Using digital technology to see angles from different angles

Part 2: Openings and turns



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Ever wondered how to use technology to teach angles? This article follows on from an earlier article published last year, providing a range of ideas for integrating technology and concrete materials with the teaching of angle concepts. The authors also provide a comprehensive list of free online games and learning objects that can be used to teach angles in an engaging way.

This article is the second of two articles in which we offer some ideas concerning the teaching of angle concepts in primary school using a combination of concrete materials and pictorial representations. In Part 1 (Host, Baynham and McMaster, 2014) we presented a teaching sequence that integrated the use of pattern blocks and the interactive whiteboard (IWB) with the teaching of angles as corners. In this article we will link the teaching of angles as corners to angles as openings and turns. As with our first article, our teaching sequence begins with an exploration of angles in the real world. We then use a combination of concrete materials, videos, digital learning objects and computer games to help students make connections between angles as corners (a static context) and angles as openings and turns (a dynamic context).

In the *Australian Curriculum: Mathematics*, angles are initially introduced in Year 1 as corners of shapes. In Year 2, students are introduced to angles as fractional turns (full-turn, half-turn and quarter-turn). Research has shown that children mentally classify corners, openings and turns as being different from each other (Prescott, Mitchelmore, and White, 2002). In many dynamic contexts, children have difficulty locating the vertex and arms of an angle of rotation (Mitchelmore, 2000). It has also been noticed by Clausen-May (2008) that students often fail to relate the dynamic action of turning to a static printed diagram.

We began our teaching of angle measurement using the context of corners (Host, Baynham and McMaster, 2014) because in this context, the vertex is easy to locate, both arms of the angle are visible and the direction in which the angle is measured does not matter. In the context of an opening or a turn, the vertex is not always so easy to locate, one or both arms of the angle need to be imagined, and the direction of measurement of a turn (clockwise or anticlockwise) is important. In Part 1:

- Angles were found in the real world.
- Angle sizes were compared directly and then indirectly by determining the number of a particular corner that would fit around a point.
- The sizes of different angles were expressed as unitary fractions, the denominator being the number of them that fitted around a point.
- The arms and vertex of an angle were identified.
- The quarter angle was called a right angle and two quarter angles were discovered to make a straight angle. Angles were classified as acute or obtuse.
- The misconception that arm length determined the size of an angle was addressed through discussion surrounding the construction of a circular protractor.

- Children were introduced to the degree as the standard unit of measure for angle size, with 360 of these units fitting around a point.
- Angles described by the fraction of revolution, were measured using a circular protractor.
- The circular protractor was used to measure angles in the real world.

The next step is to incorporate turning into the children's understanding of corners. The sequence of activities we recommend is:

1. Recognising dynamic angles in the real world.
2. Naming angles in a revolution, including reflex angles.
3. Recognising a vertex as a centre of rotation and locating an invisible arm.
4. Measuring dynamic angles.
5. Estimating angle size from any base line and in both directions.
6. Recognising that an angle can be limited or unlimited.
7. Finding and measuring difficult-to-locate dynamic angles in the real world.

1. Recognising dynamic angles in the real world

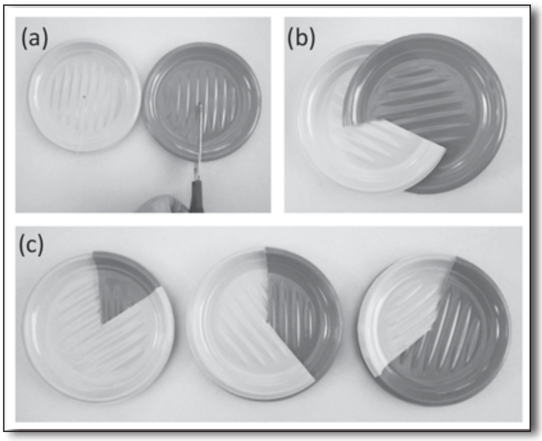
Mitchelmore and White (2000) found that children are more likely to notice openings (e.g., the opening of scissors) than turns (e.g., the turning of a wheel). Of the dynamic angle situations represented in their study, the fan was the easiest of these in which to recognise angles.

