Enhancing the teaching and learning of mathematical visual images

Lorna Quinnell
Charles Darwin University
lorna.quinnell@cdu.edu.au

The literacy demands of mathematics are very different to those in other subjects (Gough, 2007; O'Halloran, 2005; Quinnell, 2011; Rubenstein, 2007) and much has been written on the challenges that literacy in mathematics poses to learners (Abedi & Lord, 2001; Lowrie & Diezmann, 2007, 2009; Rubenstein, 2007). In particular, a diverse selection of visuals typifies the field of mathematics (Carter, Hipwell & Quinnell, 2012), placing unique literacy demands on learners. Such visuals include varied tables, graphs, diagrams and other representations, all of which are used to communicate information.

The importance of mathematical visual images is indicated by the introductory paragraph in the Statistics and Probability content strand of the Australian Curriculum, which draws attention to the importance of learners developing skills to analyse and draw inferences from data and "represent, summarise and interpret data and undertake purposeful investigations involving the collection and interpretation of data" (Australian Curriculum Assessment and Reporting Authority [ACARA], 2011, para. 6). It also indicates the importance of learners developing abilities to critically analyse statistical information.

Teachers cannot assume student knowledge of the different semiotics systems in the subject. In order to enhance learning and to cater for needs of diverse student groups, it is the responsibility of mathematics teachers to focus some teaching on this area of mathematics. This should be seen as part of the teaching of mathematics, not an extra that will take time away from the day to day teaching of the subject. In this paper, literacy pedagogies to scaffold the teaching and learning of mathematical visual images are considered. Further pedagogies to aid the establishment of links between different representations are described. The pedagogies are based upon their previous use in literacy and/or mathematics classrooms.

Strategies to scaffold learners' ability to decode and encode mathematical visuals

Although much has been written about the difficulties that mathematics learners have with decoding and encoding mathematical visuals and matching visuals with data sets (Baker, Corbett & Koedinger, 2001; Diezmann, 2005; Friel, Curcio & Bright, 2001; Lowrie & Diezmann, 2007, 2009; Wall & Benson, 2009), there appears to be little in terms of comprehensive guidelines to assist teachers and learners with these processes. This article provides a suggested list which teachers can follow to scaffold decoding and encoding of mathematical visuals, based on scholarly work done in the fields of literacy and mathematics.

Based on the Four Resources Model from literacy education (Freebody & Luke, 2003) the heuristic in Table 1 has been constructed. The *Four Resources Model* separates literacy into four competencies: code breaking, meaning making, text using, and text analysing. Code breaking focuses on syntax and includes cracking the codes of the text, including pronunciation, sentence structure, abbreviations, spelling, keys, labels, scales, and colours, for instance. Meaning making focuses on semantics or making meaning of the entire text, with reference to prior knowledge, identification of key messages, and drawing of inferences from the text. Text using includes identification of the purpose and audience of the text for instance, and includes use of varied text such as everyday text of different formats. Critical analysis focuses on uncovering the bias or possible misrepresentation of messages conveyed by text. Code breaking and meaning making have been combined in the heuristic because the codes such as the size, order, and positioning of the elements of the text, contribute to the meaning of the text. It must be noted that the order of the categories is not crucial and some of the elements may not apply to certain mathematical visual images.

The list in Table 1 draws attention to the wide range of competencies needed in terms of encoding and decoding text. It has been tried and tested as part of an educational doctorate, appearing to offer the small number of pre-service teachers, who were the participants, scaffolding in terms of decoding and encoding mathematical visuals. However the effectiveness of the use of the list does need to be investigated further and on a larger scale.

