# Using Arrays

## To Build Towards Multiplicative Thinking in the Early Years

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In this article, Lorraine Jacob and Joanne Mulligan discuss how arrays can be used to promote students' early learning in relation to multiplication and division. They provide examples of activities that can be used from Foundation to Year 5.

Young children come to understand multiplication and division through real life experiences as they group, share and partition to solve everyday problems. Research has shown that they can develop multiplication and division concepts, albeit with small numbers, through an initial understanding of fair shares, equal parts and a many-to-one count (Mulligan & Mitchelmore, 1997; Nunes, Bryant & Watson, 2009). Curricula for the early years now encourage the development of these processes alongside counting, addition and subtraction. However, structural understanding of multiplication and division is more complex and involves understanding and coordinating the quantities involved, the relationship between multiplication and division and commutativity, and how these relate to a variety of situations and representations. The purpose of this article is to show how the array can be used to enhance students' early learning about these ideas.

From the Foundation level, the *Australian Curriculum: Mathematics* (Australian Curriculum and Reporting Authority, 2012) prescribes student learning about multiplication and division in terms of solving problems and calculating strategies. For example, Year 3 students "represent and solve problems involving multiplication using efficient mental and written strategies and appropriate digital technologies" (p. 11). Year 4 students "solve word problems by using number sentences involving

multiplication and division where there is no remainder" (p. 15). In terms of calculating, the relationship between multiplication and division and commutativity appear quite early. For example, Year 3 students "recall multiplication facts of two, three, five and ten and related division facts" (p. 11). Year 4 students "recall multiplication facts up to 10 × 10 and related division facts" (p. 14) as well as develop efficient mental and written strategies "using known facts and strategies, such as commutativity, doubling and halving for multiplication, and connecting division to multiplication when there is no remainder" (p. 14). While the *Australian Curriculum: Mathematics* places emphasis at Years 2 and 3 on exploring the connection between addition and subtraction, the connection between multiplication and division—a more complex relationship—is not developed in the same way. Many researchers suggest that there should be an early emphasis on the relationship between multiplication and division and the language associated with both operations before any use of symbols or formal recording (Downton, 2008; Mulligan & Mitchelmore, 1997; Nunes & Bryant, 1996).

Regardless, the outcomes described above involve important mathematical concepts and relationships for young students' development of multiplicative thinking. Two important multiplicative concepts are captured in *First Steps in Mathematics*: Understand Operations, Key Understanding 5:

> Early understanding of multiplication and division with whole numbers requires students to think about three quantities: the whole (or total) quantity, the number of equal groups, and the amount in each group.
>
> Linking the two ideas of repeating equal quantities and partitioning a quantity into equal portions can help students to understand the connection between multiplication and division. Therefore, it is an important component of their being able to use multiplication and division flexibly to solve problems (Willis, Jacob, Devlin, Powell, Tomazos & Treacy, 2004, p. 52).

## The nature of the three quantities in a multiplicative situation

A number of studies have shown that children from the Foundation years can represent multiplicative situations and solve word problems by modelling using concrete materials, actions with fingers, drawings and visualisation. Their early use of numbers and signs often involves addition and subtraction. Even young children need to be exposed to a range of multiplication and division problems. An example, an equal group multiplication problem, and the related division problem, can be seen as follows:

> There are 8 children in each flag race team and there are 4 teams. How many children are there altogether? (Multiplication)
>
> There are 4 flag race teams and 32 children altogether in the teams. How many children are in each team? (Partition division)
>
> There are 8 children in a flag race team. How many teams can you make if there are 32 children? (Quotition division)

In order to move to using multiplication or division to represent such problems requires that students recognise the structure of the problem and coordinate three quantities. For example, in the quotition division problem students should come to recognise the whole amount is 32 children. The number in each group is 8 children. The number of groups of children is unknown. Hence the division $32 \div 8 = ?$ is required. Even then, young children may use strategies such as repeated addition, building up or repeated subtraction by building down to help them find the solution.

## The relationship between multiplication and division

However, students drawing on their pre-algebraic thinking might think of the problem semantically as "so many lots of 8 makes 32" and hence learn to represent the problem as $? \times 8 = 32$. This representation can make calculating the solution easier and needs to be encouraged. It provides a starting point for learning about inverse relationships. Later, students can draw on

this knowledge, knowing that more complex division problems can be represented as multiplications or divisions depending on their choice of calculation strategies. This idea is also important as they recall multiplication and division facts. If they know the multiplication fact $8 \times 4$ is 32 they can use that fact to work out $32 \div 8$ and $32 \div 4$.

