

Promoting reasoning through the magic V task



Leicha A. Bragg
Deakin University
<leicha.bragg@deakin.edu.au>

Esther Yook-Kin Loong
Deakin University

Wanty Widjaja
Deakin University

Colleen Vale
Deakin University

Sandra Herbert
Deakin University

Reasoning in mathematics plays a critical role in developing mathematical understandings. In this article, Bragg, Loong, Widjaja, Vale & Herbert explore an adaptation of the Magic V Task and how it was used in several classrooms to promote and develop reasoning skills.

The importance of reasoning

Mathematical reasoning is the foundation of deep understanding (Bragg, et al., 2013). Adaptive reasoning is viewed as “the glue that holds everything together, the lodestar that guides learning” (National Research Council, 2001, p. 129). The importance of reasoning is noticeable in its inclusion as an explicit learning requirement of many nations’ curriculum documents (Loong, Vale, Bragg & Herbert, 2013) including the *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2013) where it is one of four designated proficiency strands defined as:

Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false and when they compare and contrast related ideas and explain their choices. (p. 5).

Opportunities to reason should commence at the earliest opportunity for children and as they progress, their reasoning should become more

sophisticated when supported by teachers through a systematic approach (Stacey, 2013). The Mathematics Reasoning Research Group at Deakin University developed the *Mathematical Reasoning Professional Learning Research Program* [MRPLRP] to support and further teachers’ knowledge of reasoning to foster the critical engagement of their students in mathematical reasoning. In this article, we describe our adaptation of the Magic V task (<http://nrch.maths.org/6274>), in the second of two reasoning lessons demonstrated in three Victorian primary schools and one Canadian elementary school in our project to assist primary teachers to promote and support mathematical reasoning in middle and upper primary classes. See Bragg, et al. (2013) for a full description of the first reasoning lesson called “What else belongs?”

The Magic V task

The Magic V is a task which affords children an opportunity to develop and test conjectures and form generalisations (Widjaja, 2014). The Magic V explores mathematical reasoning giving children the opportunity to, “Investigate and use the properties of odd and even numbers (ACMNA071)” (ACARA, 2013, p. 30). Specific learning objectives addressing reasoning for this demonstration lesson within the MRPLRP included but were not exclusive to: use oral language for equivalence and equivalent number sentences to record, explain and justify solutions;

compare and contrast to generalise and develop ideas (conjectures); test ideas (justifying and proving); trial to form conjectures (inductive reasoning); develop a logical argument based on an understanding of equivalence and properties of odd and even numbers (deductive reasoning). There was also an emphasis on developing mathematical language, such as “equals” and “does not equal”, as the children explored and explained properties of odd and even numbers. Skills associated with problem-solving, such as applying systematic trial and error to seek solutions, were also supported in this lesson.

The lesson commences with the teacher referring to the two Vs on the board (see Figure 1a and 1b) and inviting the children to share what is the same about the Vs.

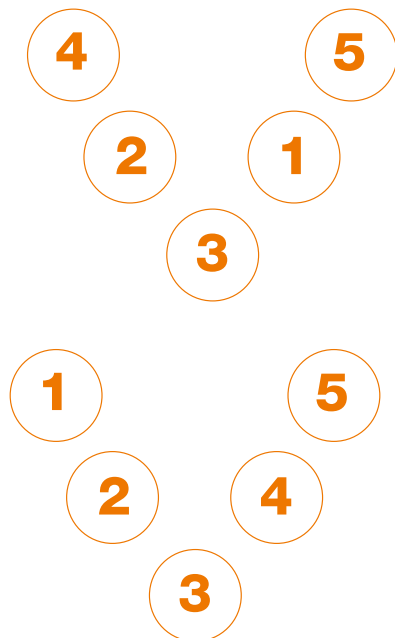


Figure 1a and 1b. Magic V and non-magic V.

