# Note-taking in a mathematics classroom

**Leong Yew Hoong, Tay Eng Guan, Quek Khiok Seng, Yap Sook Fwe, Tong Cherng Luen, Toh Wei Yeng Karen, Alexander Chia & Ong Yao Teck**
Nanyang Technological University, Singapore
yewhoong.leong@nie.edu.sg

## Introduction

We are a team of teachers and teacher educators who are deeply interested in helping mathematically-challenged students improve in their learning of mathematics. In Singapore, depending on their performance at the end of a nationwide Year 6 examination, students are channelled into three ability streams for Years 7 to 10: Express (60%), Normal Academic (25%), Normal Technical (15%). In the paper reported here, our focus is on students in the Normal Academic (NA) stream. We discussed at length the academic portrait of this targeted student group in order to be able to identify areas to attend to when designing intervention. Among others, we identified 'study habits' (SH) as one domain for careful consideration. By SH, we have in mind the set of behavioural norms that heighten productive study of mathematics, such as doing homework regularly, taking careful notes in class, being attentive when teacher is explaining, asking the right questions, among others.

Since the intervention was to be carried out during one unit (extending across five 45-minute periods) of classroom lessons, we decided to focus only on one aspect of SH: note-taking. The goal was to help students build the skill of note-taking as a way to become conscious of the thinking behind mathematical procedures. That note-taking influences academic success is also corroborated by other researchers (e.g., Carrier & Titus, 1981; Kiewra, 1987; Laidlaw, Skok & McLaughlin, 1993; Montis, 2007).

We identified the topic of 'simultaneous linear equations' as a suitable unit to infuse note-taking as a useful SH. After some revisions over a number of discussion sessions, we arrived at the note-taking template format as shown in Figure 1.

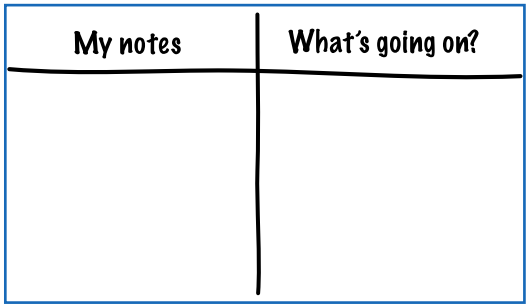


Figure 1. A template for note-taking.

## Using the note-taking template

This study involved two Year 8 NA classes in the Research School. In the first three lessons of the unit, the method of substitution was taught; the last two lessons were used to teach the method of elimination. Throughout the lessons in the unit, the teachers presented the working on the whiteboard using the template as shown in Figure 1. Under the 'My notes' column, the teachers wrote the usual working steps for the solution of simultaneous linear equations; in the 'What's going on?' column, a parallel commentary that served as a sort of meta-explanation was provided. In other words, the two columns presented a kind of division between what was 'conventional' and what was 'innovative' for the teachers (and the students); the 'My notes' column depicted what teachers were used to doing and what the students were used to seeing; the 'What's going on?' column was something novel to the teachers and students. Figure 2 is an example of what was written within these two columns in a typical lesson. For the rest of this paper, we share the outcomes of this intervention through the lenses of teacher reflections and student observations.

