just attend to the units. Now focus on the top scale (gauge) of the ruler in Figure 3, in which the unit is millimetres. It too has marks of different sizes, but it makes little sense to focus on the units (or spaces) instead of the marks as they are so small. Here the numbering indicates that each interval is 10 mm, so the mark that scaffolds halfway (e.g., between 120 and 130) is 125 mm. For this gauge, it is important again to focus on the marks, but here it is the interval, the way it is numbered, and how it is partitioned into pieces of equal size that is important.

Figure 3. Part of a metric school ruler.

Now consider Figure 2. Notice that there are four totally different scales (gauges) on the rulers. On the top ruler, there is a simple gauge in centimetres. On the bottom ruler is a gauge in inches, partitioned into halves and tenths. There are two gauges in millimetres but with different lengths of mark, and two in centimetres with millimetres nested between them. One of these latter two indicates that the large lines are centimetres, and the small lines millimetres. The other (out of view) indicates the gauge is in centimetres. Now look at the gauges at the top of each ruler. Notice that only one of the three starts with zero. Considering these are three rulers I happened to have at home (and not ones deliberately sought out), this is considerable variability, so I cast my eye wider and found other tools for measuring length in different parts of my house (Figure 4). Even more variability is evident. For example, while the top rule is fairly standard, how easy would it be to measure a line that is 13 cm long with the second tape? Where is the start point of this scale? For this tape you also might note that each centimetre is only partitioned into five pieces rather than the usual ten.

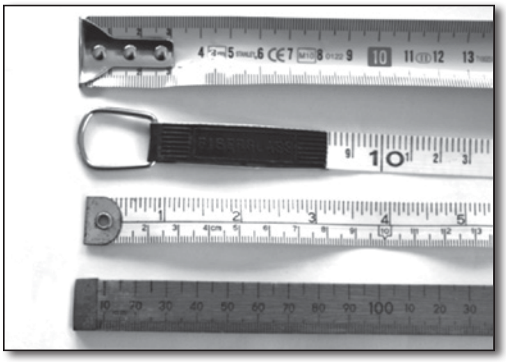


Figure 4. Another collection of linear measures from around the home.

Looking at how such instruments handle the transition at one metre was also interesting (Figure 5). For example, the top gauge restarts with 1, 2, 3, 4, …, so to measure with this tape it is important to check how many whole metres have been measured before switching to centimetres. The second gauge simply continues in centimetres (100, 101, 102, …) while the third gauge works in multiples of 100 millimetres (1000, 10, 20, 30, …).

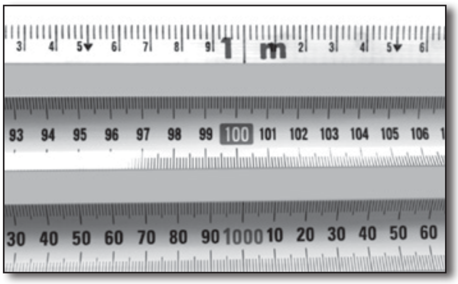


Figure 5. Transitions at one metre.

This recognition of the variety between rulers is important, as it highlights that students who learn about only one form of ruler may be assessed on something quite unfamiliar. It also begs the question: when students are learning to use a ruler to measure length, what do they need to learn? Plainly not to use the school ruler. Rather, they are starting to learn about a class of instruments, each of which may contain significant differences. We should also remember that each example from this class was created by a professional who understood the concepts involved with length measurement and who was creating a tool to meet the needs of a particular circumstance. So, given this new knowledge, how can we use it to improve teaching and learning? The next section explains some findings from research that will help.

Using research to find a way forward

Recent work undertaken by Mulligan and Mitchelmore (2009) indicates that young children's drawings suggest they may not recognise important features of mathematical structure, in particular the spatial features of connectivity and equality. For example, when asked to draw from memory a picture of an array made of nine squares of equal size, children can produce a collection of shapes of different sizes, with only some connected. Their findings also indicate that maturation alone is insufficient to overcome this issue, although teaching can. These findings, however, should not surprise us since when children are learning to count, they need to learn to discount both the space between items and any physical differences (Drake, 2010). When counting, whether items are connected (continuous) or disconnected (discrete) is irrelevant, as is the nature of the objects being counted. In another recent study in which kindergarten students were asked to draw a ruler, while the students had a notion of what ruler a looked like, many did not have equal partitioning (MacDonald, 2011). Yet identifying the need for units that connect without overlap and are of equal size is a critical starting point when creating a ruler and also for understanding the conventions used on a ruler. This suggests that spatial notions of connectivity and equality need to be in place for rulers to be meaningful.

