Using photographic images

to enhance conceptual development in situations of proportion




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Find out how to use photographic images to support the conceptual development of proportional thinking. This paper provides insight into a sequenced activity that promotes student engagement and makes links to familiar and unfamiliar contexts.

Introduction

The ability to think proportionally does not always occur naturally in students and often takes considerable time to develop (Bangert-Drowns, Hurley & Wilkinson, 2004). For students to understand situations of proportion, they need to able to use multiplicative thinking, to have a sense of co-variation and to recognise multiplicative relationships in situations of comparison (Behr, Harel, Post & Lesh, 1992; Lesh, Post & Behr, 1988). Multiplicative thinking refers to the capacity to recognise and solve problems that involve the use of multiplication or division (Siemon, Izard, Breed & Virgona, 2006). Having a sense of co-variation requires students to be able to recognise situations and solve problems in which two quantities are related in such a way that a change in one quantity results in either a directly or inversely proportional change in the other. For example, a child may understand that if they walk the same distance at twice the speed, they will take half the time.

The main proportional reasoning problem types include rate problems (involving both commonly used rates, such as speed, and rate situations in which the relationship between quantities is defined within the question, such as birthday cake slices per child); part–part–whole (e.g., ratio problems in which two complementary parts are compared with each other or the whole, such as comparing numbers of boys to girls in a class or boys to total students); and stretchers and shrinkers (growth or scale problems; see Lamon, 2005). Concepts such as relative and absolute (e.g., such as when comparing Group A: 4 people with 3 pizzas and Group B: 3 people with 2 pizzas, who has the most pizza relatively?) are also pivotal in students' understanding of proportionality. As highlighted by Van Dooren, De Bock, Verschaffel and Janssens (2003), it is also important to give students practice at differentiating proportional situations from non-proportional.

While formal instruction around ratio and proportion may not occur in mathematics curricula until early secondary schooling, research shows that the concepts and skills that are needed by students if they are to successfully engage with problems that involve ratio and proportion take a long time to develop. Exposing children to foundational concepts (such as fractions and part–part–whole comparisons), which are needed for proportional thinking, can build on the intuitive understanding that they often have of such relationships (Irwin, 1996; Parish, 2010).

Teachers often express confidence in teaching procedural skills, such as multiplicative, fractional, and relational thinking, which are required for proportional reasoning. However, the development of underlying conceptual understanding can be challenging for students and it is difficult for teachers to identify ways to assist students to recognise and deal with situations involving proportional reasoning (Sowder, Armstrong, Lamon, Simon, Sowder & Thompson, 1998; Sowder, 2007). Staples and Truxaw (2012) argued that a key aspect of focusing on conceptual understanding is the use of mathematical language. They suggested that unless teachers focus on and are aware of mathematical language demands, they might inadvertently shift a task from one that engages students in conceptual discourse to a less demanding procedural task. In this article, we describe a sequenced activity that used selected images to target the conceptual development of proportional thinking and focus on the language associated with comparison and proportional thinking. We also describe the participating teachers' perceptions of the activity.

Impetus for the development of the activity

During a two-year multi-state Australian research project, which focused on enhancing proportional reasoning, (see Hilton, Hilton, Dole & Goos, 2013; Hilton, Hilton, Dole, Goos & O'Brien, 2013; Hilton, Hilton, Dole, Goos & O'Brien, 2012), some upper primary school teachers requested assistance in finding ways of engaging their students in the underlying concepts of proportional reasoning without necessarily focusing on algorithmic or procedural skills. They felt that their students often could not recognise the difference between proportional and non-proportional situations, could not identify the different types of proportional situations, and regularly did not have the language to competently engage in or describe the proportional situations. In response to these requests from teachers, we decided to trial the use of photographic images to promote students' understanding and discussion of real life examples of varied proportional situations. This trial involved 15 upper primary school teachers. As noted earlier, this article focuses on the activity itself and the teachers' perceptions of it. Students' feedback and diagnostic test results were also collected during the study. These are beyond the scope of this article and some have been reported previously (see Hilton, Hilton, Dole & Goos, 2013).