## Using arrays to develop early multiplicative thinking

While tens-frame tools have helped young students develop additive reasoning, arrays have been used extensively to assist students develop multiplicative reasoning (Siemon, Beswick, Brady, Clark, Faragher & Warren, 2011; Booker, Bond, Sparrow & Swan, 2010; Vale & Davies, 2007; Young-Loveridge, 2005) including programs such as *Count Me In Too* (NSW Department of Education and Training, 2002) and the *Pattern and Structure Mathematics Awareness Program* [PASMAP] (Mulligan, Mitchelmore, Kemp, Marston & Highfield, 2008).

Integral to young students' learning about arrays is the idea of collinearity—that is, the recognition and coordination of rows and columns and equal sized spacing (Battista, 1999; Outhred & Mitchelmore, 2000). Teachers can use the visual pattern of an array to focus students' attention on all three quantities at once (Siemon et al., 2011): in other words, the number of groups, the number in each group and the whole amount, as well as the associated language. In addition, teachers can show that 3 rows of 4 is grouped differently to 4 rows of 3, yet the total is the same—the idea of commutativity. In word problems that may be embedded in complex language, the relationship between multiplication and division may go unnoticed by many students. The array can be used to focus students' attention on that relationship.

## Learning experiences using arrays

These learning experiences utilise class sized array cards and packs of playing cards (and involve arrays up to $5 \times 5$).

Figure 1

## Look and see groups

### F–4 small group or whole class

1. Show an array card (e.g., Figure 1). Ask: How many scones? How did you see the scones?
2. Hide the card and have students reproduce the array with counters on grid paper or a diagram.

Focus the students' attention on:

- seeing the collection in groups in different ways, e.g. "I see 2 and 2 and 2; 3 rows of 2; 2 columns of three" and so on;
- the three components of a multiplicative situation: What's the whole amount? How many groups can you see? How many in each group? Model multiplication language 3 lots of 2 is 6; 3 times 2 is 6; 2, three times is 6; and
- the idea of commutativity, that is, 2 lots of 3 is grouped differently to 3 lots of 2 yet the total is the same.

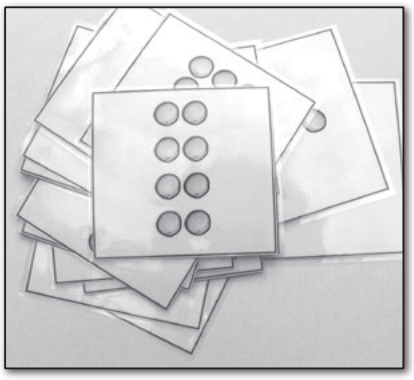


Figure 2

## Who has more?

### F–4, restrict cards to groups of 2 and/or 3 initially

1. Students share a pack of playing cards (Figure 2) and place them face down. At the same time they each turn over a card. The student with more wins and collects both cards. However, they can only collect both cards if they can say how many without counting by ones (for example: 4 and 4 is 8, 2 columns of 4 are 8, 2 fours or 4, two times and so on).
2. As students are ready they work individually with their own piles of cards, recording the total for each card and how they saw that total (for example: 4 and 4 and 4; 3 lots of 4, three fours or 4 × 3).
   Focus the students' attention on:
   - using groups rather than counting by ones;
   - using multiplication language involving the 3 quantities (for example: 2 fours are 8 or 4, two times is 8);
   - using numbers and signs as they are ready.

## Story problems

### F–4 small group or whole class

1. Show an array card (e.g., Figure 1). Ask: How many? What can you see?
2. Say: "I am going to hide the array card. I want you to imagine the array so that you can answer some story problems." Ask a multiplication, a grouping division and a sharing division story problem. Students visualise the array to answer the questions, for example: Three students had 2 scones each. How many scones altogether? There are 6 scones and 3 students. How many scones each? There are 6 scones. If we put 2 scones on a plate how many plates will we need? Focus students' attention on how visualising the array can help think about each situation.