The Four Resources heuristic in Table 1 was used in the research to guide understanding of the graph in Figure 1. In terms of code breaking and meaning making, brief skimming of the graph allowed the participants to identify the key message of the graph, which was to depict emergency waiting times of 2773 patients in Australian hospitals. Prior experience of hospital emergency rooms aided the participants' comprehension of

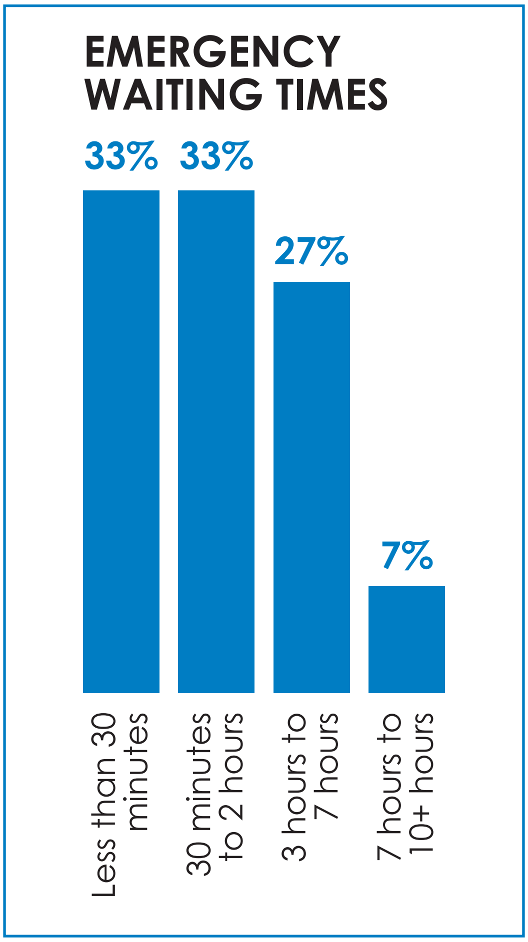


Figure 1. "Public health system at 'breaking point'". Source: Quest Newspapers YourVote online survey of 2773 respondents in Australia (Davis, 16 February 2012).

Table 1. Four Resources Model for decoding/encoding mathematical visual images.

Four Resources	Key points
Code breaking and meaning making	• When decoding mathematical visuals focus on overall meaning of the visual by skimming then move on to scanning and close reading to identify specific details of the codes and conventions of the text such as titles, labels, scales, keys, and for instance. • When encoding data, ensure careful choice of the type of representation. Note that a bar graph is not always an appropriate choice. Matching graphs with data sets can be a useful scaffolding step. • Check students' understanding of scales • Draw on prior knowledge of familiar visuals to scaffold understanding of unfamiliar visuals • Give special attention to deciding whether directionality in reading a visual is important • Consider whether information in a visual needs to be understood or used in combination with information in prose for instance • Consider information gained by reading directly from a mathematical visual, information gained by reading between the data such as by comparing data points or investigating relationships in the data and information gained by reading beyond the data, which can include making predictions, inferences, extrapolations (Monteiro & Ainley, 2003). Use of all three categories encourages deeper meaning-making of the text. Use of graphs with no scales and units on the axes can encourage learners to focus on the meaning of a graph and develop abilities to read between and beyond the data (Ontario Ministry or Education, n.d.). • Focussed teaching is needed in which learners are exposed to varied and repeated practice, including time spent on more complicated graph-types such as stem-and leaf graphs
Text using	• Think about the context of the mathematical visuals and use diverse, carefully chosen, and practical visuals or data from various sources including everyday sources (Watson, 2000; Watson & Callingham, 2003). Examples include graphs from the media such as graphs showing banking information, exchange rates, or bus timetables. Choice of context plays a crucial role in reader's comprehension of the content of visuals, which may be affected by the reader's prior experience and cultural background (Avgerinou & Petterson, 2011). Careful consideration of the context can minimise problems that may be related to the contexts that underpin graphs (Monteiro & Ainley, 2003).
Text analysing	• Consider the purpose of the text and identify possibly misleading elements or factors which contribute to misleading messages being given. • In terms of encoding of mathematical visual images, Mackinlay's (1999) suggestion of the importance of two criteria to ensure a complete and unbiased representation, are helpful. He stated that all facts need to be represented and only given facts should be represented. This is a first step in terms of insuring an unbiased representation but fails to highlight all the factors that may cause bias in a representation. • Extend understanding by including construction of representations for varied purposes

