Beyond NAPLAN testing…
Nurturing mathematical talent

Kumudini Dharmadasa
University of Tasmania
kumudin@hilbert.maths.utas.edu.au
Anna Nakos
Temple Christian College, SA
anna.nakos@templecc.sa.edu.au
John Bament
O'Loughlin Catholic College, NT
johnbament@hotmail.com
Andy Edwards
Queensland Studies Authority
andy.edwards@qsa.qld.edu.au
Howard Reeves
Hobart, Tas.
reeves@netspace.net.au

The first issue of *The Australian Mathematics Teacher* in 2013 (AMT Volume 69 Number 1 2013) carried an interesting perspective on mathematical giftedness, *Mathematical giftedness: A creative scenario* by Yogesh Sharma. It provided a comprehensive research review of aspects of giftedness and some ideas for teachers to use with their students.

In this article, we, the contributing authors, provide strategies and suggest access to resources for teachers to nurture mathematical talent in their classrooms and schools. Furthermore, there are suggestions for school districts and jurisdictions to engage teachers in professional learning to increase their personal confidence and mathematical competence to equip them to deliver such programs for their students.

How are schools and teachers catering for their mathematically talented students? Are these students being exposed to the richness of topics described in the *Australian Curriculum: Mathematics* document? What resources are teachers using with talented mathematics students? The serious risk is that the curriculum is being restricted to those topics and ideas likely to be tested in the NAPLAN tests, and talented students are not being extended by school programs that do not focus on higher order mathematical goals and outcomes.

In all States and Territories, there is individual support for mathematically talented students and support materials for teachers keen to nurture such talent. Firstly, in each capital city talented students meet regularly—often weekly—in groups with volunteer teachers and academics, part of a network of such people fostered and supported by the Australian Mathematical Olympiad Committee (AMOC), itself part of the Australian Mathematics Trust (AMT) which amongst other activities conducts the annual Australian Mathematics Competition (AMC).

University of Tasmania lecturer Dr Kumudini Dharmadasa coordinates and teaches in the Hobart after-school student program four days a week for students at different ability/age levels. Kumudini, the AMOC Director for Tasmania, takes up the story of one of her 2012 Year 6 students.

> Thomas joined the primary group of our Maths Extended program when he was in Year 3. His analytical and computational skills clearly set him apart from his peers; it was obvious that this is a talent that has to be nurtured in order for it to reach its maximal potential.
>
> In Year 5 he grasped the concept of modular arithmetic with ease; he was able to prove symmetry, reflexivity and transitivity properties of congruence relations with no difficulty. He worked out, all by himself, "how to find the unit digit of 13^{947} using modular arithmetic".
>
> While the audience was a little tired at the end of a viewing of a video on *Infinity and Beyond*, Thomas, bright-eyed and excited, said, "Oh! How wonderful!" This enthusiasm and excitement about these concepts led him to make two (PowerPoint) presentations on those topics at our annual Presentation Evening, which was held in front of an audience that included mathematicians at the University.
>
> Last year he was fascinated about mathematical conjectures. Being asked to find out more about conjectures, he commented that he understood the ideas behind most of them (e.g., Twin Prime Conjecture, etc.), apart from the one which talks about 'irreducible polynomials'!

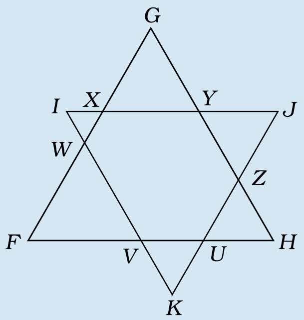
Anna Nakos is the South Australian Director of the Australian Mathematics Competition and teaches at Temple Christian College in Adelaide. Anna uses the AMT's Mathematics Challenge for Young Australians and Enrichment materials with talented students in her school.

