Using concept maps to show 'connections' in measurement: An example from the Australian Curriculum

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> The *Australian Curriculum: Mathematics* ensures that the links between the various components of mathematics … are made clear" (Rationale ACARA, 2012).

Introduction

As teachers we want our students to understand mathematics. Within the *Australian Curriculum: Mathematics* the Understanding proficiency strand states, "Students build understanding when they connect related ideas, when they represent concepts in different ways, when they identify commonalities and differences between aspects of content, when they describe their thinking mathematically and when they interpret mathematical information" (ACARA, 2012). Concept maps can be used to organise and represent the connections in knowledge graphically which allows teachers to assess the connections that students make between concepts (Novak & Cañas, 2008). This article explores the use of concept maps with a Year 9 mathematics class to determine their understanding and the connections that they make between concepts within a measurement unit. The findings show the importance for teachers of Year 7 to help their students to see the connections between these concepts when they are first introduced.

Background

Schoenfeld (1992) states "doing mathematics [is] an act of making sense" (p. 18). If students are to do mathematics and make sense of it, they need to be given opportunities to think, talk and argue about the mathematical concept in question. Meaningful learning occurs when connections and patterns are found and these can be tied to what the students already know. Teachers can help students with this by developing their metacognitive skills so that they have the "knowledge about knowing and learning" (Woolfolk & Margetts, 2010, p. 290). This can be achieved by asking students questions about the processes of doing mathematics and by asking students to reflect on their learning.

One way to encourage reflection and to allow teachers to observe the links that students are making is to have students draw a concept map that demonstrates their connections. If students are asked to work collaboratively to construct a group concept map, they are given the opportunity to discuss the mathematics. By explaining and justifying their individual understanding to the group, as they work their way to consensus, the students are able to talk their way to a deeper understanding (Marshman, 2010).

Meaningful learning, as opposed to rote learning, requires the learner to incorporate new concepts and propositions with their existing ideas in meaningful ways (Novak, 2010). These concepts and propositions are built into a hierarchical cognitive structure. This builds confidence so "the learner feels in control of the knowledge acquired and capable of using this knowledge in problem solving or facilitating further meaningful learning" (p. 22).

The use of concept maps as a teaching strategy in science was developed by Novak (1990) to organise and represent knowledge. Concept maps (Novak & Cañas, 2008) are created by writing concepts in ovals or boxes and then linking these with a line. Linking words or phrases are added to the lines to indicate the relationship or connection between the concepts. Concept maps are hierarchical with the overarching ideas at the top. Concepts may be cross-linked between sections of the concept map. More cross-links indicate more connections between the concepts and hence an enhanced conceptual understanding. Novak and Gowin (1984 in Novak, 2010) demonstrate by their research that they can also be used to help students learn how to learn.

Planning a unit of work with concept maps allows the teacher to clarify the key concepts that they believe are important, and the connections and relationships between concepts. This then enables them to sequence the learning activities in a logical way so that their students can gradually build on their knowledge. Teachers may also share their concept maps with their students to reinforce understanding, thus indicating to the students the relationships that they perceive as important. Also teachers can give key parts of their concept maps as "expert skeleton concept maps" (Novak, 2010) to their students to scaffold their learning in a new area.

Jin and Wong (2010) analysed the concept maps of 48 Chinese Year 8 students, using social network analysis to measure the number of connections drawn between each of the concepts across the map. The students worked individually with a list of eleven concepts on triangles. The results for their concept maps were similar to their results on a specially designed conceptual understanding test indicating that concept maps were a valid approach to determining students' conceptual understanding in mathematics. Afamasaga-Fuata'I (2007) used an exploratory teaching experiment to explore the development of Samoan students' understanding of mathematics content from previous courses. She showed that across the semester the concept maps became more integrated and differentiated as students justified their connections and negotiated meanings with their peers. In this case concept maps were used to deepen students' conceptual mathematics understanding.

When students generate their own concept maps, either individually or collaboratively, it provides an opportunity for the teacher to check students' understandings and diagnose possible misconceptions. Alternately, they could be used to formally evaluate learning as suggested by Williams (1998) and Stoddart, Abrams, Gasper and Canaday (2010).

