Enhancing Mathematical Communication

Bag of Tricks game



Sitti Maesuri Patahuddin
University of Canberra
<Sitti.Patahuddin@canberra.edu.au>

Ajay Ramful
University of Canberra
<Ajay.Ramful@canberra.edu.au>

Jane Greenlees
Charles Sturt University
<jgreenlees@csu.edu.au>

An engaging activity which prompts students to listen, talk, reason and write about geometrical properties. The 'Bag of Tricks' encourages students to clarify their thoughts and communicate precisely using accurate mathematical language.

Mathematical communication, expressed in oral, visual and written forms or gestures is an important mediating tool for enabling students to appropriate the conventions of mathematics as they make sense of the underlying concepts (National Council of Teachers of Mathematics, 2000). Thus, it is important for students to use precise and accurate mathematical terminologies from the primary level onwards. It is also crucial for students to make precise mathematical arguments. The *Bag of Tricks* activity presented in this article prompts students to listen, talk and write about geometrical ideas. Consequently, it allows students to communicate their ideas and organise their thinking. This article describes an engaging activity to teach closely related concepts in geometry. We illustrate the design and implementation of the *Bag of Tricks* activity and outline the benefits that we experienced in helping students differentiate between closely related geometrical concepts.

In the Australian primary curriculum, students are expected to describe the features of three-dimensional objects, such as "make models of three-dimensional objects and describe key features" (Australian Curriculum Assessment and Reporting Authority (ACARA), 2014). It is known that geometrical concepts such as vertices, edges and faces, or prisms and pyramids can be confusing for students (Koester, 2003). The *Bag of Tricks* activity that we trialled with our students proved to be productive in enhancing their understanding of the features of three-dimensional objects. Importantly, it enabled students to communicate their mathematical thinking and deepen their understanding. For instance, it helps students to make the clear distinction between edges and vertices as well as prisms and pyramids.

Bag of Tricks: Procedures of the game

- This activity is preferably conducted in groups of 4–5 students.
- Teacher puts a three-dimensional object (e.g., triangular-based pyramid, triangular-based prism) into an opaque bag for each group so that children cannot see.
- One child from each group is called up and asked to feel/touch the object in the bag to identify its features. This child then returns to his/her group to inform the other children as to what three-dimensional object is in the bag.
- Teacher encourages students to work quietly so that other groups will not be able to hear their answers.
- Each group writes as many features about the object that is in the bag as they can on a sheet of paper. This may take 5–10 minutes.

- Teacher asks a representative from each group to come to the front of the class with their list of features.

- Each nominated group member presents the features listed by the group to the class while holding the object. This modelling activity allows the other students to observe the features.

- Together with the class, teacher verifies and records the stated features for each group on the board. This gives the teacher opportunities to clarify the features to the whole class. Preferably, each group presents a different object.

- Simultaneously the teacher draws a table on the board to record a tally of scores when each group present the features of the three-dimensional object. One point is allocated for a correct property. Points are awarded only for valid features.

What preparation is needed for the activity?

Teaching preparation is always the key for success. The *Bag of Tricks* game requires the following considerations:

- Identify pre-requisite knowledge that students need to be able to perform the activity.

- Anticipate the mathematical terminologies you want students to use for the selected three-dimensional object.

- Identify and anticipate types of incorrect/inappropriate language that may occur as a result of inference from common language. For example, students tend to use the familiar word 'corner' rather than the mathematical term 'vertex', or 'side' rather than 'face'.

- Provide a summary that outlines the features of each three-dimensional object. This can be used as a guide for teachers during the activities or to summarise the lesson.

Benefits of the game: Instances from a vignette

Our reflections on the implementation of the *Bag of Tricks* game enable us to highlight a number of observable benefits. We describe these benefits through the following classroom vignette.

In this lesson, the teacher, Amy, was promoting the development of the concepts of three-dimensional geometry for her students. The objectives of her lesson was to enable her students to use the correct mathematical terminology associated with prisms and pyramids; to recognise the differences between prisms and pyramids; and to identify and articulate the features of prisms and pyramids.