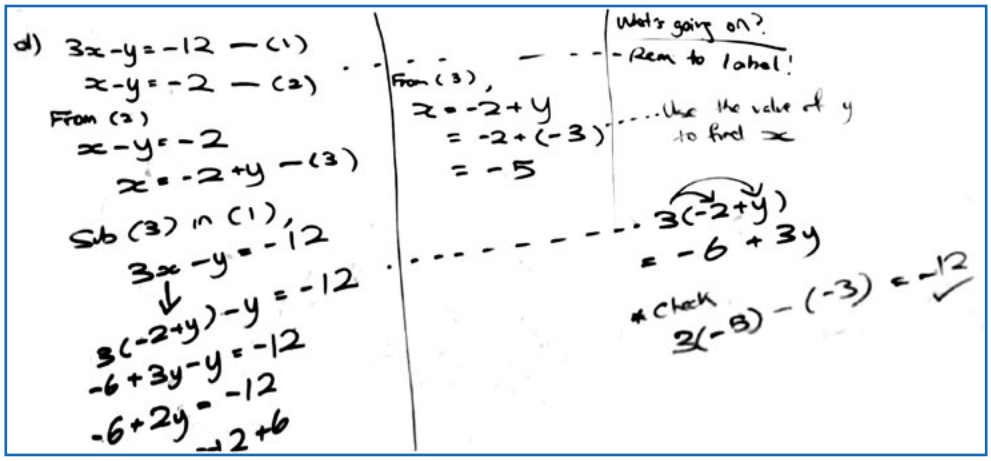


Figure 2. An example of how the template was used.

## Teachers' reflections

Both of the teachers of the respective classes were surprised that what they initially thought was a minor tweak to the way they presented working on the whiteboard made a significant difference to their instruction and to the students' learning (the latter will be discussed in greater detail in the next section). One teacher, George, expressed that although the original intention of the innovation was to encourage students' note-taking, he found the practice of including the 'What's going on?' column helpful for him and his teaching as well:

1. It "forced" him to think of the underlying basis for the working steps that he had become so familiar with, and thus unquestioned, over the years.
2. It allowed him to make visible his thinking processes which in the past had remained purely verbal and unwritten.

By writing it down, the underlying thinking attained a state of permanence which allowed students to refer to it more easily.

The other teacher, Thomas, found the innovation transformed his lesson-planning process. In accommodating a 'What's going on?' column, he found

that he needed to plan carefully what should be included on the whiteboard and how the information should be organised—a part of lesson-planning to which he had not paid particular attention in the past. By more careful selection and arrangement of the working and the rationale, he could better mediate the structuring of the lesson content for the students.

At the end of the module, a test was administered to check how well students had learnt the skill of solving simultaneous linear equations in two variables. The items in the test were similar to the ones they had learnt and practiced during the unit. When the teachers were asked how they thought the students had performed—based on the test results—both teachers expressed satisfaction at the outcomes. In email correspondence, George mentioned that, "[The results were] better than I expected," and Thomas, separately and not privy to George's response, said that, "It certainly helped most of the weaker students."

## Observation of students

Throughout the module, students were given handouts containing examples which also used a template consisting of the 'My notes' and "What's going on?" columns similar to the ones shown on the whiteboard by teachers (see Figure 1). The almost identical layout between the whiteboard organisation and the student handout structure was a deliberate design to provide ease in the students' note-taking efforts. The intention was that students, through taking down notes from the whiteboard presentation, would learn what was important to take special note of in the topic, as well as learn the importance of thinking about the rationale behind working steps.

The teachers and teacher educators who participated in this project sat in on some of the lessons in the module to observe how students took to the note-taking emphasis that was designed. In the next section of this paper, we will focus in greater detail on what we learned from close observation of student Shafiq throughout all the lessons. We chose to highlight Shafiq because his case demonstrated what we did not expect from a student whose instructional history revealed poor overall mathematics performance.

## Student Shafiq

In the first two lessons (over a continuous 90-minute duration), Shafiq was rather slow to pick up the note-taking exercise. There was a discernible lag time between what teachers wrote on the whiteboard and what he took down on his handout. At times, he only started to take down the notes written on the whiteboard only after the teacher had completed the entire working for the question. As to the 'What's going on?' column, there was even greater resistance. It was only after the teacher expressly instructed the class to "take down what is written in this column" that Shafiq complied. Also, glaring manipulation mistakes and conceptual errors showed up when he attempted examples on his own. Examples include:

1. For pairs of simultaneous equations $y = 6 - 2x$ and $-4x + 3y = 23$, he substituted the first into the second and wrote $-4(6 - 2x) = 23$.
2. He simplified $-3y - 2y = -y$.

In short, at the end of the first two lessons, Shafiq's responses in class fitted the typical academic portrait of low achievers in mathematics: poor

note-taking skills and a lack of prerequisite knowledge resulting in barriers to productive classroom learning.