The study

Participants

This paper describes my experiences with a Year 9 mathematics class. This was part of a larger action research project using Collective Argumentation to scaffold learning with collaboration (Brown, 2007, Brown and Renshaw, 2006; Marshman, 2010). This is a technique where students individually represent their thoughts about, or solution to the problem or question. This ensures each student has thought about the problem before they co-operatively compare their response with their group. The students take turns to explain and justify their solution, as the group works towards consensus. The understanding and solution that they agree collaboratively on is then shared with the class for discussion and validation.

In my class, students were used to working in friendship groups and were aware that they would be chosen randomly to share with the class their concept map and the thinking that led to it. During the whole year each of the students and I kept journals for reflection. Students were given stimulus questions to encourage the process.

We were completing a term-long (10 week) investigation of building a swimming pool. The students designed a swimming pool composed of at least two three-dimensional shapes, and the task included:

- making a scale model,
- doing a three-dimensional drawing,
- drawing a dimensioned plan and elevation to scale, and
- calculation of:
  - the area of their pool to be tiled;
  - the capacity of their pool; and,
  - the area of the pool cover.

Within the 'using units of measurement' strand of the *Australian Curriculum: Mathematics*, students would need an understanding of length, area, surface area, volume and capacity. Figure 1 shows my concept map drawn as part of my planning. The concept map shows that for this section of the unit there are many connections to be made between the different

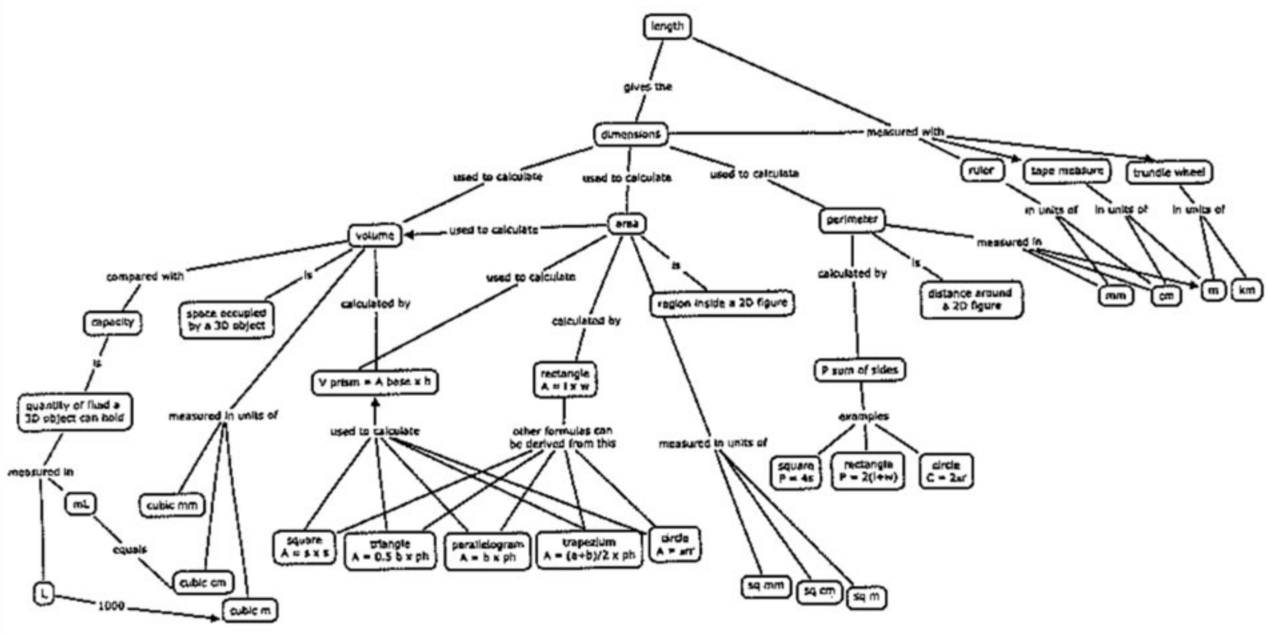


Figure 1. Teacher-generated concept map for the key measurement concepts in the unit.

mathematical concepts of this investigation, for example between length and area, and area and volume.

The teaching–facilitating learning

I saw my role in this class as facilitating learning. I wanted the students to explore how area builds on length (the dimensions) and then how area extends into volume in three dimensions. Therefore, during this investigation it was important that students developed a conceptual understanding of the different parts of measurement and understood how they were related. The students worked collaboratively in friendship groups as when I chose the group it required significantly more effort to keep the students on task and produced no better results.

