Using digital technology to see angles from different angles

Part 1: Corners



Erin Host
Sydney University
<erin.host1@gmail.com>

Emily Baynham
Sydney University
<mily.baynham@gmail.com>

Heather McMaster
Sydney University
<heather@workingmaths.net>

In Part 1 of their article, Erin Host, Emily Baynham and Heather McMaster use a combination of digital technology and concrete materials to explore the concept of 'corners'. They provide a practical, easy to follow sequence of activities that builds on students' understandings.

The concept of an angle is implicit in many mathematics topics in the Australian Curriculum. In primary school, students need to understand the circular part–whole model of fractions, read analogue time, interpret sector graphs (pie charts), use compass directions and reason geometrically. In high school, students continue to reason geometrically with angles. They also need to recognise the relationship between the angle and the arc of a sector, understand gradient, interpret compass bearings and angles of inclination and use trigonometry.

An angle is clearly a multifaceted concept that is difficult to define. Some authors use a static definition (e.g., an angle is a pair of rays with a common end point) and some use a dynamic definition (e.g., an angle is an amount of turning about a point between two lines). Initially children see static and dynamic angle contexts differently, so beginning with a definition of an angle is unhelpful. What is important is that children eventually recognise angles in both static and dynamic contexts, irrespective of any formal definition (White & Mitchelmore, 1998).

Prescott, Mitchelmore and White (2002) found that children progressively generalise and abstract the concept of an angle through experiencing a variety of physical angle contexts. They found that there are five 'clusters' of physical angle concepts generalised by students. In order of difficulty, these contexts are corners, openings, turns, slopes and directions. Corners and slopes contexts are static, whereas opening and turning contexts are dynamic. Direction contexts can be both. Contexts which involve a change in direction (e.g., a rebounding ball) are perhaps the most difficult to understand because the vertex moves.

In primary school, the major focus is on corners, openings and turns, so we will limit our discussion to these three types of contexts. In this article we will present a teaching sequence that integrates use of the interactive whiteboard (IWB) with the teaching of angles as corners. In a subsequent article (Part 2) we will link the teaching of angles as corners to the teaching of dynamic angles (openings and turns). Within each type of context, our teaching sequence begins with an exploration of angles in the real world. We then use a combination of concrete materials and pictorial representations to help students make connections and reach important generalisations about angles.

According to Prescott, Mitchelmore and White (2002), students' initial difficulties with angles are:

- recognising angles in the physical environment,
- knowing what they are measuring,
- positioning the baseline and the vertex of a protractor when measuring angle size,
- reading the correct scale on a protractor.

Based on this research, the sequence of activities we recommend is:

1. Recognising static angles in the real world.
2. Directly comparing angle sizes.
3. Fitting one angle size into another.
4. Fitting the same-sized angles around a point and naming angle sizes as fractions of a revolution.
5. Defining the arms and vertex of an angle and making a circular protractor.
6. Defining a degree and using a circular protractor.
7. Estimating and measuring static angles in the real world.

Lessons may be made up of one or more activities in this sequence. They are all based around the use of a bendable straw (to show angles in the real world), a standard set of pattern blocks (comprised of the basic six shapes shown below), and the same shapes represented on an interactive white board (IWB).

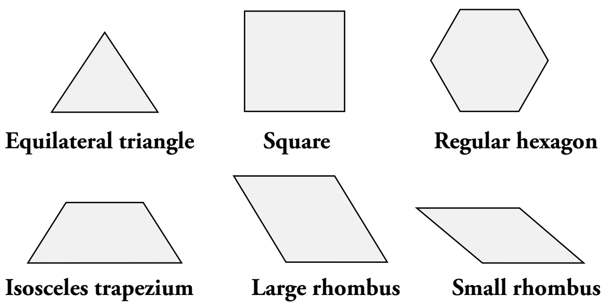

Figure 1. The 6 basic shapes.

