Mental Computation Strategies for Addition: There's more than one way to skin a cat

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Marlene Chesney describes a piece of research where the participants were asked to complete a calculation, 16 + 8, and then asked to describe how they solved it. The diversity of invented strategies will be of interest to teachers along with the recommendations that are made. So "how do *you* solve 16 + 8?"

Introduction

Much has been written about mental computation and addition. In the past, many teachers have focussed on teaching the standard algorithm for addition (add the units column, put down the units, carry the tens, and so on); the addition algorithm might be successfully used by students, but its use does not necessarily imply that the concepts behind it have been understood. The great benefits in the use of mental computations are that the student needs to think and understand the numbers used to generate a strategy (McIntosh, 2004, p. 7). Northcote and McIntosh (1999) indicated that 85% of mathematical calculations done by adults involved some mental calculations.

McIntosh (2005) conducted a large study called Developing Computation which involved 34 teachers. By the end of the project, the teachers' views changed considerably: early introduction of formal algorithms in Years 2 to 4 ended and instead mental computation and students' informal written methods were developed.

The purpose of the current study was to see what mental computational methods school children and adults used.

The study

Participants were asked to attempt five questions mentally—one each on addition, subtraction, multiplication, division and fractions—and then to explain their strategy. In this article only the data for the addition question are discussed.

The 30 participants were a convenience sample comprising students in Year 2 (8 students), Year 3 (6 students), Years 6 to 11 (4 students), and 12 adults aged 50 to mid-80s. The addition question 16 + 8 was chosen as it had been used in Callingham and Watson's (2008, p. 67) research. The terms 'add,' 'plus' or 'and' were used. Most participants did not see 16 + 8 written down and the response time was not restricted. Participants were asked to explain how they did the calculation mentally and were only offered a pen and paper if they could not do the problem mentally. Names have been changed.

Results

All of the Year 6 to 11 students and the adults gave the correct answer. Three of the Years 2–3 students gave incorrect answers, however two of these self corrected. The strategies used are summarised in Table 1. Participants have been grouped into older (Years 6–11 and adults) and Years 2–3.

In the following sections, each strategy is examined in detail, in order of sophistication.

Counting strategies

Low level strategies

There were three key counting strategies which were only used by Year 2–3 students: counting all, counting on and skip counting.

Counting all

Only one Year 2 student, Anna, used the *counting all* strategy, counting individual units from 1 to 16 and then counting on another 8. She lost track of her fingers and gave an initial incorrect answer of 20. When given the opportunity to write, Anna produced the following drawing (Figure 1) and then counted these lines getting the correct answer of 24.

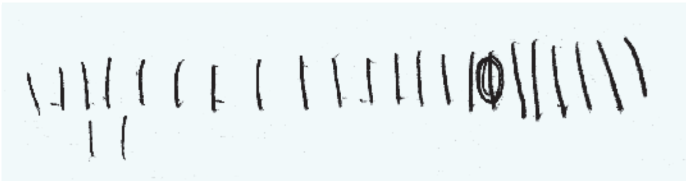

Figure 1. Anna's counting all strategy.

Counting on

Five Year 2–3 students used a *counting on* strategy, starting with the larger number, 16, and counting on 8, saying 17, 18, … 24, by either touching or looking at their fingers as they counted. One student made an error using this method. No student started at 8 and counted on 16.

Table 1. Summary of strategies used for 16 + 8.

		Strategies for 16 +8	Year 2–3	Years 6–11 & Adults	Sub Total	Total	
Low Level strategies	Counting	Counting all	1	0	1	8	8
		Counting on	5	0	5		
		Skip counting	2	0	2		
High Level strategies	Bridging 10	Bridging 10	3	5	8	8	22
	Known number facts	Splitting one/both numbers and using known multiplication tables/10 facts	2	6	8	8	
	Other	Split 10's	1	2	3	6	
		Compensation	0	1	1		
		No explanation given	0	2	2		
			14	16	30	30	30

Skip counting

Skip counting was used by two Year 2–3 students. Both Ken and Jane used their knowledge of multiplication facts to assist them in calculating 16 + 8. Ken, Year 2, started at 16 and counted on in twos, clearly explaining his method as: "16 + 2 = 18, then 18 + 2 = 20, 20 + 2 = 22, and 22 + 2 = 24." Ken had recognised that 8 was an even number and hence a multiple of 2, and used this knowledge. Jane, Year 3, identified that 16 and 8 were multiples of 4 (or were in the four-times tables) and used this information to say, "4, 8, 12, 16, then 20, 24."