Lessons usually began with a question for investigation. The students then worked collaboratively to solve the question and present their findings to the class for discussion. This way we built a class understanding for the key ideas. For example when asked, "Why is the area of a rectangle given by length × width?" they were given centimetre grid paper, and drew various size arrays on it (as shown in Figure 2a). This enabled them to see 'visually' how the formula was derived. This was followed by other investigations to derive the formulas for the areas of other shapes. Cutting the shapes and rearranging them into rectangles could be used to determine the formula. This is shown in Figure 2b for the parallelogram and 2c for the circle. In each case the students were required to cut and manipulate the pieces to produce a rectangle. After each small investigation, groups of students would present their findings to the class for discussion. In this way the students produced their own knowledge with the intention of developing conceptual understanding.

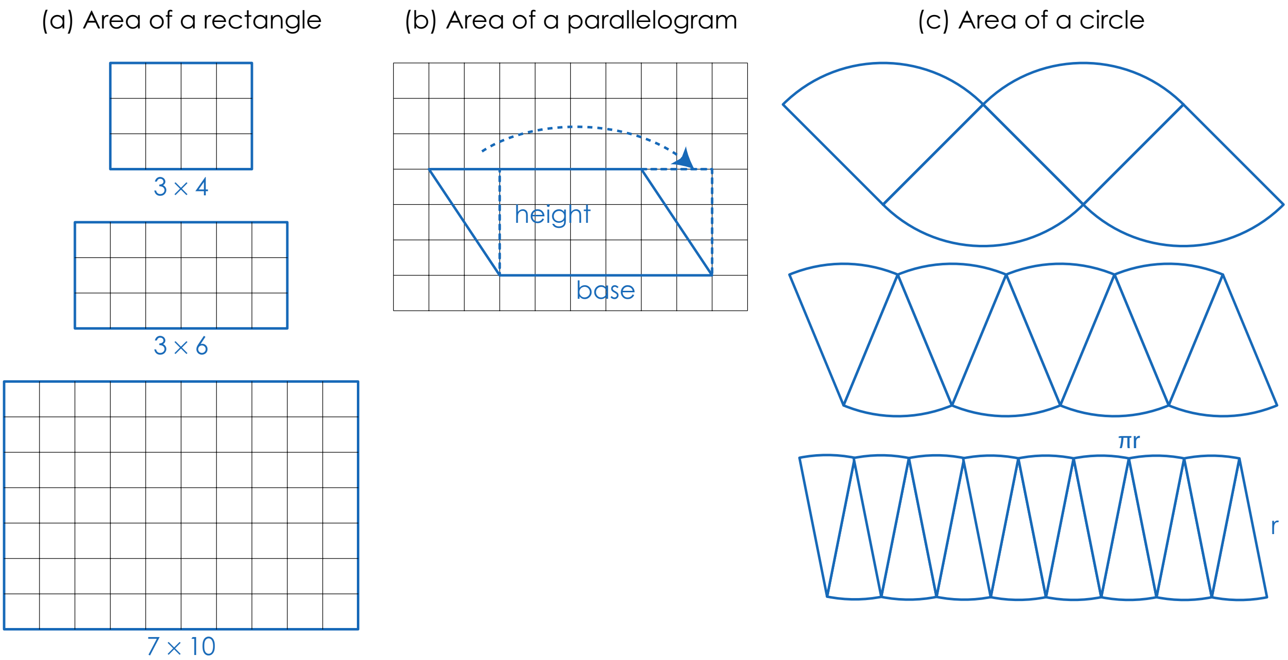


Figure 2. Developing the formula for area of (a) a rectangle, (b) a parallelogram and (c) a circle.

Beginning with a rectangular prism, the students determined volume formulas. Different sized prisms were built with multi-link cubes and the area of the base was determined as the area of the rectangle by counting the number of blocks. This was then compared to the array model where the number of blocks was calculated by multiplying the number of blocks in the length by the number of blocks in the width (which was how the area formula was developed.) As each layer of height is added, the same number of blocks, as in the base, is added. This provided an opportunity for linking area and volume. This repeated addition of the layers can be simplified to multiplication and so generalised to $V_{prism} = A_{base} \times \text{height}$. This is shown in Figure 3.