- Play a video of a fan dance that shows an opening fan as well as turns of the fan and angles made by the dancer's body movements. For example, play the YouTube video "*Beautiful Japanese Fan Dance*" (https://www.youtube.com/watch?v=-NJhVZtNFDw). You might even make a video of a child fan dancing. Pause the video in one place and ask the children what angles they can see. Mark them. Pause the video again a little later and ask them if them if the original angle has changed. They don't need to be made aware of turns as angles at this stage.

2. Naming angles in a revolution

In Part 1 of this article (Host et al, 2014) acute, obtuse and right angles were named in a static context. This learning can now be transferred to the real-life dynamic context they have observed (the fan) and extended to the naming of reflex angles and straight angles using a circular fan and concrete materials (plastic plates).

- Give the children two plates which make "plate angles" (Figure 1). The plates in the photograph show two different angles: a "dark-coloured angle" and a "light-coloured angle". Decide which colour will indicate the angle of interest in the lesson. Ask the children to show you the vertex and the arms of this angle (e.g., the dark-coloured angle) on their plates. Tell them that the vertex of their angle is also called the centre of rotation because the arms of the angle can be turned without moving this point.



'Plate angles' are made using two plastic plates of different colours. (a) Cut each plate along a radius. (b) Fit the plates together. (c) Make different angles by turning one plate over the other. Alternatively, use two white plates by marking the cut edges with a permanent marker to show the arms of the angle, and by drawing a small circle around the centre of one plate to indicate the angle of interest.

Figure 1.

- Show them a fan (like the one in the video) opening at different angles. For each angle you make, ask them to make that angle as a 'plate angle' and hold it up to show you. Then ask them if they think the angle is acute, obtuse or a right angle.
- Introduce the idea of a reflex angle by showing them an angle made by a circular fan and asking them to show the same angle on their plates.

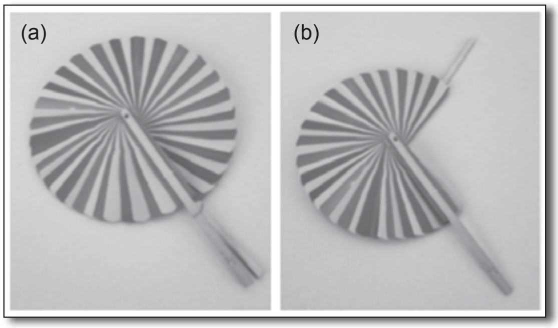


Figure 2. (a) A circular fan, (b) The fan showing a reflex angle.

If a dark-coloured angle in Figure 1(c) is an acute angle or an obtuse angle, they can see that the other angle (the light-coloured angle) is a reflex angle. Ask them to show you a reflex angle, and then look at the range of sizes of reflex angles (the smallest reflex angle being as close as possible to 180° and the largest reflex angle being as close as possible to 360°). Also ask them what the two angles on the plate are called if they are both the same size. Because two straight angles make a revolution, the size of a straight angle is 360°÷ 2 = 180°.

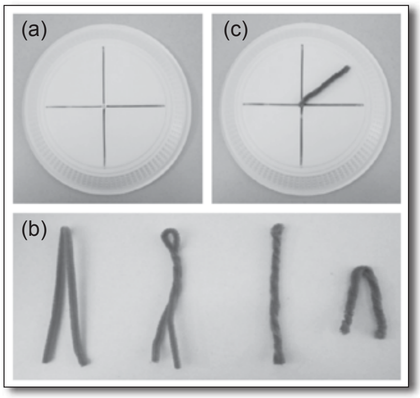
3. Recognising a vertex as a centre of rotation and locating an invisible arm

An opening door is a difficult situation in which to see an angle because when one arm (the bottom edge of the door) moves, the original arm location is invisible (White and Mitchelmore, 1998). This context can be made easier by first observing a door between two rooms with different flooring, so the floor edge trim marks the location of the angle's baseline. If there is no floor edge trim, this line can then be marked with masking tape.