> The AMC problem-solving questions meet the requirements of the *Australian Curriculum* giving students the opportunity to develop higher-order critical and creative thinking. Of course, there are other types of problem solving—such as open-ended tasks, experimental design investigations, or mathematical modelling of real world situations—and a balance of all different types of task is required.
>
> I give students the AMC questions as a whole paper from previous years about a month before the AMC. To make it more interesting, we have some time challenges. I ask the students to try to complete questions 1–10 in less than 10 minutes, keeping a record of their time for each paper to see if they keep getting a personal best! Then questions 11 to 15 in under 10 minutes, and questions 16 to 20 in under 20 minutes. Of the first 20 questions, we find who has completed the most consecutively correct questions and they achieve the 'Prudence Award' for that year's paper—a chocolate! Of the last 10 questions, the students are asked to choose any question they feel they can try and any that are answered correctly are celebrated in the class. I try to commend highly students who have a correct solution. I ask those students to show the class how they solved the question and ask for others who may have tried a different strategy. I often use the term 'budding mathematicians' to describe those who are developing their problem-solving skills.

J22 (2010)
Consider the sentence:
THIS IS ONE GREAT CHALLENGE IN MATHEMATICS
Every minute, the first letter of each word is moved to the other end of the word. After how many minutes will the original sentence first reappear?
(A) 422 (B) 880 (C) 1264
(D) 1800 (E) 1980

J25 (2009)
A palindromic number is a 'symetrical' number which reads the same forwards as backwards. For example, 55, 101 and 8668 are palindromic numbers. There are 90 four-digit palindromic numbers. How many of these four digit palindromic numbers are divisible by 7?
(A) 7 (B) 9 (C) 14 (D) 18 (E) 21

J29 (2009)
The sides of two equilateral triangles *FGH* and *IJK* meet at points *X*, *Y*, *Z*, *U*, *V* and *W* as shown, where $IJ \parallel FH$.



The perimeters of triangles *IXW*, *IJK* and *FGH* are 100 cm, 500 cm and 700 cm respectively, and $\angle GXY = 60°$. What is the perimeter, in centimetres, of ΔZHU?

J26 (2002)
One hundred and twenty 5 cent coins are placed in a row. Every second coin is then replaced with a 10 cent coin. Every third coin is then replaced with a 20 cent coin. Every fourth coin is then replaced with a 50 cent coin. Finally, every fifth coin is replaced with a dollar coin. The total value of the 120 coins in a row is now
(A) \$40 (B) \$44 (C) \$44.40
(D) \$46 (E) \$48

J11 (2004)
Which of the following represents 360 as a product of its prime factors?
(A) $3^3 \times 2^2 \times 5$
(B) $2^3 \times 3^2 \times 5$
(C) $2^2 \times 9 \times 10$
(D) $2^3 \times 5 \times 9$
(E) $2^5 \times 3^2$

The second way I use AMC questions is to support the topic I may be teaching at the time. There are lots of questions in the first 20 that fit the topics. However I leave these for the later practice of the full paper and focus on the harder questions 21 to 30. For example if I am teaching:

- geometry, students can try: J24 (2000), J21 (2001), J22 (2007), 26 (2008), J29 (2009);
- measurement: J24 (2008), J20 (2009), J27 (2010);
- ratio/rates: J26 (2000), J23 (2008);
- lowest common multiple: J26 (2000), J22 (2010);
- conditions of side length of a triangle: J21 (2010).

Other times, I specifically want to teach a mathematical idea and explain how this can be a useful tool in problem solving. The AMC papers have a wealth of questions that link mathematical topics such as counting techniques and questions where using algebra as a tool can help solve the question. Once I have taught the mathematics, we work on a variety of AMC questions that require this. For example, when teaching:

- writing numbers as powers of primes, and using the power to find the number of factors the number has, students then work on J11 (2004), J20 (2007), J22 (2000), J27 (2000), J30 (2001), J22 (2005), J10 (2010), J23 (2010), J24 (2010);
- using a tree diagram: J29 (2000), J30 (2004);
- using the digits of a number to write its value, e.g., the two-digit number *ab* has value $10a + b$. This technique is useful in J21 (2007) and J23 (2007), J25 (2009);
- the sum of the numbers 1 to *n*: J29 (2008).