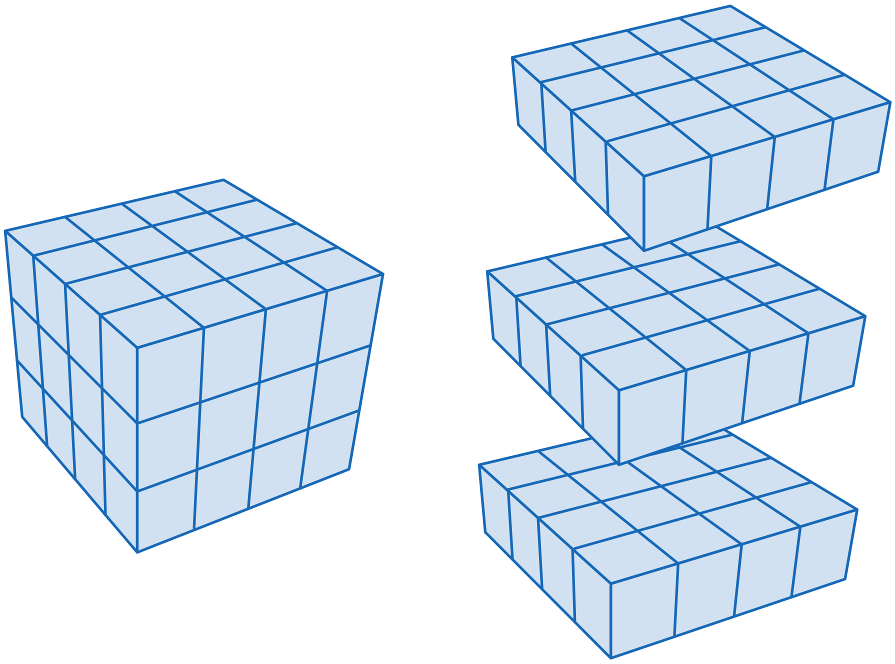


Figure 3. Developing a formula for the volume of a prism.

To develop formulas for the volume of a pyramid or a cone and to determine capacity, students were given empty relational three-dimensional objects (prisms and the corresponding pyramid with the same base dimensions and height) and measuring cylinders. There was a tap outside the classroom, which they could use to fill the three-dimensional objects with water, measure the capacity and determine the relationship between the capacities of the prism and pyramid and hence the formulas for volume of a pyramid and cone.

Towards the end of the unit of work, after the students had built their scale models of the swimming pools, I asked the students the focus question (Novak & Cañas, 2008), “What are the connections between length, area and volume?” and to develop a concept map showing these connections. This was a single lesson activity. Students presented their concept maps to the class and discussed the connections they made. Students were given a sheet with a list of measurement words written in boxes as a parking lot (the list of possible concepts that could be moved onto the concept map) (Novak & Cañas, 2008). This allowed the students to cut out the words and move them around as they developed their concept maps. They were given an A3 sheet to produce it on. If students were permitted to write their concept maps onto a piece of paper, it would be unlikely that they would change the placement of concepts following discussions. It was much easier for the students to move the pieces round during the discussions, which is what they did. A number of empty boxes were also provided so that students could include their own words. The students worked in friendship groups to produce a consensus concept map. This meant the students engaged in mathematical discussions with their peers. Individual students were sharing, explaining and justifying their understandings, so the group as a whole could deepen their knowledge and understanding.

Students' work

Unlike Jin and Wong (2010) who trained their students to develop concept maps on their own without scaffolded guidance, we drew a whole class concept map first and this was their first attempt on their own. Many of these students found this a novel task as can be seen from the comments made by a student is his reflective journal:

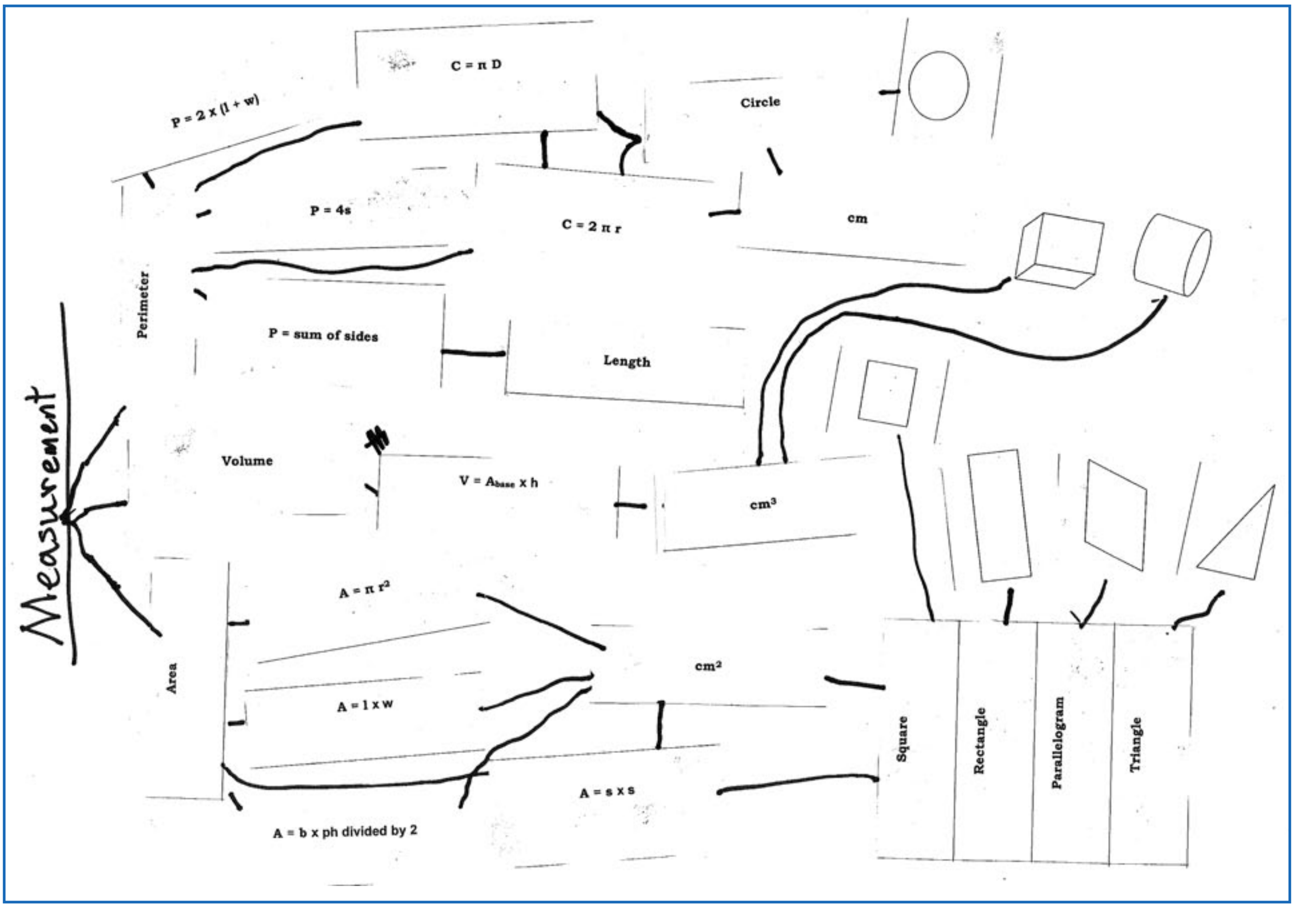
> What new thing/s do you know or can do after today's lesson?
> *Look at things differently.*
>
> How do you feel you worked during today's lesson?
> *Well because I was on the job the whole time.*

There were, however, some other students who were still reluctant to fully engage with the thinking and were just producing the necessary products. This is seen in my reflection along with my frustration at the difficulty at trying to change some students' ways of thinking.

> Why can't we just learn the formulas? Why do we have to think? What is the purpose? ... These students are very much stuck in ruts. They are used to thinking about maths one way and it is very difficult to try and change their way of thinking (Teacher journal, June 2007).