The decision to combine images (visual representations) and discussion (verbal/linguistic representations) was based on research findings that have identified a range of learning advantages associated with having students learn through the use of multiple representations. These include allowing students to communicate their understanding to others, to make connections between familiar and unfamiliar or novel situations and concepts, and to clarify and refine their ideas (Ainsworth, 2008; Chittleborough & Treagust, 2008; Lesgold, 1998; Prain, Tytler & Peterson, 2009). According to Lemke (2002), it is important for teachers to use a combination of natural language and mathematical and visual representations when engaging students in mathematics learning.

The activity

A series of photographic images portraying differing proportional contexts was selected and accompanying discussion prompts were designed. The purpose of the activity was to provide students with opportunities to consider the underlying concepts and develop language associated with the inherent proportional situation in the images. The images represented the main proportional reasoning problem types identified by Lamon (2005) and each problem type was presented more than once to allow students and teachers to re-engage with them on multiple occasions. It is important that students are explicitly taught mathematical language so that they can clarify and communicate their thinking (Queensland Studies Authority, 2013). The integration of mathematical language in the discussion prompts for the images allowed the teachers to address this need. Some examples of the images and accompanying prompts follow. These examples represent some of the different proportional reasoning problem types.

Example 1: Relative and absolute

The image in Figure 1 shows hippopotamuses (very large powerful animals) standing next to an elephant (an even larger powerful animal). The image was used to illustrate that while a hippopotamus is a very large animal (absolute), it appears to be relatively small in the photo when compared to the elephant.

In this example, the accompanying prompts attempted to elicit notions of relative size with suggestions for questions that included:

1. Hippopotamuses grow to approximately 3.5 m in length and can weigh up to 1800 kg which compared to humans is quite large. Why do the hippos not look so large in this photograph?
2. Could we say that hippopotamuses are relatively large compared to humans but relatively small compared to elephants?
3. Give other examples where something can be thought of as large or small depending on the situation? Use the word 'relatively'.

Figure 1. Image of hippos and elephant demonstrating concept of relative size.

Example 2: Fractional thinking, part-part-whole relationships

A photograph of a lizard, which is missing its tail, shown in Figure 2, was used to prompt discussion about part–part–whole fractional concepts and to allow students to use fractions and percentages in their explanations.

Figure 2. Image showing a lizard with missing tail demonstrating concepts of part–part–whole fractional thinking.

The script included the following:

1. What fraction of the lizard's length do you think is missing, and what fraction of its original length do you think is present (part of a whole expressed as a fraction or percentage)?
2. Do these two responses add to a whole?
3. How many times longer do you think the existing part is than the missing part and vice versa (part–part relationship)?

Example 3: Ratio

The image in Figure 3 shows herds of zebra and wildebeest massing at the crocodile infested Mara River in Kenya. They are waiting to cross the river. They often wait for many hours until thousands of animals arrive and then they cross at the same time. The purpose of this image was to engage the students in ideas related to ratio. It also provided teachers with the opportunity to engage students in considering ratio in a context involving chance—the odds of being attacked are decreased when the number of animals increases.

Figure 3. Animals crossing a crocodile infested river demonstrating ratio concepts.

The script elicited notions of ratio through the following prompts:

1. Why do you think the animals do not cross as soon as they arrive at the riverbank?
2. By waiting for other animals to arrive, what happens to their chance of survival when they cross the river? (Changing ratios.)
3. What is meant by safety in numbers? (Draw out the concept of ratio.)
4. Have you (students) ever been in a situation where you felt safer in a crowd? (When and why?)

The implementation process

The teachers used 15 photographs and discussion prompts to engage the students in discussion. The concepts were repeated in subsequent weeks to reinforce the ideas and language. A description of each image with its inherent proportional reasoning concept is shown in Table 1.

One new image was projected on the whiteboard each day for three weeks. It was intended that the activity only take around 10 minutes per day. In Table 1, a sample discussion prompt has been included for each image with relevant strands of the Australian Curriculum; however, the nature of the discussion prompts and the students' responses will influence which strand or strands are relevant to each image. The discussion prompts were provided as a guide for the teachers; however, they were optional for the teachers to use. If the teachers felt confident with the language and the underlying concepts portrayed in the photographs, they could choose to use their own ideas. Similarly, if the students' ideas went beyond the suggested prompts, the teachers were free to extend or expand the discussion or to extend the time allocated to the activity.