Often as a class we discuss the importance of reading the question carefully and the fact that a question can be answered using different strategies. A great example to give to the class is J26 (2002). If students do not read it carefully, they miss the key word "then": "every 3rd coin is *then* replaced with..." After students work on this question we discuss the fact that it required a systematic approach working forwards, or it can be done working backwards with students demonstrating both ways on the board to the class. It is also interesting to note that the strategy 'solve a simpler problem' can be used. Asking students why looking at a set of the first 10 and then extending this to the 120 coins will not work, whereas looking at the first 15 does.

In addition to Anna's work with her students, she conducts professional development sessions in Adelaide for Independent sector teachers who want to develop confidence and mathematical competence to work with their talented students. As with the following professional development being conducted in Tasmania by the Department of Education, a focus is the use of Australian Mathematics Trust challenge and enrichment materials.

> In Tasmania for the last two years the Education Department, AMT and the University of Tasmania has supported a teacher professional development program to develop teacher confidence to provide a learning program and work with talented students in their classrooms and schools. In Hobart and Launceston, Years 5 to 8 teachers have participated in four half-day sessions to become familiar with the AMT Challenge and Enrichment materials and strategies to use them. Specifically, teachers were expected personally to undertake the Newton program materials (suitable for Years 5–8 students) with opportunities to discuss 'the mathematics' of this material and share solutions. The spaced learning model allowed teachers to study some of the content and solve related problems between sessions. The aim was to develop confidence in teachers to run enrichment programs in their own schools. We suspect it is lack of teacher confidence (and perhaps lack of familiarity) with 'the mathematics' that makes teachers reluctant to provide enrichment for their schools' talented students.
>
> At the first session, teachers were asked to 'name for themselves' some students in their class/school who should be being 'stretched' more by their mathematics program. By the time the group met for its second session (a fortnight later), some participating teachers had initiated enrichment programs in their schools!

John Bament is the NT Director of the Australian Mathematics Competition and teaches at O'Loughlin Catholic College in Darwin.

> Problem solving is a part of life. Solving issues such as: what type of mortgage to get; the transporting of kids to their sporting, academic and social engagements; resolving a problem at work; are examples of real-life problems where logical and mathematical skills and reasoning can be of assistance. A lot of the answers cannot be found immediately—although we sometimes wish they could! It can take time to analyse problems systematically to filter through the various scenarios.
>
> This is why I feel that problem solving is so important for our students. It is not just about mathematics, it is about taking the time to understand and analyse a problem, reflect on the skills they have, or need to acquire, to produce a viable and relevant solution.
>
> Many teachers and students see activities such as the AMT Mathematics Challenge for Young Australians (MCYA) as being suitable for only their most able students, and this is true to some extent: the Challenge and Enrichment Stages target the top 20% of Year 5–10 students. I feel that it is also important for all students to have some exposure to such questions. In my experience, students regularly surprise themselves by having the ability to solve a problem, often in various and ingenious ways.
>
> Teachers can sometimes feel intimidated by such mathematical problems due to not having the confidence or skill set to support their students in undertaking such activities. This is what workshops like the Nurturing Mathematical Talent, which are occurring in Tasmania and the Northern Territory in Terms 2 and 3, aim to address.

In Darwin, Dr Ian Roberts has kindly offered to run 'teacher only' followed by 'students and their teacher' workshops to explore the use of enrichment materials. The teachers work collaboratively to develop their knowledge and confidence in using such resources, and then work with their students in a supportive environment.

Problem solving should not be seen as an add-on to regular classroom teaching but an approach to enrich students learning and develop necessary skills for the future.

Finally, here is a suggestion from Andy Edwards (Queensland Studies Authority, following 32 years in secondary mathematics classrooms) about how he used Australian Mathematics Competition questions in his classroom:

When I taught elective maths classes to Years 9 and 10 (extra four periods per week for a semester with a differentiated syllabus from their normal six periods per week maths classes), I would use old AMC questions in my problem-solving group work.