- Show them an open door and ask them to show you the angle of opening. They will need to find the vertex and arms of this angle. The vertex is found by locating the centre of rotation and the arms are found by finding the two positions of the door. Tell them the arm of the angle that didn't move (i.e. the original position) is called the baseline.
- To link openings with turns, show them a dial on a set of kitchen scales and ask them to find the vertex and arms of an angle made by the needle when something is weighed. Ask them where else they have seen angles with only one arm visible (e.g., car windscreen wipers).

4. Measuring dynamic angles

The glossary of the *Australian Curriculum* gives the definition of angle as static (rays meeting at a point) but the unit of measure is defined in the curriculum dynamically as "an amount of turning". To extend children's understanding from the measurement of static angles to the measurement of dynamic angles, use a "dial plate" (Figure 3).



A 'dial plate' is made from a white plastic dinner plate (with a smooth surface underneath) and a long pipe cleaner. (a) Turn the plate upside down and with a permanent marker, mark two diameters that are perpendicular to each other. Make a hole in the centre. (b) Bend a long pipe cleaner in half and twist the two halves together. Bend this doubled pipe cleaner. (c) Thread the doubled pipe cleaner 'needle' through the hole in the plate so one end is on the top and the other end is underneath the plate.

Figure 3.

- Ask children to place the needle over one of the marked lines and write the word "baseline" along this arm of the angle.
- Show them a clock and the arm positions on the clock, numbered from 1 to 12. Tell them that on a clock, the baseline is an imaginary line which goes from the centre to the 12 position. Tell them that their dial plate is like a clock with the baseline drawn in at the 12 position. Ask what angle is made if the needle moves from the 12 position to the 3 position (90°), from the 12 position to the 6 position (180°), and from the 12 position to the 9 position (270°).
- Explain that the direction in which the arms of a clock turn is called the clockwise direction and the opposite direction is called the anticlockwise direction.

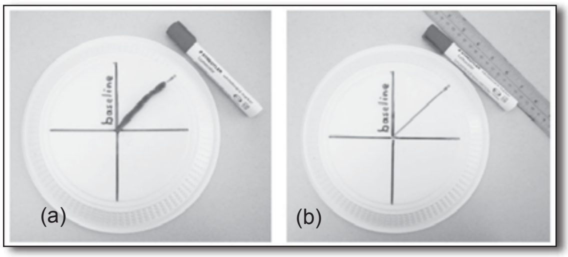
Table 1. Examples of free computer games to improve estimation of angle size.

Name and Website	Description	Arms Shown	Baseline/ Starting Position	Is Game Direction Dependent?	Interval of Degrees	Max. Degrees	Timed?
Kung Fu angles Level 1 http://www.bbc.co.uk/keyskills/flash/kfa/kfa.shtml	Ninja stands in "Circle of Death" and is attacked by enemy from any point on the circumference. Enter the number of degrees the ninja must turn clockwise to punch enemy.	Both. Vertex not shown.	Each turn begins facing 0° (vertical baseline).	Yes: must be clockwise.	Intervals of 10° marked, but numbers not shown.	360°	No
Kung Fu Angles Level 2 (Access not dependent on winning Level 1)		Both. Vertex not shown.	Any: each turn begins from end position of previous kick.	Yes: must be clockwise.	Intervals of 10° marked, but numbers not shown.	360°	No
Kung Fu Angles Level 3 (Access not dependent on winning previous levels)		Both. Vertex not shown.	Any: each turn begins from end position of previous kick.	Yes: must be clockwise.	Intervals of 10° marked, but numbers not shown.	360°	10 seconds.
Asteroids http://homepage.ntlworld.com/john-paul.g/swf/asteroids.swf	Rocket blows up incoming asteroids.	None	Any: each turn begins from position of previous shot.	Yes: clockwise or anticlockwise.	Begins 0° or 180°. Progresses to 0°, 180°, 90°; and then 60°, 120°, 180°.	360°	About 3 seconds per shot.
Alien Angles http://www.mathplayground.com/alienangles.html	Rescue aliens by sending a launcher. Student estimates angle the computer generates.	One initially. Player creates second.	Each turn begins facing 0°. Horizontal baseline.	No: anticlockwise by default.	Intervals of 1°: estimates must be accurate to the nearest 5°.	180°	No
Banana Hunt http://www.oswego.org/ocsd-web/games/bananahunt/bhunt.html	Drag the second arm to find bananas. Student estimates angle the computer generates.	Both	Horizontal baseline.	No: clockwise by default.	Intervals of 1°: accuracy is rewarded.	360°	No
Helicopter Shootdown–Tank Angle Measurement http://www.xpmath.com/forums/arcade.php?do=play&gameid=74	Drag the tank barrel to shoot helicopters, which appear at different distances from the tank.	Both	Each turn begins facing 0°. Horizontal baseline.	No: clockwise by default.	Intervals of 10°.	90°	120 seconds to shoot as many as possible.
UFO Attack: Space Angle Measurement http://www.xpmath.com/forums/arcade.php?do=play&gameid=75	Drag the barrel to shoot UFOs, which appear at different distances from the tank.					180°	
A Tangled Web http://www.mangahigh.com/en_au/games/atangledweb	Guide Itzi through a maze by turning a cog.	Both	Any	No	Intervals of 5°.	360°	No