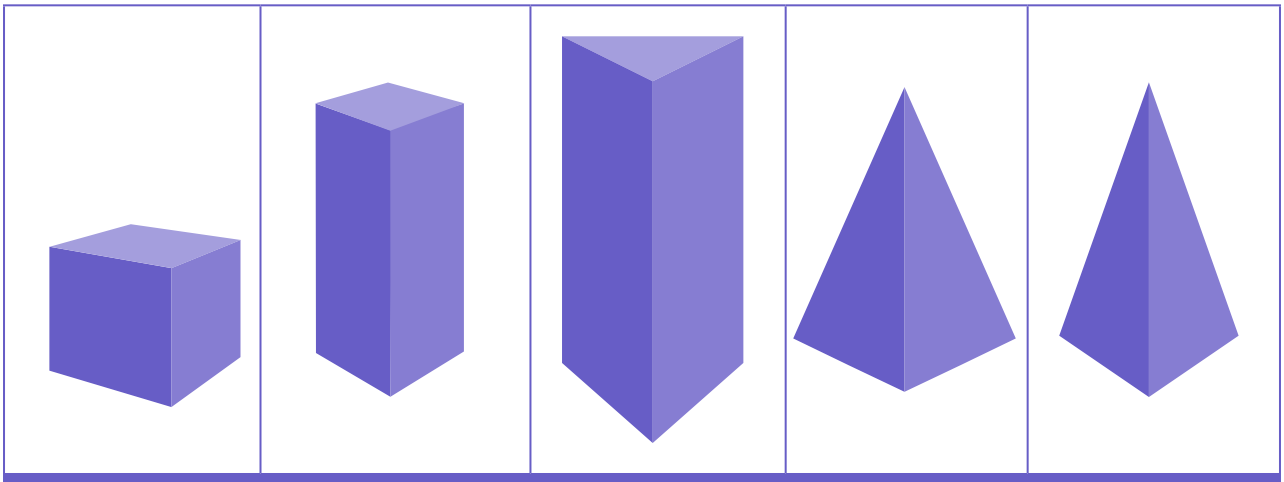


Figure 1.

Amy set her class into six groups. She called one student from each group and asked them to touch and feel a three-dimensional object in an opaque bag in front of the class. Each child was asked to cover her/his eyes. Amy gave each child a bag with a model inside namely a prism or a pyramid as in Figure 1a–1e. These students enthusiastically explored the assigned objects. Once the students felt confident of what the object was, they returned the bag to the teacher.

The following is an illustration of the students' interactions as they attempted to describe the features of a square-based pyramid (in Group 1) and of a triangular-based prism (Group 2).

Students in both groups freely expressed their ideas about a square-based pyramid and a triangular-based prism using their common language. For example, they described the square-based pyramid as consisting of a square and four triangles. However, they sometimes used inappropriate mathematical language such as "corners" instead of "vertex" or "vertices".

Benefit 1: The Bag of Tricks activity promotes spatial visualisation

The first advantage of this game is that since the three-dimensional object is not given to them physically in the group, they are stimulated to visualise its spatial arrangement. Note that spatial visualisation refers to "understanding and performing imagined movement of two- and three-dimensional objects. To do this, you need to be able to create a mental image and manipulate it" (Clements, 2010, p. 75). We observed how they used gestures to count the number of edges in the square-based pyramid. When explaining to his group, one student re-drew the shape in the air, where his classmates were expected to visualise the outline of the shape.

"Wait. Let me count them again, one, two, three,.., only 8 edges (using his finger to imagine moving along the edges of the object). Because the bottom part has 4 edges and the top to the bottom has 4 edges. In total, there are 8 edges."

The Measurement and Geometry strand of the Australian Curriculum also promotes spatial visualisation. Thus, it is important that teachers provide opportunities for students to actively explore mathematical concepts associated with three-dimensional objects and to develop spatial visualisation skills. Activities such as *Bag of Tricks* are necessary to allow students the opportunity to develop visualisation skills, which are essential, not only in a mathematics setting, but also for a variety of everyday experiences.

Benefit 2: The game promotes verbal and written communication in mathematics

The language of mathematics is an important tool for developing mathematical understanding (Thompson & Rubenstein, 2000). Another benefit of this game is that it prompts students to pose questions to clarify the terms used by their friends. For example, with regard to the square-based pyramid, two students in Group 1 clarified their understanding as follows.

S1 It has five faces.

S2 Face! Isn't it this part [S2 points to the cover of his book]? Instead of this [She points to the edges of his book]?

The *Bag of Tricks* game also requires students to describe the three-dimensional object verbally and in writing, as illustrated from the following script of Group 1.