Typical responses are “Number 1 to 5 are used in both Vs”, “5, 3 and 2 are in the same spots” and “All the numbers add up to 15.” Once the sameness of the two Vs is exhausted, the teacher invites the children to share what is different about the two Vs. The typical initial response is, “4 and 2 are in different spots”, and is often followed by a student noticing, “The arms on one V don’t add to the same as the arms on the other V.” Such a response helps draw the children’s focus to the essence of what makes one V magical and the other non-magical, that is, a Magic V is when the numbers on one arm add up to the same as the numbers on the other arm. For example in Figure

1a, the arm $4 + 2 + 3$ is equal to the sum of the arm $5 + 1 + 3$; both arms are equal to 9. Whereas in Figure 1b, the arm $1 + 2 + 3$ is not equal to the sum of the arm $5 + 4 + 3$.

If the students have failed to notice the symmetry of the arms in the Magic V, the teacher can offer enabling prompts such as, “I think there is something interesting going on with these Vs. We have been thinking about the numbers just by themselves. Can you think about ways we might add these numbers? [Pause] Can you add the numbers in the arms? What do you notice?” Once the total of the arms have been explored the teacher points to the Magic V and explains this is the preferred V, and the Magic V will be the focus of the lesson and removes the non-magical V from the board. This discussion establishes the rule for testing or verifying whether a V is magic or not and links to specific actions in the *Australian Curriculum: Mathematics* definition of reasoning, such as, “analysing, proving, evaluating, explaining” (ACARA, 2013, p. 5)

The teacher poses the challenge to the children, “I wonder if we can make more Magic Vs with the numbers 1 to 5?” In pairs, the children are given a set of numbers, a Magic V mat (see Figure 2) and a record sheet to record all the Magic Vs they discover. The teacher reminds the children of the classroom norms for working together, such as sharing materials; the task of recording; explaining their thinking to each other; and checking and convincing each other that they have found a Magic V before they commence finding the next one.

The teacher records the various strategies adopted by the children to collate the Magic Vs. Key observational points for the teacher to note is if the children are specialising, that is, trying out particular cases.



Figure 2. Children discovering multiple Magic Vs with the set of numbers 1 to 5.

Students will initially explore possibilities with placing the numbers into the circles, checking if the two totals are equal, then moving specific numbers around to ensure the two arms balance. The children may begin to notice relationships and consider switching positions of the numbers to change the order but keep the same combination; that is, the children may realise the commutative property for addition as they explore all possible Magic Vs with the same number at the vertex. Conversely, the children may consider different combinations and begin to generalise from the specific examples thus employing inductive reasoning (Holton, Stacey, & FitzSimons, 2012), that is, notice common attributes in the combinations or placement of numbers in the vertex. For example, that their Magic Vs have an odd number at the vertex. With a systematic approach children can find 24 different Magic Vs from the set of numbers 1 to 5. You can create eight Magic Vs with the number 1 at the vertex and the arms equalling 8, eight Magic Vs with the number 3 at the vertex and the arms equalling 9, and eight Magic Vs with the number 5 at the vertex and the arms equalling 10.

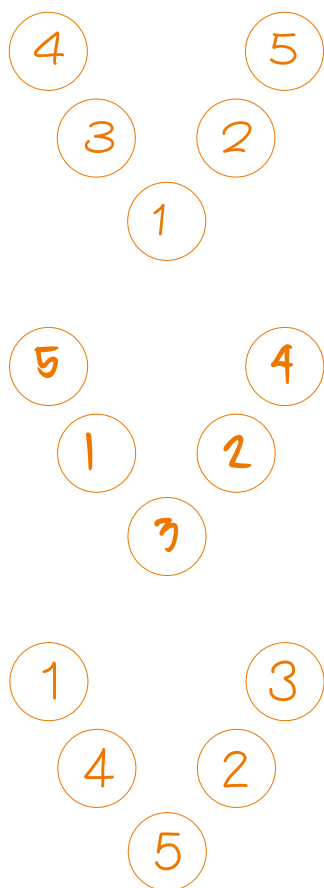


Figure 3. Children's Magic Vs on the board.