In Lesson 3, Shafiq's struggles with the algebraic manipulation persisted, not helped by the examples that became harder as compared to the previous lessons. However, Shafiq had begun to take down notes more readily. In addition, he would fill in the 'What's going on?' column with "Label", "Find $x$ and $y$", and "Check" before the teacher prompted to do so. In one question somewhere in the middle of the lesson, he managed to obtain the correct solution—and did the check as well—after considerable struggle. There was a discernible smile of satisfaction on his face and heightened interest thereafter.

In Lesson 4, the teacher introduced the method of elimination to solve simultaneous linear equations. Right from the start of the lesson, Shafiq took the notes (including the right column) readily. It was clear that he found the method of elimination easier as it is less demanding in terms of algebraic manipulation compared to the substitution method learnt in the previous three lessons. As such, he proceeded smoothly and successfully through the examples given—at various points, he was even ahead of the teacher. He would begin attempts at a question by first writing "Label" and "Find $x$ and $y$" on the 'What's going on?' column before proceeding with the actual working.

Lesson 5 was essentially a homework-revision lesson. The teacher-observer was surprised to see that Shafiq's homework pages were completed successfully, with a substantial amount of notes written in the right column. This prompted a short exchange between the teacher-observer (TO) and Shafiq:

| | |
|---|---|
| TO: | Did you do these on your own? |
| Shafiq: | Yeah. |
| TO: | How did you manage? |
| Shafiq: | I looked at the notes. |

Shafiq did well in the test too. He scored 11 out of 15—much better than the teacher expected.

Looking at Shafiq's learning journey, we see that he started slowly with note-taking but gradually found that the notes helped him follow the working steps. That provided him with the foundation for success which kick-started the success–motivation–more success virtuous cycle. As educators, it is very encouraging to see such improvements in students.

## Conclusion

When we started off, we honestly did not know if the manner in which we emphasised note-taking would really help the students. We did a small 'innovation' as we did not want to make too drastic a change, especially when we were not sure if it would ultimately benefit the students. As it turned out, we were happy that a little tweak—by adding a 'What's going on?' column and a constant emphasis on the importance of note-taking—was helpful in at least two ways:

1. The teachers felt that space was made on the whiteboard to make visible their thinking process and meta-explanation.

2. This, together with the handouts which reflected the two columns, helped students keep track of and monitor the working process, as well as the sense-making behind the procedures.

This benefitted them in the learning of a topic, such as solving simultaneous linear equations, which students at this year level generally find challenging.

## References


Carrier, C. & Titus, A. (1981). Effects of note taking pretraining and test mode expectations on learning from lectures. *American Educational Research Journal, 18*(4), 385–397.

Kiewra, K. A. (1987). Note taking and review: The research and its implications. *Instructional Science, 16*(3), 233–249.

Laidlaw, E. N., Skok, R. L. & McLaughlin, T. F. (1993). The effects of note taking and self-questioning on quiz performance. *Science Education, 77*, 75–82.

Montis, K. K. (2007). Guided notes: An interactive method for success in secondary and college mathematics classrooms. *Focus on Learning Problems in Mathematics, 29*(3), 55–68.


From Helen Prochazka's

## SCRAPBOOK

### Programmed beauty

In an interview with Shapeways Community, French computer science professor, Francesco de Comite talks about his computer-generated art.

Five years ago, I began to work on picture distortions (using Gimp... I soon realized that programming images could give me more freedom in conceiving images, and I learned PovRay.

Ray-tracing software, and the ability to put some programming in their code, opens a wide range of possibilities as everything you can imagine can be brought to life as soon as you can write a program to generate it.

I am not an artist, I can't draw something beautiful on my own. I am a programmer; I can only render the intrinsic beauty of mathematics. I like to make my own renders of known mathematical shapes, and experiment with abstract formulas to see how I can use them to render mathematical gems.

What makes the beauty of a mathematical shape is, that there is some order, some symmetry, some hidden rule behind it. You then have a choice to either try to find the rule (and maybe reproduce the shape), or just stay and wonder. You don't need to be an astronomer to enjoy a starry night.