S1 I think we also need to talk about the position of the vertices, the edges, and the faces. Are they next to each other or what?

S2 There is one vertex at the top and 4 vertices at the bottom.

S3 But what if you tilt it or you turn it around 90 degrees?

S2 It's okay; we can say one vertex on the left and 4 vertices on the right.

S4 What should we write here?

S2 Won't it be better to write things that are definitely correct such as "One of the faces is a rectangle and the other faces are triangles?"

When the students in Group 2 discussed the positions of the parts (vertices, edges, and faces) of a triangular prism, they experienced difficulties in writing the features when the orientation of the object changed.

S5 Let's also talk about the positions among each other.

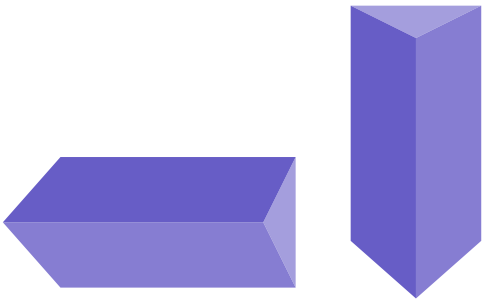
S6 When I touched, I held it this way [by gesture, see Figure 2a], so you can write that the left side and the right side are triangles.

S7 But what if you turn it around like this [by gesture, See Figure 2b], this (one of the triangular faces) could be at the top.

S8 This is tricky, because if the base is a triangle, the top part is also a triangle. But if the base is a rectangle, then the top part is only an edge.

S6 Why don't we write what you've just said!

This transcript also highlights how spatial visualisation is used in identifying and describing the three-dimensional object.



(a) (b)

Figure 2.

Communicating ideas in mathematics is an important way for students to construct their own mathematical knowledge and understandings. The activity *Bag of Tricks* promotes communication through the use of group work and encourages students to listen to one another and value each other's perceptions and interpretations.

Benefit 3: The activity motivates students to be interested in the assigned task

Since an incentive was attached to this game in the form of a group score, students showed their enthusiasm to do the best for their team. This was indicated by their engagement throughout the lesson as could be observed during the activity. The students were still engaged even after the lesson was over. By presenting mathematics concepts in a fun and enjoyable way, we provide opportunities for students to remember the appropriate mathematical terms. Research has also indicated that students who use manipulatives, such as those used in the *Bag of Tricks* activity, enjoyed learning mathematics (Moyer, 2001).

Benefits for the teacher

This game is not only beneficial for students but also beneficial for the teacher. This activity creates opportunities for teachers to assess students' mathematics such as their prior knowledge, and can thus inform future instruction. This game also facilitates the teacher's task of making mathematical concepts more accessible to students. As we witnessed ourselves, this activity engages students in mathematical conversations and enhances their understanding about the features of three-dimensional objects and the differences between prisms and pyramids.

Conclusion

The success of the *Bags of Tricks* activity depends on both careful preparation and implementation. It requires the teacher to prompt students' thinking. For example, the teacher could ask the following questions: "When you touched the three-dimensional object, what was the first thing that came to your mind?" or "When you were feeling the three-dimensional object, what were you looking for to help you decide the name of the object? To successfully accomplish this game, each group of students are required to work collaboratively. The most noticeable feature of the *Bags of Tricks* activity that we experienced was the dynamic interaction among students in the class, the richness in students' mathematical conversations and the joyful learning environment that it generated.

Reference

Australian Curriculum Assessment and Reporting Authority (ACARA). (2014). *Foundation to Year 10 Curriculum: mathematics.* Retrieved 13 November, 2014, from http://www.australiancurriculum.edu.au/mathematics/curriculum/f-10?layout=1

Clements, D. H. (2010). Geometric and spatial thinking in young children. In J. V. Copley (Ed.), *Mathematics in the early years* (pp. 66–79). USA: The National Council of Teachers of Mathematics, INC.

Koester, B. (2003). Prisms and Pyramids: Constructing Three-Dimensional Models to Build Understanding. *Teaching Children Mathematics*, 436–442.

Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics, 47*(2), 175–197.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Thompson, D. R. & Rubenstein, R. N. (2000). Learning mathematics vocabulary: Potential pitfalls and instructional strategies. *The Mathematics Teacher,* 568–574.