A second useful idea can be found in Lehrer (2003), who notes that when students are introduced to length measurement problems involving a half in the answer, they are often not sure how to make their count. With an object $2\frac{1}{2}$ units long, some students will count a third unit and then put a half to indicate that only half of this unit is needed, giving a response of $3\frac{1}{2}$ units.

The third piece of our jigsaw comes from Sfard (1991), who suggests that historically, many common mathematical concepts used today began life in processes that were useful for getting answers to particular problems. In some cases such a procedural approach lasted many centuries before the answers became objects (structures) with meaning in their own right. She suggests that this process is mirrored in individual learning. For example, when learning to count, a young child's initial conception of the number 3 is as the answer to a count of a set of items. It is only with lots of repetition that the child starts to see that 3 is always the result when counting a set of that size, and it is at that point that 3 starts to make sense as a number in its own right. This new object can then be explored and can start to be used mathematically. Sfard also points out that the same is true for students learning about rational number. For many, fractions arise from processes, such as from undertaking divisions (or sharing out objects fairly); thus they may not consider fractions to be numbers during early stages of their conceptual development. Putting these three pieces together suggests a way to improve understanding of rulers and how to use them to measure length.

Ideas for teaching and learning about rulers

Measurement is a practical topic where students can learn while attempting to answer problems that interest or challenge. Will this desk fit through that door? How can we compare the size of those trees? How big is your arm span (and how does it compare to your height)? However, giving classroom rulers to students and teaching how to use them is not enough—rulers are simply too variable for a 'one size fits all' approach. Students must develop conceptual understanding. Here the mathematical actions of teachers are important as there are a number of key understandings that need to be actively developed. Contexts for measurement should also be carefully chosen as length measurement is a situated activity; that is, the choice of tool and unit is dependent on what is being measured. For example, it is very unlikely that a standard classroom ruler can be used accurately to measure the girth of trees or the length of the classroom.

If we apply Sfard's (1991) ideas to learning about rulers, it suggests that initially students need to be given plenty of practical experiences that lead to the creation of rulers (so the measure is the result of a process). Such activities would help establish the need for connectivity and equality—the connected units of a ruler. My exploration of rulers suggests that any practice given needs to include developing the idea of using short units for measuring small things, and long units for measuring big things. Measuring items like the length of a model car and the classroom could serve. Keeping Lehrer (2003) and Sfard (1991) in mind, initially giving practice with measuring things that result in whole number answers seems important (so part units are not present) or can be answered through the use of language rather than with decimals or fractions ("a bit more than three" or "nearly two"). Once these initial ideas are in place, carefully scaffold challenges. For example, introduce lengths that do not have such answers, rather require a half unit. Struggling with learning how to deal with this problem will help structure students' thinking so they start to see the ruler as an object, and in doing this are starting to think beyond the process of counting to treating the ruler as a trusted tool (an object) that helps get the answer.

Each mental struggle you introduce should lead to progressive improvements in the students' use of rulers, and an improvement to their understanding of length measurement. For example, counting the units one by one is an inefficient process (especially with a large count) so numbering the units (spaces) is a logical improvement to their rulers. However, when a problem requiring a half unit is introduced, numbering the units becomes problematic. It is at this stage that numbering the ends of the units makes sense, because this allows half units to be marked on the ruler. Suddenly the meaning of the marks on a ruler needs to be considered, and the question of what number to put at the start of the ruler also arises, as does the concept that measurement of length is not a count: it is a measure from here to there, an interval. However, be aware that at this stage, rulers that show a single scale (e.g., whole centimetres) or self-constructed rulers made of paper are all that students need to use; the ruler that is the tool of a builder or dressmaker is still too complex to introduce meaningfully.

Each time that measurement is revisited, keep using carefully constructed practical tasks but challenge students to be more and more accurate. For example, scaffold the introduction of quarter units and, if using large units, even eighths. This is an important development as it helps establish fractions as answers to measurement problems (the result of a process), and growing familiarity with obtaining fractional answers will then help students accept that fractions are numbers that live between the whole numbers (a point worth reinforcing regularly).

As you move to measuring random objects in the environment, the push for even more accuracy should lead to the idea of mixing units. For example, to measure a door, start out by using metres (because a door is big), but swap to centimetres when whole metres do not fit (so it is 1 m, 90 cm high—one this long and 90 that long). It is at this point that the nesting of units on the metric school ruler should start to make sense—this understanding being based on how units work. However, discussions about still wanting to make it easy to find the halfway mark need to be held, and conventions for doing this discussed. Such developments should now mean that commercial rulers of various sorts are potentially meaningful objects – but not on their own. Hand out a variety of rulers and tapes at the same time, not just one sort. Get students to focus on the variability, explore why it exists, and how it affects the process of measurement. For example, ask questions such as: Why would someone want a ruler in millimetres? (Won't the numbers be really big?) How easy is it to use? What problems might arise when trying to use that ruler? Why isn't there a number at the start of this ruler? If I make up a line from 13 centimetre-long units, can you use the ruler to check how long it is? What do you get as your answer? Why is there a bigger mark between 120 and 130 mm? Why on a 30-metre tape might we find that between each metre it gives a count in centimetres from one to 99? How do we measure something that is longer than our ruler? What can we do if we need more accuracy than the ruler provides? I believe these are the sorts of questions and tasks that students need to be able to answer if they are to understand rulers and use them effectively.