Although students thought differently about the task, they were not able to make the connections that would demonstrate a deeper understanding. None of the groups chose to add extra key words or definitions. Most groups of students sorted and classified the concepts but did not take the task any further. In most cases the concepts were sorted according to perimeter, area and volume, which is the traditional way they were taught. The students then listed the respective formulas and perhaps the units generally used in classroom measurement (cm, cm^2, and cm^3) under the headings of perimeter, area and volume. The names of shapes and perhaps a diagram of the actual shape were included with the formulas. These were often not drawn as a concept map but presented as a list. One group sorted their map as square, circle and rectangle; listing the area formula before the perimeter. None of these groups made any connection between length, area or volume and if they drew linking lines they did not write words on them to show a relationship.

The most detailed concept map drawn is shown in Figure 4(a). The students attempted to make links; for example they connected the perimeter formulas and the area formulas with the correct units. They have not, however, made connections between the key ideas; that is, no connection was made between length and area, or area and volume. This can be seen more clearly in Figure 4(b) where the concept map has been redrawn keeping the same links that the students made but reorganising it so the concepts



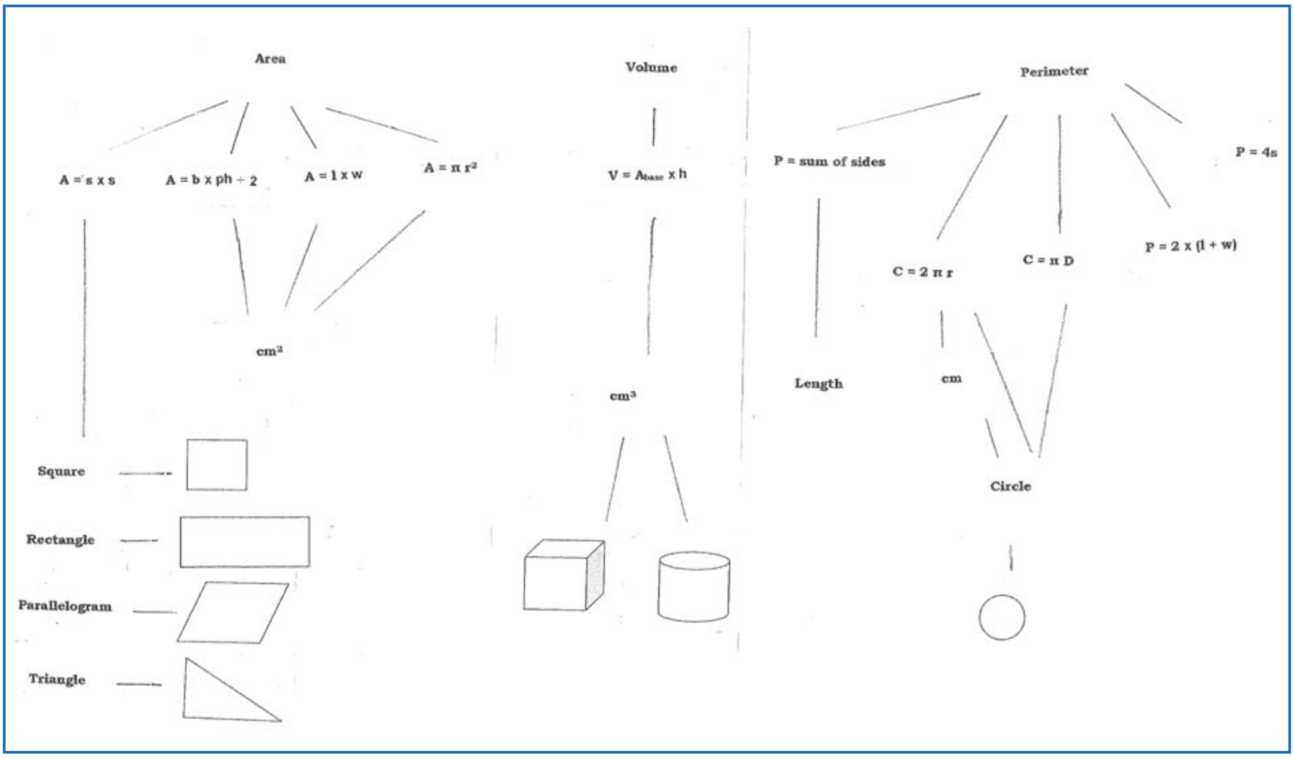


Figure 4(a) shows the original student concept map and (b) shows a rearranged version. This shows exactly the same words and connections. They have been re-oriented onto landscape for ease of viewing.

that have been linked are closer together. Once this was done the concept map became three distinct concept maps.