1. Recognising static angles in the real world

Give each child a bendable straw and ask them to show you a corner they can see in the classroom by bending it and laying the arms along the sides of the angle. Make sure they find corners of objects (three dimensions) such as the angle of a stool leg to the seat (shown below), as well as angles on surfaces (two dimensions) such as the corner of a table. Students may record the corner they find by taping their straw onto the corner and photographing it. These results may then be shared with the whole class on an IWB, giving opportunity for immediate discussion and clarification.

Figure 2. Stool angles.

2. Directly comparing angle sizes

The questions below and the discussions surrounding them are aided by pictures of the pattern block shapes being moved around on an IWB.

- Make sure that students know the names of the two-dimensional shapes represented by the top faces of the pattern blocks. Tell them that you will be talking about the corners of the top faces of the pattern blocks (two dimensions) and that you will be calling these corners angles.
- Ask them to compare the angles of different pattern blocks by placing one on top of the other. They are to do the following:

  (a) Find which angles are the same size (e.g. the base angles of the isosceles trapezium and the angles of the equilateral triangle are the same). Ask whether the sides of an angle have to be the same for the angles to be the same size.

  (b) Find which pattern blocks have angles that are not all the same size.(These will be the trapezium and the rhombuses.)

  (c) For the blocks whose angles are all the same size (the regular polygons), place them in order of angle size. They should find that of an angle of an equilateral triangle is less than an angle of a square which is less than an angle of a regular hexagon. They might also notice that the greater the number of sides a regular polygon has, the larger its angles are.

3. Fitting one angle size into another

- Ask the students to find pattern block angles that 'go into' each other. They could find, for example, that two of the equilateral triangle angles go into one regular hexagon angle. Show this on the IWB.
- On the IWB, change the relative size of the equilateral triangles in relation to the regular hexagon. Ask the students whether they think there will now be more, fewer or the same number of equilateral triangle angles in a regular hexagon angle. This leads to the realisation that when shapes are enlarged or reduced, their angle sizes stay the same.

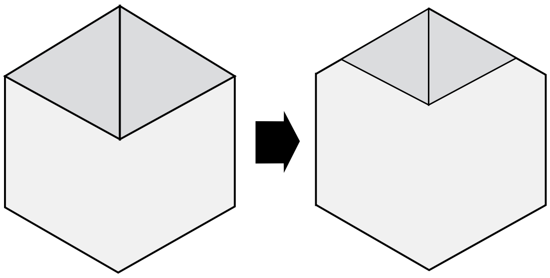

Figure 3. Two equilateral triangle angles go into one regular hexagon angle.

4. Fitting the same-sized angles around a point and naming angle sizes as fractions of a revolution

- Using the same-sized angles of a shape, ask the students to fit them around a point marked on a piece of paper. For each type of block and angle, they are to count the number that fit.

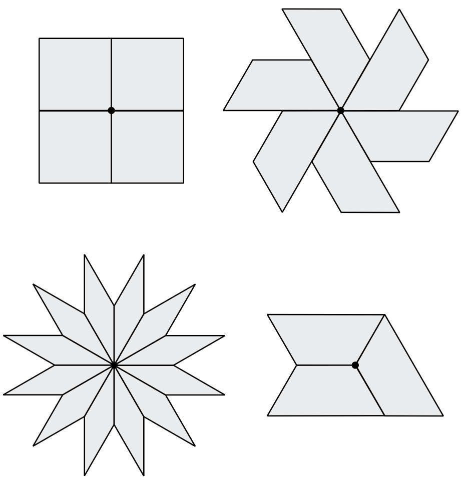

Figure 4. Shapes around a point.

- Make a table on the IWB into which the students' results are to be entered. Include a third column for the 'angle name'. The angle name is the fraction of a revolution made by that angle (Mitchelmore, 2000). A completed table is shown below.

Table 1. Shape the angle is in.