Final words

Rulers of one form or another are common tools, readily found in most households. Students encounter them at an early age and learn that they are used to measure length. However, assessment results indicate that learning to use a ruler is not easy. Various fixes that have been mooted over time seem to have had little impact on results. This article suggests that one reason for this is that rulers are a class of instruments that come in a variety of forms. Therefore, to use any particular ruler, students need to understand the variability between rulers and be able to adjust for it. Examining the nature of the gauges on rulers has also been enlightening. It has shown that students not only have to understand the concept of a unit (something this long) but that we use different units in different circumstances. When coming to read the ruler we have also identified that it is important to understand the role of the spaces (units are intervals that go from here to here) and the scaffolding information provided by the marks. Noting how the gauge has been numbered and how each interval has been partitioned has also been identified as important. It seems that rulers are complex tools, so it is not surprising that coming to understand them is not simple.

Summary

- Remember that when first introducing rulers, students will already have some knowledge of length measurement, such as being able to compare lengths. In all probability, they have also seen rulers but do not know how they work.
- Take your time. Research shows that even by Grade 8 some students are still confused when measuring length.
- Use contexts and problems that provide a reason to measure, but make sure they match the sophistication of measurement understanding of your students.
- Initially focus on creating simple (paper) rulers with a single unit, and only measure objects that give whole number answers or can be answered with statements such as “a bit more than three” or “nearly two”.
- Students need to create rulers with small units to measure short things and big units to measure long things.
- Push for accuracy. Scaffold the introduction of half units then quarter units. Over time, introduce students to the conventional notations used on commercial rulers, in particular using zero to indicate the start point for the measure.
- Keep pushing. Try swapping to smaller units to become even more accurate, and to nest units on their rulers.
- Do not introduce your students to just one commercial ruler. Get them to use what they have learned about measuring length and creating rulers to explore how to use the various types.

References

Bragg, P. & Outhred, L. (2000). Students’ knowledge of length units: Do they know more than rules about rulers? In T. Nakahara & M. Koyama (Eds), *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education* (pp. 97–104). Hiroshima: Hiroshima University.

Drake, M. (2010). Teaching practical measurement. In R. Averill & R. Harvey (Eds), *Teaching primary school mathematics and statistics: Evidence-based practice* (pp. 101–112). Wellington: NZCER.

Hiebert, J. (1984). Why do some children have trouble learning measurement concepts. *Arithmetic Teacher, March*, 19–24.

Kloosterman, P., Rutledge, Z. & Kenney, P. (2009). Exploring results of the NAEP: 1980s to the present: Results of the long-term trend assessment (LTT) for middle grades show positive advancement. *Mathematics Teaching in the Middle School, 14*(6) 357–365.

Lehrer, R. (2003). Developing understanding of measurement. In J. Kilpatrick, W. Martin & D. Schifter (Eds), *A research companion to principles and standards for school mathematics* (pp. 179–192). Reston, VA: NCTM.

Males, L., Sweeny, S., Gilbertson, N. & Gonulates, F. (2011). *Activities to support conceptual understanding of spatial measurement.* Retrieved from https://www.msu.edu/~maleslor/presentations.html

MacDonald, A. (2011). Young children’s representations of their developing measurement understandings. In J. Clark, B. Kissane, J. Mousley, T. Spencer & S. Thornton (Eds), *Mathematics: Traditions and [new] practices* (Proceedings of the AAMT–MERGA conference). Adelaide: AAMT & MERGA.

Mulligan, J. & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal, 21*(2), 33–49. doi:10.1007/BF03217544

National Center for Education Statistics. (2012). *2003 mathematics item* (Block M6, item 18). Retrieved from http://nces.ed.gov/nationsreportcard/itmrlsx/search.aspx?subject=mathematics

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics, 22*(1), 1–36. doi:10.1007/BF00302715

US Department of Education, Institute of Education Sciences, National Center for Education Statistics, NAEP, Mathematics Assessment. (2003). *Use a scale to find a length.* (Mathematics item, grade 8, block M6, item 18.) Retrieved from http://nces.ed.gov/nationsreportcard/itmrlsx/search.aspx?subject=mathematics