The teachers' perceptions of the activity

During the activity, the teachers recorded their ideas as to the success or usefulness of each image in eliciting student understanding of the concepts of proportional reasoning. At the conclusion of the activity, we asked the teachers about its benefits for student learning. They emphasised a number of different aspects.

Student engagement and interest

The unanimous response from teachers was that using the photographs as discussion starters was very effective for engaging their students with proportional reasoning concepts. The teachers noted that students participated in the discussions and found them enjoyable. This positive disposition towards mathematical discussions was a powerful outcome of the activity. Teachers noted students' "willingness to discuss the images" and their "interest in the activity" and that the "images certainly engaged the children and generated a lot of discussion".

The teachers also noted that "students had an open-ended opportunity to engage in discussion about the ideas that the images provoked" and "(the images) led to discussion and debate… it allowed the discussion to be open and illustrated different ways to answer the same question".

Language and conceptual development

The benefit to students' language skills and conceptual understanding of proportional reasoning was regularly noted by the teachers. For example, one teacher stated, "By the end of the images, they (students) were used to thinking proportionally and were using language such as relative, proportion, and ratio", while another reported, "They (students) began to use the language, terms and phrases of proportional reasoning". Throughout the activity, the students were using visual representations of proportional concepts (the images) and engaging in verbal representations through which varying notions about the concepts were shared and discussed. The teachers' observations that discussion of concepts led to powerful learning, reflect research findings regarding the benefits of learning through multiple representations mentioned earlier.

The teachers felt that the sustained (daily for three weeks) engagement with proportional reasoning concepts in the classroom allowed significant conceptual and language development to occur over time. For example, "concept development was a slow progression— a little bit but often" and "it got better as it went on because now they could look at a picture and identify what type of (proportional) situation was involved".

Others stated, "the benefit was the regular use of the language of proportion and the regular familiarity with the general concepts of proportion" and "by the end, they (students) were able to express themselves mathematically using proportional reasoning words". This is an example of the social construction of knowledge emphasised by Kozma and Russell (2005) and Lemke (2001). This approach to engaging students in the use of mathematical language also helped to address the problem noted by McKendree, Small, Stenning and Conlon (2002) that teachers often assume students' understanding of the language associated with mathematical concepts.

Table 1. List of image types and concepts used during the intervention

Image	Inherent proportional reasoning concept	Sample question/task	Australian Curriculum: Mathematics strand(s)
Statue with person standing beside it.	Relative size of statue to person; estimation of height of statue using person as benchmark.	How many times taller is the statue than the man? (Linear scale.)	Measurement and Geometry
Incorrectly drawn graph.	Non-proportional (seemingly proportional but not).	Represent this information proportionally.	Statistics and Probability
Lizard missing its tail (see Figure 2).	Part–part–whole (fractional thinking).		Number and Algebra
Elevator sign: total load and person capacity.	Multiplicative thinking (the average mass of persons used in calculating elevator loads varies).	Compare the different lift notices. Are they all based on the same assumptions (e.g., average mass)?	Number and Algebra Statistics and Probability
Large herds of animals crossing a crocodile infested river (see Figure 3).	Ratio (safety in numbers).		Number and Algebra Statistics and Probability
Elephants and hippos (see Figure 1.)	Relative thinking.		Number and Algebra
Nails needed for attaching fence palings.	Non proportional situation (Seemingly proportional situation).	Why would using more and more nails to attach fence palings not multiply their holding strength?	Number and Algebra
Medieval bread (1.5 m long, 0.5 m wide and 0.3 m high).	Fractional thinking (part-part-whole relating modern-day bread size to very large medieval loaf).	If you wanted to buy a piece of medieval bread of a size similar to a modern day loaf, what fraction of the medieval loaf might you buy? (Answers can vary but reasoning should be provided.)	Number and Algebra Measurement and Geometry
Christmas lights hung across a street in multiple rows.	Multiplicative thinking (using multiplicative strategy to estimate extremely large numbers).	Can you suggest a multiplicative method that would allow you to estimate the number of Christmas lights hung across the street?	Number and Algebra
Town Map.	Scale (using scale to determine times and distances).	Discuss why is it important that a walking map for tourists has a scale on it.	Measurement and Geometry
Museum Plaque.	(WW1 casualties) (see Figure 4)	Absolute/relative (using casualty numbers in absolute and relative terms).	Number and Algebra Statistics and Probability
Small car with large man beside it.	Relative thinking, scale/rate	Why or how would the relative size of a vehicle influence its uses (e.g., family car, pizza delivery car, builder's vehicle)?	Measurement and Geometry
Empty African classroom showing a portion of furniture.	Multiplicative thinking (considering the seating capacity of a classroom set out in rows).	How would the seating capacity of the room vary according to the age and size of the students?	Number and Algebra
Animal size.	Disproportional situations (images from nature illustrating that sometimes animals must instinctively consider proportional situations).	How do animals sometimes have to instinctively think proportionally (e.g., in places or spaces in which they live)?	Measurement and Geometry
Photo of a fish with a matchbox beside it.	Using an object of known size to benchmark the size of another object – relative thinking/scale.	Why is an object with a known size sometimes included in photographs?	Measurement and Geometry