the graph. Close reading allowed identification of codes which needed to be understood. These included the layout of the bar graph with percentages given. Each bar corresponded to a range of emergency waiting times given in minutes or hours with the range of waiting times extending from zero to 10+ hours. The heights of the bars showed the percentage of patients who waited in the range of times displayed at the bottom of each bar. For instance, 7% of patients waited more than 7 hours and 66% of the patients were treated within the first 2 hours. It was evident that such meaning making took place in a random fashion, unlike much reading in which text is processed from left to right. When combining the information in the title and graph, the message was that emergency waiting times were unacceptable. However, moving on to text using, the participants realised that the graph, which was printed in a newspaper, may have conveyed misleading messages. In terms of critical analysis, it was realised that the emotive heading "Public health system at breaking point", stressed the unacceptable nature of emergency waiting times. However, as noted by the participants, only 2773 patients in Australia were surveyed in an online survey, which was not likely to have drawn a representative sample of all patients in emergency in Australia. In fact, it may have been more likely to have drawn responses from dissatisfied patients. Closer perusal of the graph indicated discrepancies such as the fact that although the percentages added up to 100%, there appeared to have been no patients who waited between 2 and 3 hours. And, although the bars were the same width they represented different time periods, thus distorting the data. It was also unclear why the last bar was labelled 7 hours to 10+ hours not simply 7+ hours. Questioning the representation further, it was unclear how many of the patients, in particular those who waited the longest were in fact emergencies.

In order to scaffold understanding of such a graph, posing questions that read the data, read between the data, and read beyond the data, can be used (Monteiro & Ainley, 2003). In terms of reading the data, learners may be asked to identify the percentage of patients who waited between 30 minutes and 2 hours. Slightly more complex, in terms of reading between the data, a question could be asked such as: What percentage of patients waited more than 3 hours? At an even more complex level, in terms of reading beyond the data, students may be asked to critically analyse the data for instance. This is discussed at length in the next example.

The graph in Figure 2 depicts the growth of money in a banking account. In terms of code breaking, the labels, scales, two dimensional graph, and use of bars the heights of which represent the sum of money in 2010 and 2011, need consideration. Meaning making leads to an understanding of the total message of the text built upon prior knowledge of banking and interest rates and on information taken from the title, labels, and graph. Such a graph could be used to advertise a banking account and a possible inference would be that the depicted growth of money between 2010 and 2011 would continue into future years, which is not necessarily the case. As indicated both in the aforementioned ACARA statement and in the heuristic above, critical analysis of the graph is important. The graph highlights some of the factors that contribute to bias in messages given to the reader. These include an emotive title, three dimensional bars of different colours and widths, information given for only a two-year period, and a y-axis which begins at a non-zero number. In Figure 2 it appeared that the second bar was twice the height of the first, which was obviously a misrepresentation of $100 and $105. This difference was exaggerated by use of three dimensional bars and use of different widths and colours for the bars. Such distortions can be found in graphs designed for advertising.

In order to scaffold student knowledge and understanding of mathematical visual images, varied practice is required. Pedagogies that aid such knowledge construction are described below. In particular, pedagogies which make links between diverse representations have been chosen.