The students were asked specifically to draw a concept map showing the connections between length, area and volume. Because the students have not thought about these measurement ideas conceptually means that they have three distinct concept maps. In this case the students are thinking procedurally, that is, in terms of the algorithms or rules, formal language and symbolic representation that can be used to complete mathematical tasks (Engelbrecht, Harding & Potgieter, 2005). Had they thought conceptually they would have been drawing on their internally constructed relationships and the links between length area and volume (Engelbrecht, Harding & Potgieter, 2005). Curry and Outhred (2005) describe these relationships:

- length, area and volume are spatial with the attribute conserved;
- measurement is about iterating the unit, and the inverse relationship between the size and number of units;
- in length the unit is iterated in one dimension, with area the unit is iterated in two dimensions and for volume the unit is iterated in three dimensions.
- for area and volume there is a multiplicative relationship with the length of sides.

Another reason that students have not thought conceptually is that they do not understand length, area and volume conceptually. Instead the students have considered perimeter, area and volume as formulas only. This is particularly evident as in all but one of the nine concept maps students considered perimeter as the most important one-dimensional concept rather than length. Most did not even include length in their concept maps.

Both primary and secondary school students have poor understanding of area and often confuse area and perimeter formulas (Outhred & Mitchelmore, 2000). Simon and Blume (1994) claim that this is because students have learnt the formula by rote. In this case there was an overt attempt at developing students' conceptual understanding.

What are the implications for teaching?

Concept maps have been shown to be a powerful way to access student thinking. The inability of any of the students to make connections between length, area and volume in any of the concept maps was disappointing, particularly when this was an inquiry-based classroom where the students had spent considerable amounts of time investigating formulas by connecting with previous formulas. In my journal I wrote "I had a number of discussions with students about some of the connections, for example, reminding students how we had derived formulas from exploring shapes, and how we had used blocks to show how the formulas built on each other that volume of a prism is area of base × height, for example, area of a square is $A = s^2$ and the volume of a cube is $V = s^3$." (teacher journal, June). From these discussions it was clear that students had not learned key concepts despite the inquiry based pedagogy that was used.

Students had been told most of these formulas in earlier years, though they had forgotten them. So although there was a definite attempt in the teaching to encourage the students to understand why the formulas worked and how to derive them, it did not change these students basic learning pattern for maths. They still expected to memorise a formula rather than truly understand and apply the concept expressed in the formula. Many students were interested in where the formulas came from, particularly for the circle. For example after finding the formula for a trapezium one student wrote in their journal, "by using grid paper we had a better understanding of what was happening" A few students actively resisted having to think mathematically, as indicated by journal comments such as: "Why can't we just learn the formulas?", "Why do we have to think?", "What is the purpose?", etc.

It may be that using grid paper is not a good choice of concrete materials for exploring area of a rectangle. Doig, Cheeseman and Lindsey (1995) suggest that students should be manipulating tiles to cover the area to be measured so that they are aware of the two-dimensional nature and the need to count rather than having the pre-drawn grid.

By Year 9, it is difficult to change student perceptions of what mathematics is. Therefore, for students to make these connections, we need first to ensure that they understand the concepts, and then to make them visible when initially introducing the formulas.

The *Australian Curriculum: Mathematics* addresses this issue with its statement "Students build understanding when they connect related ideas, when they represent concepts in different ways, when they identify commonalities and differences between aspects of content, when they describe their thinking mathematically and when they interpret mathematical information" (ACARA, 2012). It is teachers of Year Seven who first "Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving" (ACARA, 2012) and "Calculate volumes of rectangular prisms" (ACARA, 2012) so these are the teachers who have the potential to help future students to see the connections between length, area and volume and teach for conceptual understanding and application in problem solving .

The teachers' concept map is a powerful tool for planning, and by sharing this with their students they may be able to provide a visual picture of the connections between concepts. Obviously one day may not be enough to change old habits. The lesson described in this paper needs to be extended by asking students to draw concept maps at the beginning of a unit of work and then gradually refine their concept map as the unit progresses. This may encourage students to think more deeply about concepts and how they link together.

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From Helen Prochazka's

Scrapbook

Pi goes on and on and on
And e is just as cursed
I wonder – which is larger
When their digits are reversed?

Author unknown