Making connections

The teachers noted several ways in which the activity helped students make connections. Firstly, they noted that engaging the students with different images but similar proportional reasoning concepts over the three-week period allowed the students to make connections between similar concepts in different images, which they felt helped the students to begin to recognise the different concepts involved. For example, one teacher said that when an image involving relative thinking was shown in Week 2, his students noted that the thinking and language were similar to an image discussed in Week 1.

Another student learning benefit noted by teachers was that the photographs allowed the students to use and make connections to their background knowledge and previous experiences. One teacher summarised her perception of this learning benefit by saying, "I really enjoyed that they (students) would look at a picture and think mathematically".

Some images provided a simple and powerful opportunity to engage students in concepts and language through unfamiliar contexts. For example, the plaque in Figure 4 shows the total number of soldiers mobilised from each country during World War 1 with the total killed, followed by the percentage losses. The discussion prompts focused on the absolute and relative losses. For instance, Germany (Deutsches Reich) had the greatest number of soldiers mobilised and killed but they did not have the greatest losses relative to soldiers mobilised; Bulgaria (Balgarija) had the highest relative losses at 22%. While some teachers noted that students initially had difficulty understanding the difference between relative and absolute, many teachers reported that this image elicited great interest and provided an excellent opportunity to engage the students in these concepts.

Conclusion

The photographs used in this activity represented authentic situations. The teachers noted that they interested the students and encouraged a logical link between mathematics and the real world. By engaging students in the proportional reasoning concepts in this new way, the teachers had broadened the students' exposure to the concepts through visual and oral modes. This powerful strategy requires students to make connections between visual images, mathematical concepts and mathematical language and helps them to construct their understanding of the mathematical relationships involved without the need to engage in algorithmic or procedural aspects. Indeed, through this activity, the teachers reported becoming more aware of the need for a general broadening of focus from procedural teaching and learning to include a greater emphasis on concept and language development.

The teachers suggested that this was a useful and valuable approach, not only for engaging students in concepts associated with proportional reasoning but in other mathematical topics or even beyond mathematics. Many of the teachers reported plans to extend the activity by using photographs that they or their students would generate as well as using the idea to enhance learning of broader mathematical concepts.

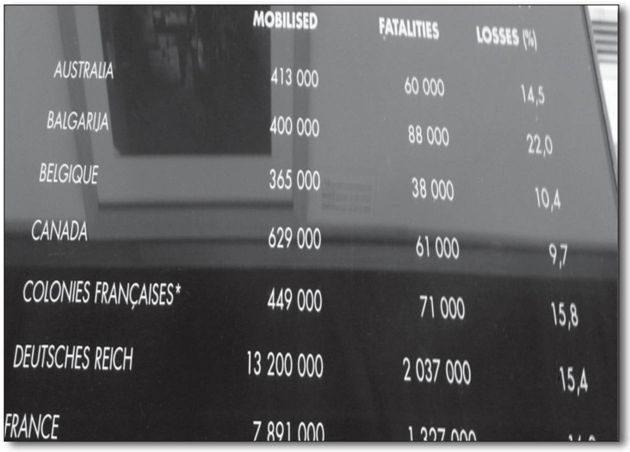

Figure 4. Museum plaque showing absolute and relative World War 1 information

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