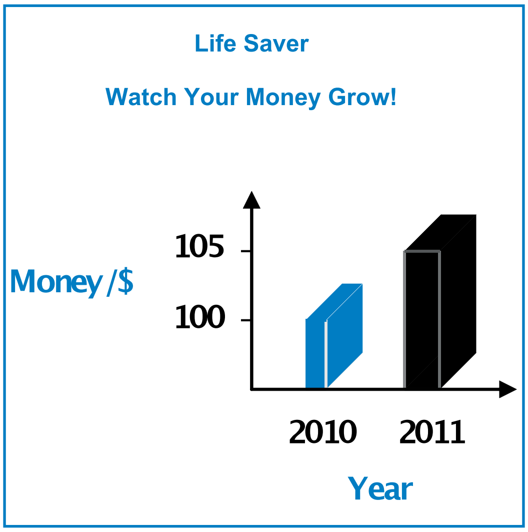


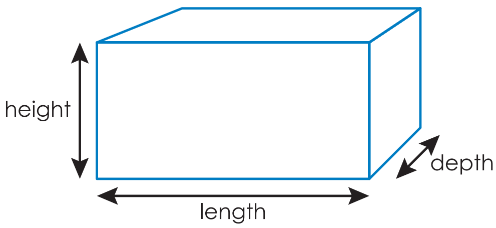
Figure 2. Example of bias in a statistical graph.

Strategies to aid links to be established between different semiotic systems

Mathematical text contains many unique features and is often a combination of various elements such as written text, tables, graphs, and symbolic notation (e.g. Figure 3). Learners require the ability to shift their focus between such semiotic systems and integrate meaning from different parts of the text (Bossé & Faulconer, 2008; Lemke, 2003), including uncovering meaning in visuals which may not be in the written text, for instance (Unsworth & Chan, 2009). Often the process occurs in a non-sequential fashion, using scanning to locate and comprehend titles, captions, and labels, and to identify key points (O'Halloran, 2005). Thus, practice is needed to encourage learners to make links between the different semiotic systems.

A postage company uses the formula below to calculate parcel sizes.

size = depth (*d*) + height (*h*) + length (*l*)



The maximum size allowed for parcels is 110 cm. Suppose you need to post the following four parcels. Are any of them too big? Explain.

Parcel	*d*	*h*	*l*
A	35	45	35
B	25	20	45
C	20	50	40
D	25	35	35

Figure 3. Example of combination of semiotics systems in mathematical text.

Practice with using different semiotic systems and shifting focus between systems can be gained in many ways including completion of questions similar to that shown in Figure 3. Another powerful way of linking knowledge of varied representations is what has been called a verbal and visual word association diagram (Barton, Heidema & Jordon, 2002; Gay, 2002). Such diagrams consist of a rectangle divided into four sections: the first giving the word or phrase; the second a context, characteristic, or connection; the third a diagram; and the forth an informal definition. The actual categories are flexible with the possibility of using tables, graphs, examples, non examples, associations, and attributes.

In the research, verbal and visual word association diagrams were used to represent concepts such as hypotenuse, prime number, or square based pyramid. Categories including the term, a diagram, an informal definition, and an association were used. The participants filled in the four categories, then collaborated with others and made use of dictionaries such as the Origo Handbook (Anderson et al., 2008) or the online mathematics dictionary (Eather, 2011), to modify and improve on their first efforts. An example of one participant's verbal and visual word association diagram to depict the word hypotenuse is shown in Figure 4. When verbal and visual word association diagrams were used to represent the concept prime number some of the participants needed to modify their examples to omit the number 1 and modify their definitions after referring to the mathematical dictionary. At first a diagram for prime number posed problems until, with scaffolding, the participants realised that prime numbers can only be represented by a one dimensional array. Similarly scaffolding was needed in the association category in some instances, for example an association for square based pyramids could be the pyramids in Egypt. The success of the activity was that the participants were able to construct new knowledge with the help of scaffolding by building on their original knowledge and linking ideas, in line with constructivist learning (Bruner, 2006a, 2006b; Vygotsky, 1978). Although such strategies help to structure information by building on prior knowledge, and make links between different representations, including verbal, symbolic, and diagrammatic representations, they do rely on prior knowledge of a concept (Ewing Monroe & Orme, 2002).