5. Estimating angle size from any base line and in both directions

When measuring an angle with a protractor, children need to think about the direction of movement from the arm they have chosen to be the baseline. The ability to estimate enables them to determine whether their measurement is reasonable.

- Ask them to estimate a 45° angle by turning the needle in a clockwise direction from the baseline, mark its new position on the rim of the dial plate, then rule a line with a whiteboard marker to show the arm (Table 1). Give them a circular protractor with which to measure the angle made by the turn. There are often two scales on a circular protractor. Ask them why there are two scales, revealing that one measures the angle in a clockwise direction and the other measures in an anticlockwise direction from the baseline. Measure to find out which children made the best estimate of the 45° angle, and ask them how they made their estimate. These children would have realised that 45 was half of 90. Repeat this process with angles of 30°, 60°, 120° and 240°. This discussion should use fractions language (e.g., 30° is one-third of a right angle, 60° is two-thirds of a right angle, 30° and 60° together make a right angle).

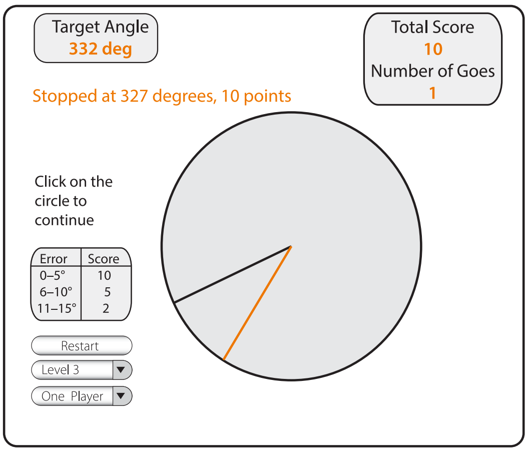


(a) The needle is moved to an estimated angle and its position marked on the dial plate.
(b) The needle is removed and the arm ruled in. The angle of turn is then measured.

Figure 4.

Discuss how the size of a reflex angle can be determined by measuring an acute angle or an obtuse angle and subtracting from 360°, and how the size of an obtuse angle can be determined by finding an acute angle and subtracting from 180°. The permanently marked arms on their dial plate should aid this discussion.

- Give them a semicircular protractor with which to measure some reflex angles. From discussing their methods, they should reach the generalisation that angles measured in opposite directions from the baseline add to 360° and angles in a straight angle add to 180°. The fact that angles on a straight line add to 180° can lead to them deducing that vertically opposite angles are equal. This finding could be supported using dynamic geometry software such as *GeoGebra*, thereby satisfying the Year 6 outcome in the *Australian Curriculum: Mathematics* that children "Investigate, with and without the use of digital technologies, angles on a straight line, angles at a point, and vertically opposite angles; use the results to find unknown angles (ACMMG141)".
- Estimating angles using digital learning objects (Figure 5) and online games. (Table 1) is an engaging way of practicing estimation skills and consolidating understandings of angles as turns. These games are freely available on the Internet. They differ as to whether one or both rays are shown, the location of the baseline, the direction of turning, the angle positions marked, and whether movement is through a full 360° or less.