I had cut-and-pasted about six or seven different question-sets, each with about 13 questions, complete with all their multiple-choice options.

I would break the class into 6–8 groups of three (depending on how many students were in the class) and give them each the same set of 13 questions to do. Three students per group was a good number because it meant everyone in the group had to pull their weight at least to some extent.

The first six questions in the set were from the easy part of the AMC paper (Q1–10) and paid three points each and nothing for wrong answers. The next five questions were Q11–20 type questions and were worth 4 each; the last two were from Q21–30 and were worth 5 points each.

The groups had about 30 minutes to come up with its list of 13 agreed-upon responses which they had negotiated, discussed and argued. The lists were then collected and redistributed for correction among the other groups; that is, the answer list (e.g., 1. D, 2. E, 3. B, 4. B, 5. C, 6. A, etc.) produced by 'Anna, Andy and Bruce' was given to another group 'Janine, Kevin and Giovanna' for marking.

The correct answers were read out, with interesting questions discussed; every group would be given a score and the list returned to the 'owners'. Best group got this week's set of mini-Mars Bars (or whatever) and the corrected lists went back to the teacher.

That would usually take up until the end of the period but it was what happened next that made things interesting. Every student from each group would be given their group's score which would be added to their personal cumulative score from previous rounds. So if 'Anna, Andy and Bruce' got $5 \times 3 + 4 \times 4 + 1 \times 5 = 36$ and the other three questions wrong, Anna's cumulative score would go up from 103 to 139, Bruce's would go up from 78 to 114 and Andy's would go up from 56 to 92. After everyone's score was updated, I would find the average cumulative score and make up next week's set of groups so that each group had an approximately equal total cumulative score.

So if the class's average was 120, the next week Anna might find herself in a group with students whose scores were 99 and 123, Bruce might go in with kids with scores of 122 and 126 and Andy would have the company of a 153 and a 112.

A spreadsheet that I produced to keep the scores was updated for the following week.

The result was that, in general, all students found themselves partnered by two students who they had not worked with before, but the overall problem-solving power of the group was approximately equal (according to one measure of this anyway.)

Students learned two things from this:

(i) socially, they had to learn how to learn to get on with fellow students they might not otherwise ever have had much to do with;

(ii) more powerfully, they were exposed to other students' ways of thinking about solving problems and they had to articulate why their answer was right (or be convinced and accept why it was not).

These are things students may not often get many chances to do in regular classes and yet they are extremely good ways to strengthen students' ability, versatility and confidence in the often-neglected area of problem-solving.

Students remained interested because the weekly prizes got shared around a fair bit and those who were not very good at problem-solving would often get better explanations from the better students in their group than they ever would have from me, their teacher. This meant that every student had a chance at the weekly prize, even if they were a bit of a 'tail-end Charlie'.

At report times, I would look at my spreadsheet and give the best 10 students A for their problem-solving; the next best—however many—B; and the least-deserving cases, Cs.

Nobody ever failed because all of them—even the least-numerically successful—had solved some problems; and become better at solving problems than they had originally been.

All thanks to some beautiful, carefully-selected questions from the AMC!

The available time for teachers to plan programs, develop teaching strategies, choose materials and plan lessons is limited. Do we allocate sufficient time to plan worthwhile extension programs to challenge what might be a relatively small group of mathematically-talented students in our classes? We believe that the approaches and materials described in this article provide a starting point and a source of teaching resources for teachers facing this challenge—the challenge of catering for students with mathematical talent. We encourage teachers to take up the challenge and give these ideas and materials a go.

From Helen Prochazka's

Scrapbook

"Many intelligent people after an average of 1500 hours of instruction over eleven years of schooling, still regard mathematics as a meaningless activity for which they have no aptitude… It is difficult to imagine how a subject could have achieved for itself such an appalling image as it now has in the popular mind… To think that all our effort has led to a situation of fear and loathing is depressing."

A. Orton & G. T. Wain in Issues in Teaching Mathematics (1994)