## Flash quiz

### Years 2–4 small group or whole class

1. Show an array card. Say: "I am going to hide the card. I want you to imagine the array so that you can answer my quiz questions." Ask multiplication, grouping division and sharing division questions that are not embedded in story (for example: What's 3 lots of 2? Six shared between 3? How many lots of 2 in 6?).
2. In time progress to incorporate 'times table' language (for example: What's 2 threes? How many threes in 6, 6 divided by 2 and so on).

Focus the students' attention on how they can visualise the array to answer both multiplication and division questions.

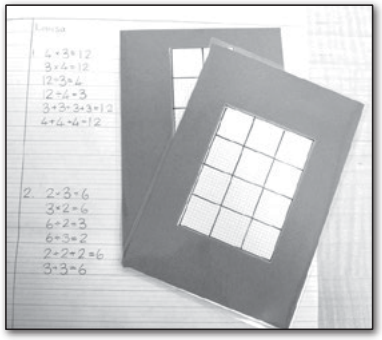


Figure 3

## Array number sentences

### Years 2–4 small group or whole class, then individual work

1. Show an array card and have students call out a number story. Accept additions but encourage multiplication and division stories (for example: lots of 4 are 12; 12 shared between 3 is 4 and so on). Show each response as a number sentence.
2. Students work individually with a set of array cards (Figure 3) and write number sentences for each array.
   Focus the students' attention on:
   - writing two multiplication and two division number sentences for each; and
   - explaining the connection between the digits in, say, 12 ÷ 3 = 4 to their array.

Name:

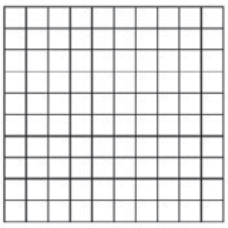
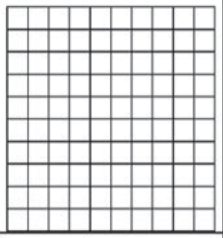
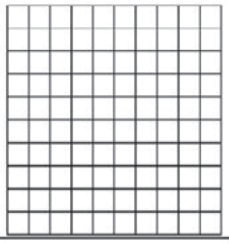
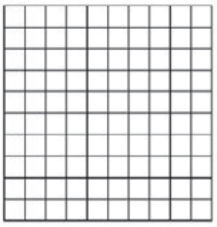
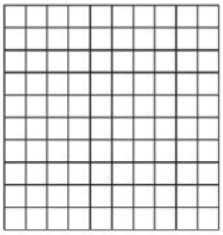
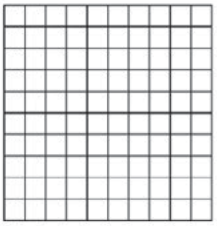
| Story | Diagram | Number Sentence | Story | Diagram | Number Sentence |
|---|---|---|---|---|---|
| The farmer puts 4 lambs in each yard. He has 3 yards. How many lambs does he have? Answer: |  | | Four fish are put into 2 fish tanks. How many fish are there? Answer: |  | |
| The farmer has 12 lambs. He wants to put 3 lambs in each yard. How many yards will he need? Answer: |  | | Jimmy has 8 fish. He wants to put 4 fish into each tank. How many tanks will he need? Answer: |  | |
| The farmer has 12 lambs. He has 3 yards. He wants the same number of lambs in each yard. How many lambs will he put in each yard? Answer: |  | | Jimmy has 8 fish He has 2 tanks to house his fish? He wants the same number of fish in each tank. How many fish will he put in each tank? Answer: |  | |

Figure 4

## Array story problems

### Years 3–5 individual or pairs

1. Use a thinkboard format (e.g. Figure 4) to have students represent a multiplication problem and the associated partition and quotition division problems as an array and a number sentence.
   Focus the students' attention on:
   - how to represent a division as an array knowing the whole amount and either the number of groups or the number in each group; and
   - how the same array represents the multiplication and the associated division problems.

## Implications for teaching

The *Australian Curriculum: Mathematics* focuses on early multiplicative strategies such as grouping and sharing. However careful pedagogical scaffolding is needed to encourage development of important multiplicative concepts, along with early algebraic thinking. Arrays provide a vehicle for teachers to focus students' attention on the nature of the quantities involved, the associated language, the relationship between multiplication and division and commutativity. Most of the learning experiences provided can be carried out without using numbers and signs. However, even students from the Foundation year may be able to construct number sentences with the support of arrays. Visualising arrays is encouraged in order that students can draw on mental images of small number arrays as they later begin to use flexible strategies for more complex calculations.

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