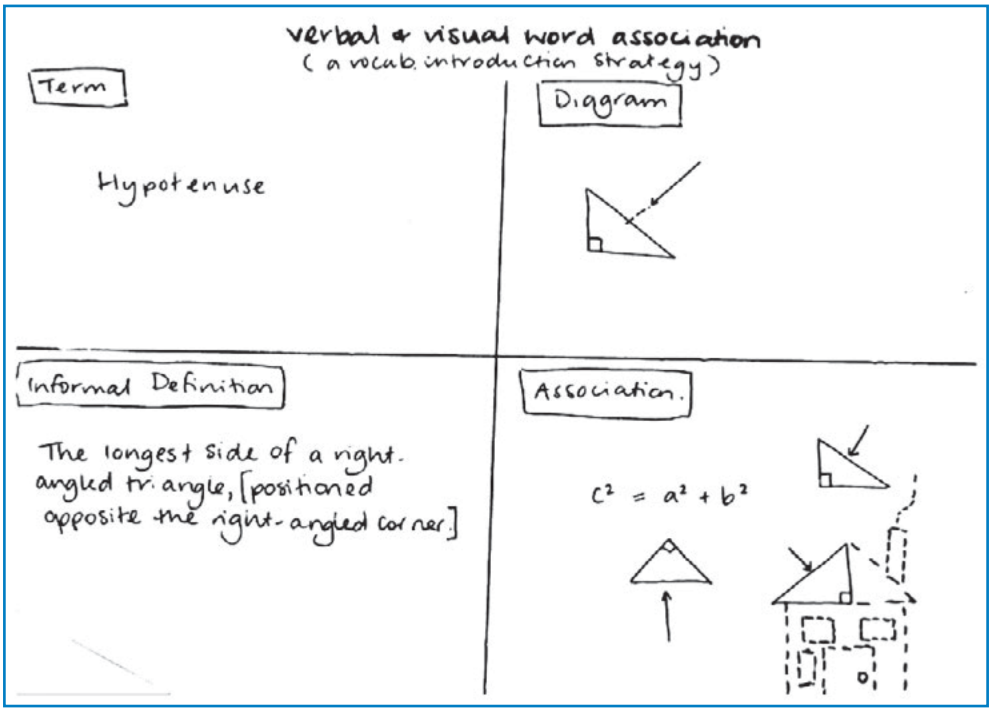


Figure 4. Verbal and visual word association diagram showing the concept hypotenuse.

Students can also be required to represent information in multiple formats. For example, suppose the price of apples is $2.50 per kilogram. This information can be represented verbally, in a table or graph, or as a formula. Another useful idea for introducing students to multiple representations is seen in Chick (2004). In this case students were supplied with multivariate data, making possible multiple varied representations to represent the data. In Chick (2004) the list of data included name, favourite activity, number of fast food meals eaten per week, and number of hours of exercise per week. As discussed by Chick, such data can encourage rich discussion and diverse representations, in the process of finding and displaying messages and relationships in the data in convincing ways.

Conclusions

Literacy is acknowledged to play a vital role in the learning of mathematics, in which learners "read, view, analyse and interpret the mathematics represented by text, pictures, symbols, tables, graphs and technological displays… [and] communicate in various ways for example, orally, visually, electronically, symbolically and graphically" (Queensland Studies Authority, 2004, p. 5). The uniqueness of the literacy demands of mathematics, compared to the literacy demands of other subjects, points to the need for focused teaching in this area of mathematics. We owe it to our students to focus attention on the many semiotic systems which constitute mathematical text, including the use of symbols and abbreviations, the use of varied visuals, and the use of written and oral text. Some strategies, which can be used to scaffold the teaching and learning of literacy in mathematics, have been described in this article, with particular focus on mathematical visual images. These can be used by teachers to enhance understanding of literacy in mathematics, and thereby enhance the learning of mathematics.

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