After ten minutes has elapsed the teacher makes an announcement, "I am going to give you a large blank V sheet [the same style as Figure 1a but without the numbers] and I would like you to record one of your Magic Vs on the sheet with a coloured marker and place it on the board. Check that you have a different Magic V to one already displayed." Once each pair has added their Magic V to the board (see Figure 3) the whole class come together to share their experiences and what they have noticed about the Magic V.

The teacher asks a pair of students, "Point to the Magic V you have placed on the board. How do you know this is a Magic V?" Depending on the response of the children, the teacher either notes on the board examples of number sentences with or without the vertex included; such as $3 + 4 = 5 + 2$ or $5 + 1 = 4 + 2$ without the vertex, or alternatively $3 + 4 + 1 = 5 + 2 + 1$ or $5 + 1 + 3 = 4 + 2 + 3$ with the vertex included. The absence of the vertex in the first number sentences format emphasises that the number in the vertex could be ignored, thereby drawing the focus of the children's noticing to the arms.

The teacher asks children to share their process for selecting numbers in particular spaces. "Share with us. How did you find your Magic V?" Anticipated responses from the children include, "We mixed the numbers around until we found one", or "We switched the numbers in order like this...". The second comment denotes a systematic approach to the discovering and recording process. The teacher now shifts the children's attention to the board to examine the Magic Vs created by the class, "Look at all these different Magic Vs. What do you notice about the Magic Vs we found?" The teacher records the children's observations and conjectures on the board. Responses include: "These two Magic Vs are the same but with the numbers swapped," "All the numbers in the vertex [bottom/ corner/ point] are 1, 3 or 5", "All the numbers in the vertex are odd numbers," "It's impossible to have 2 [or 4] in the vertex," "It's impossible to have an even number in the vertex." Noticing what is the same about the Magic Vs is a necessary step for the children to form conjectures about the properties of Magic Vs as illustrated in these statements. The development of students' capacity to form and test conjectures is a central component of reasoning (Carpenter, Franke, & Levi,

2003). When a child notices that the vertex only has 1, 3 or 5 (odd numbers), the teacher moves the children's examples on the board to collate those with the same number in the vertex [1, 3 or 5] together thus forming three groups.

The teacher challenges the children to consider this conjecture, "[Child's name] noticed that all the numbers at the vertex are odd. I wonder why all our examples are odd? I wonder if they could be even?" The teacher shares the following questions on a worksheet with the children, "Sam said 'It is impossible to make a Magic V with an even number at the bottom with the set of numbers 1 to 5'. Is Sam right? Explain why or why not? [You can use sentences, number sentences and drawings in your explanation.]"

The children return to their desks to test Sam's conjecture while the teacher roves to observe the children's reasoning and to provide enabling prompts if required. Enabling prompts may consist of the following comments and actions. "What happened when you tried 2 or 4 at the bottom of the V? I wonder why 2 and 4 does not work?" The teacher covers the number at the vertex of the V to focus students on the 4 other numbers and asks "What do you notice about these numbers in the arms of the V?" [pause] "What do the other 4 numbers add up to?" [pause] "How is this different when it is an odd number at the bottom/vertex?" The teacher may remove the numbers in the arms off the sheet, leaving a number at the vertex, and place the four remaining numbers in a row. "What do you notice about the total when you add these numbers together: 1, 2, 4 and 5; 2, 3, 4 and 5; 1, 2, 3 and 4?" [pause] "How is this different for 1, 3, 4 and 5; and 1, 2, 3 and 5?"

During this roving time the teacher records observations such as: How are students testing the conjecture? Are they exploring combinations of odd and even numbers? Are they trialling different combinations with even numbers in the vertex? Are they recording number sentences? Are they exploring expressions for equivalence? Do they ignore the number in the vertex? Are they thinking about the four numbers that could be placed in the arms? Are they thinking about the total of all numbers? What explanations are students developing? What language are they using when forming a conclusion?