Shape the angle is in	Number of angles	Angle name
Equilateral triangle angle	6	sixth angle
Square angle	4	quarter angle
Regular hexagon angle	3	third angle
Trapezium – small angle	6	sixth angle
Trapezium – large angle	3	third angle
Large rhombus – small angle	6	sixth angle
Large rhombus – large angle	3	third angle
Small rhombus – small angle	12	twelfth angle
Small rhombus – large angle	?	?

- The large angle of the small rhombus (150°) will not fit into a revolution. Talk about what you could call this angle. Ask whether it could be made up from smaller-sized angles. Which ones? How many of them? Could you call it a five-twelfths angle?

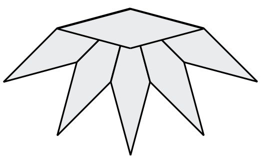

Figure 5

- Could the other angles in the table be measured as a number of twelfth angles? Add a fourth column to the table and ask them to rewrite each angle as a number of twelfth angles.
- Tell them that quarter angles (the angles of a square) are called right angles. Look for quarter angles (in different orientations) on a compass pictured on the IWB.

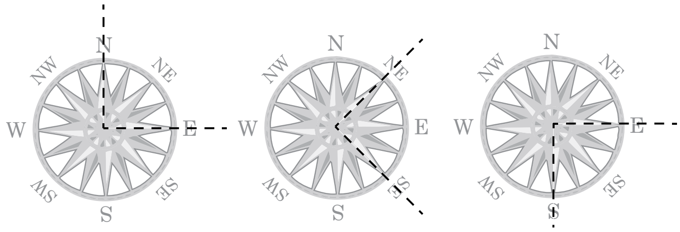

Figure 6. Compass with right angles.

- Tell them that a two-quarter angle is called a straight angle (because the two sides make a straight line). Tell them that the angles less than a quarter angle are called acute angles and the angles bigger than a right angle and smaller than a straight angle are called obtuse angles. Ask them to classify the angles in the table above as acute angles, right angles or obtuse angles.
- Discuss whether an angle can be bigger than a straight angle. For example, what would a three-quarter angle look like? Tell them that these angles are called reflex angles.

5. Defining the arms and vertex of an angle and making a protractor

- On the IWB, draw along the two sides of a pattern block angle to show what is meant by the vertex of an angle (the point) and the arms of an angle (the sides).
- Move the same angle around its vertex so there are no gaps or overlaps. Ask the students to create the same type of drawing on a piece of paper using a twelfth angle of a pattern block. Have them begin by marking a point on the paper where all the vertices will meet, then ask them to draw the arms of the angle.

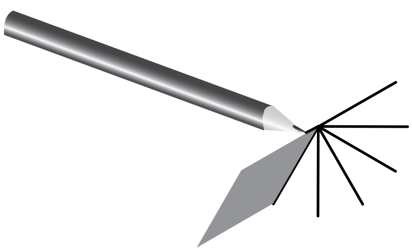

Figure 7. Draw the arms of the angle.

- Ask them to use a ruler to extend all the arms in their drawing to the edges of their page. They should end up with 12 arms that together make six straight lines that cross at the marked point.
- Ask whether the angles are still twelfth angles after their arms have been extended.
- Ask whether the lines have divided their paper into areas that are the same size.

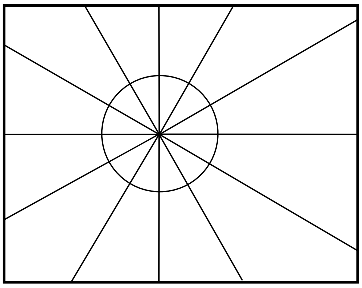

Figure 8. Extend the arms to the edges of the page.