This digital learning object at Cambridge University's NRICH website (http://nrich.maths.org/1235) tests one's ability to estimate the size angles made by turns from different baselines and in either direction. It can be played individually, in pairs or as a class, where the class tell the player out the front when to click on the circle as the one of the arms turns.

Figure 5.

6. Recognising that an angle can be limited or unlimited

The angles children encounter in computer games and by making "plate angles" are limited to 90°, 180° or one revolution. However dynamic angles in the real world are often unlimited. Hence discussion needs to be had concerning the maximum size of an angle.

- Discuss the maximum size of angles that can be made by different types of fans (including ceiling fans) and different types of doors (including revolving doors).
- Make pin wheels or spin different shapes around the plate and determine orders of rotation of the shapes, thereby integrating the topic of angles with that of rotational symmetry. A dial plate can be turned into a spinner (Figure 6).

To make the dial plate into a spinner plate, take out the pipe cleaner 'needle' and curl it around a pen or pencil pushed through the hole in the centre of the plate. Spin the needle around the pencil by flicking it with an index finger.

Figure 6.

Using a wheel or a video of a wheel, discuss the measurement of an angle made by its rotation. Because both arms of the angle need to be imagined, some point of reference (anywhere on the wheel) is required. Of the angle situations they researched, Mitchelmore and White (2000) found the wheel to be the situation children were least likely to associate with static angles (corners).

7. Finding and measuring difficult to locate dynamic angles in the real world.

The arms and vertices of angles are easier to see in computer games than in real world contexts. At this last stage, it is therefore necessary to return to real world contexts.

- Show videos or photographs of difficult-to-locate dynamic angles such as scissors opening. Children often mistakenly think the inner edges of the scissor's blades mark the arms of the angle, rather than two imaginary lines that meet at the pivot. Other examples of difficult angle contexts are a falling tree or a crowbar being used to lift a rock. Children need to be taught to look for the pivot.
- Replay the video of a fan dance, asking children to see if they can find angles they may not have noticed before, such as body angles and turns.

Children can further explore body angles using *Microsoft XBox Kinect* with the skeleton view (Figure 7) which displays their own body angles on the screen. They can also explore turns using programming software such as *Scratch* (a free downloadable program developed by the MIT Media Lab). With *Scratch* (Figure 8), children use a simple code to direct the movement of a sprite (a creature image) around a screen using turns of various sizes. There is an option on *Scratch* that enables the movement of the sprite to be traced, thereby helping children understand angles as directional change—a concept which many secondary school students find difficult.

We believe that the measuring of an angle is better taught initially in a static context and this concept then transferred to dynamic contexts. There are some angle concepts that are only applicable to dynamic contexts, namely that the vertex is the centre of rotation, that an angle measured in a clockwise direction from a baseline is not the same as one measured in an anticlockwise direction, and that the size of an angle is not limited to 360°. Dynamic contexts are also useful for establishing that one or both arms of an angle may need to be imagined. Difficult angle situations in the secondary school curriculum such as the measurement of slope from the horizontal and the measurement of change in direction from the existing path, require the use of imaginary arms.

This photograph shows a child playing with *Microsoft Xbox Kinect* using skeleton view. The skeleton on the screen has been drawn over the image of the child.
Sang1938/CC-BY-SA3.0

Figure 7.

In summary, videos and photographs of real world contexts over which angles can be marked, integrated with concrete resources, digital learning objects and engaging computer games, can enable children to make the crucial links between angle situations.

This photograph shows a 'sprite' in the program *Scratch*. Programming the creature's motion requires an understanding of angles as turns.
Wikipedia/CC-BY-SA3.0
A recent version of *Kinect* called *Kinect2 Scratch* allows data from the *Kinect* controller to be sent to *Scratch*.

Figure 8.

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