Higher-level strategies

Bridging 10

Bridging 10 is a strategy where part of the second number is used to make the first number up to the next multiple of 10, and then the remainder of the second number is added.

Example: 16 + 8 = 16 + (4 + 4) = (16 + 4) + 4 = 20 + 4 = 24. Here 4 was removed from the 8 and then added to the 16, to make 20; then the remaining 4 was added on.

In this investigation, 26.6% (8 of 30) of all participants used bridging 10 in their mental computation; 21% (3 of 14) in Years 2-3 and 31.3% (5 of 16) of the older participants.

Using known number facts

Multiplication or 10 facts were used by 37.5% (6 of 16) of older participants compared to only 14% (2 of 14) Year 2–3 participants. Many of the adults commented that they had learnt their multiplication facts by rote as school children whereas the Year 2–3 students were only beginning to learn multiplication, so they did not have these tools in their kit of skills. Facts relating to multiples of 8 were used by 3 adults (16 + 8 = 2 × 8 + 1 × 8 = 3 × 8 = 24) and there were five other methods used which included use of multiples of 7, double 6, and 10 facts.

Craig, a Year 3 student, was the only student who split both numbers and then regrouped. He said, "I know that 6 and 4 is 10, so then I added another 10 which is 20, then I added the left over 4." Craig had the mental facility to visualise the splitting of the digits and regrouping them. This regrouping of the 6 and 4 was again using knowledge of 10 facts, which is so important for the development of mental computations.

His method was:
$$\begin{aligned} & 16 + 8 \\ &= 10 + 6 + 4 + 4 \\ &= 10 + (6 + 4) + 4 \\ &= 10 + 10 + 4 \\ &= 20 + 4 \\ &= 24 \end{aligned}$$

Other strategies

These fell into two categories: Split 10s and the compensation method.

Split 10s

Split 10s is also known as 'adding units first.' Both the tens and units columns are added separately and the two totals combined. Two older and one Year 2–3 student used this method: 16 + 8: 6 + 8 = 14, 10 + 14 = 24. This was the closest strategy to the formal algorithm.

The compensation method

An example of the compensation method is: 16 + 8 = 16 + 10 – 2 = 26 – 2 = 24. It is called compensation as a larger number than required is added on, the 10, and to compensate for this 2 must be subtracted. This method was only used by one adult subject.

Finally, two adults were unable to articulate an explanation of how they arrived at the correct total.

Discussion

Twenty-nine of the 30 participants successfully answered 16 + 8. However, by giving ample time and listening carefully to the explanations, insights into the participants' number fact knowledge and thinking strategies were obtained. Not one participant used the formal written addition algorithm in their mental computation.

Is there a difference between the groups?

Table 1 shows that over half of the Year 2–3 (8/14 = 57%) students used the low level strategies of counting all, counting on and skip counting. Irons (2002, p. 24) suggests that reliance on counting should decrease as children develop, but some counting may continue for small 'distances' (2 or 3). None of the Year 6–11 students or adults used these low level strategies. High level strategies were used by less than half (43%) of the Year 2–3 students but by the entire older group. By early high school, all participants had moved on from simple counting strategies to the more sophisticated ones, including bridging 10 and use of known number facts.

Over a quarter (26.6%) of participants (five older participants and three Year 2–3 students) used bridging 10; all of these used the method: 16 + 8 = 16 + (4 + 4) = (16 + 4) + 4 = 20 + 4 = 24. They connected the knowledge 4 + 4 = 8 or $\frac{1}{2}$ of 8 is 4, or 2 × 4 = 8 with the 10s fact 6 + 4 =10 to achieve the correct answer.