After approximately 10 minutes of time for students to explore Sam's conjecture the teacher brings the children back to the floor to share findings in a final discussion. The teacher asks the following question, "Thumbs up if you agreed with Sam? Thumbs down if you disagreed with Sam? Thumbs sideways if you are not sure or not convinced?" and quickly and effectively gauges the general feeling of the class. Of the children who are not sure or not convinced the teacher invites them to share why. Typically these unconvinced children seek more proof and respond with, "I need to test more combinations."

On the board the teacher places movable numbers from 1 to 5 (seen in Figure 2 and backed with blu-tac) and two blank Vs on the board, one with a 2 at the vertex and the other with a 4. The children are invited to use this support material to demonstrate their justifications. The teacher selects students who have a range of different approaches, e.g., trial by exhausting all possibilities, students who notice the arms maybe odd on one side and even on other side, or students who notice you need to divide the remaining numbers by two. Figure 4 illustrates one pair's justification of the Sam conjecture.

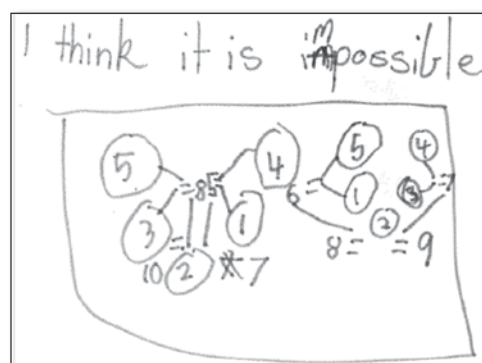


Figure 4. Justification of the Sam conjecture.

The teacher invites the first selected pair who have the least complex reason for agreeing with Sam, using trial and error, up to the board to share their reason why even numbers do not work for this set of numbers. The teacher records all responses on the board. The teacher opens an invitation to the class, "Can anyone else add to this explanation?" Through inviting children to "add to this explanation" the work from the previous pair is not diminished but rather each pair's offering is viewed as collaborating to building a shared understanding together.

The teacher targets children who have a more complex reason than trial and error, “Why did you think it was not possible?” The teacher continues to select pairs whose complexity towards advanced thinking would progress as follows:

1. Trial and error. Some or all of the number sentences are listed.
2. Noticing one arm is 1 number different to the other arm, e.g., 8 vs 9.
3. Noticing even numbers cannot make a Magic V because they make an even and an odd number on each arm.
4. Identifies total of four numbers is odd when an even number is at the vertex; whereas total of four numbers is even when an odd number is at the vertex.
5. Generalises that when an even number is at the vertex, you cannot divide the total of the remaining numbers in two evenly and therefore Sam is correct.

The teacher continues until the reasons are exhausted or the justification is complete (see Widjaja, 2014, for an analysis of the students’ justifications), that is, the total of the four numbers must equal an even number so that they can divide evenly by two leaving a whole number. Therefore, the vertex must be an odd number, leaving two odd and two even numbers on the arms.

At the conclusion of the lesson, the children are invited to consolidate and reflect upon their reasoning in their mathematics journal and share any of their own questions for further exploration.

Conclusion

The Magic V lesson tasks affords children with opportunities to reason through carefully selected prompts and models. The teacher uses the final shared discussion time to emphasise the importance of comparing and contrasting for forming conjectures; encouraging children to develop skills in testing, proving, and justifying conjectures, and building and consolidating

children’s understanding of why an odd number is at the vertex of the Magic V for this set of numbers. Each discussion provides further opportunity for children to learn from their peers’ reasoning, examples, conjectures, explanations and to use and connect mathematical ideas. The children’s noticing of testing, justifying, proving and generalisations emerges from looking for the differences and similarities within the solutions, and noticing and sharing these solutions with each other. For further tasks to support reasoning, please review the Australian Association of Mathematics Teachers *Top Drawer Teachers* site (2013).

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