- Discuss what shape you could make the paper so that the same-sized angle produces the same area. Aid this discussion by placing different shapes over the paper pictured on the IWB. Eventually they should realise that you can only do this if the shape is circular and the vertex of the angles is in the centre of the circle. Ask them whether the size of the circle matters.
- Mark the places where the arms of the angles meet the circumference of the circle. Ask whether the distance between each mark around the circumference is the same. Is this distance related to the size of the angle at the centre of the circle?

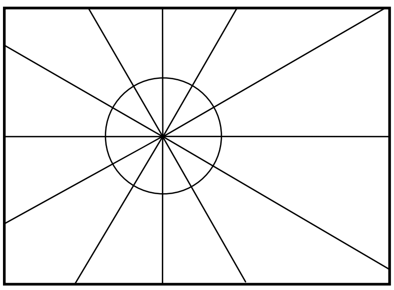

Figure 9. Vertex of the angles is at the centre of the circle.

Tell them that what they have made is called a protractor. This protractor can be used to measure the size of an angle. It will tell you how many twelfth angles large it is.

6. Using a circular protractor and defining a degree

- On the IWB, show a circular protractor that has degree markings on it but no numbers on the scale. Tell them that protractors usually measure angles in units called degrees. A degree is a 360th angle because 360 of them go into one revolution.

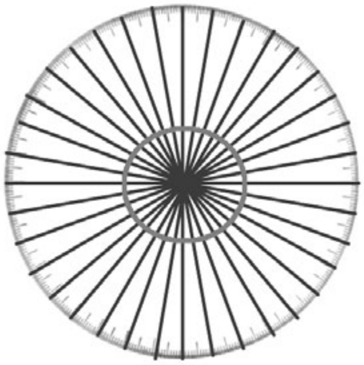

Figure 10. Protractor.

- Discuss why a degree is a useful unit to use. Is a measurement of an angle in degrees more accurate than a measurement of an angle as a number of twelfth angles?
- Does a degree fit into a twelfth angle?
- Show a twelfth angle on top of the protractor. Ask how many degrees is a twelfth angle.

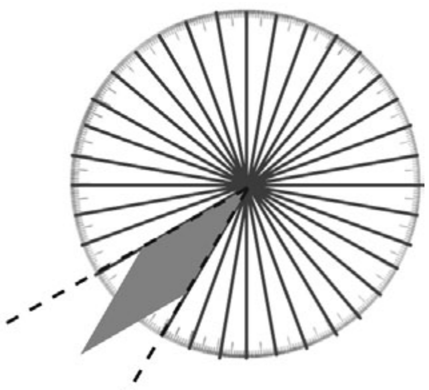

Figure 11. Rewrite fraction angles as degree angles.

- Using a numberless circular protractor printed on paper (like the one above), a pencil, a rubber and pattern blocks (if necessary) ask the students to re-write different fraction angles as a number of degree angles and record their results into a table like the one above in the next column.

Table 2. Fraction angles and number of degrees.

Fractional angle name	Number of degrees
twelfth angle	30
five-twelfths angle	150
sixth angle	60
third angle	120
two-thirds angle	240
quarter angle	90
eighth angle	45
tenth angle	36
twentieth angle	18
straight angle	180

7. Estimating and measuring static angles in the real world

Show students the photographs obtained in the first lesson. Ask students to estimate the size of their angles to the nearest 10 degrees. Then ask them to measure their angles using the numberless circular protractor. Note that in this activity, the vertex needs to be positioned at the centre of the protractor but the line that is used as the baseline doesn't matter.

In our next article (Part 2) the students' understanding of a degree is extended to enable them to measure angles using a circular protractor with two scales going in opposite directions, so a correct positioning of the base line of the protractor is necessary. The types of physical contexts used in Part 2 are openings and turns where the vertex is a pivot, the arms of the angles may or may not be visible, the turn can be clockwise or anti-clockwise and the amount of turning can be greater than a revolution.

References

Mitchelmore, M. (2000). Teaching angle measurement without turning. *Australian Primary Mathematics Classroom 5*(2), 4–8.

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