Bill's error

One Year 2 student, Bill, made a counting error using the counting on method, getting 23 as the answer. Bill said, "Well… I started with the 16 and counted on 8 more in my head." When Bill had another go using his fingers, he still got 23. It was observed by the interviewer that when Bill added on the 8, he started counting from 16 instead of 17; this was an inefficient use of fingers. McIntosh (2006, p. 9) states that "teachers need to observe how they (the student) count and keep track of their counting". This observation was the vital evidence required to distinguish this error being labelled 'a slip', whereas in fact it was the use of an incorrect counting procedure. This information needs to be explained to the student, thus preventing the student continuing with the same error throughout primary school and beyond. This error of 'off by one' was also prevalent in Callingham and Watson's (2008, p. 63) research.

An example of the diversity of strategies

One Year 2 student, Jane, approached 16 + 8 in a non intuitive manner. She said, "I like 7 times tables. I know 3 × 7 is 21 and then I add on 2 and 1." The procedure was 16 + 8 = (2 × 7) + 2 + (1 × 7) +1= (3 × 7) + 2 + 1= 21 + 2 +1 = 24. Initially Jane gave the answer of 23, but then remembered the one and said 24. Jane had a good grasp of seven times tables and drew on these to solve the problem. She showed flexibility in her thinking and was using an invented strategy.

Two older participants used double 6 and the knowledge that for addition, numbers can be split and then regrouped in different ways; this is the associative law. The first three steps in both were identical, but then the regrouping differed.

$$
\begin{aligned}
& 16 + 8 \\
&= (10 + 6) + 8 \\
&= 10 + (6 + 6) + 2 \\
&= 10 + 12 \quad + \quad 2
\end{aligned}
$$

= 10 + 14 = 24	= 22 + 2 = 24

Another adult used knowledge of 10 facts to remove 2 from the 16 and add it to the 8 to make 10. He did 10 + 10 + 4 = 24 which is another bridging 10 strategy.

Helping students use more sophisticated strategies

The use of fingers and other concrete materials to aid calculations is significant before tuition in addition, and continues to be so during children's early years of schooling (Nunes & Bryant, 1996). Manipulatives such as counters, dominoes, 'ten grids', empty number lines and 'bundles of 10 straws' can enable children to develop a robust understanding of the processes of addition and subtraction, which is necessary prior to moving on to written and/or mental addition algorithms. Figure 2 shows the use of an empty number line for 16 + 8.

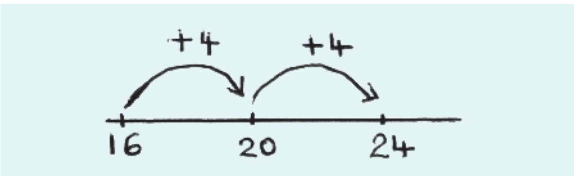

Figure 2

The use of fingers for questions such as 20 + 10 is not an efficient strategy, but strategies using the number five can be encouraged (Wright, Martland & Stafford, 2006). Once a student is confident with 5 + 5 = 10, they can be shown 20 + 10: 20 + 5 = 25, 25 + 5 = 30; five along with ten are essential reference points (Wright et al., 2006).

McIntosh (2006, p. 9) recommended that children need to be encouraged to stop using counting on and back by ones, due to its inefficiency, and to move on to more efficient strategies: "using doubles and near doubles, bridging ten, adding tens, using compatible numbers and using related known facts."

To move a student from counting on to more sophisticated methods, certain skills are required.

- The first is an understanding of place value; that is, that 16 means 10 plus 6. Ask the student to count 16 counters from a pile. Then ask, "Can you explain why we write this number as 16? Show by regrouping the counters."
- Adding tens and units numbers: 20 + 6 = 26.
- Splitting two-digit numbers into component parts: 37 = 30 + 7.
- Knowledge of 10 facts or 'friends of 10' or compatible numbers. These are pairs of numbers which total 10. Example: 4 + 6 =10. These 10 facts are the foundation for the bridging 10 method. Concrete objects such as counters and ten frames aid in the teaching of these concepts and should be available. 'Make compatible' games also provide great reinforcement (Dole & McIntosh, 2004).
- That 2 + 8 = 8 + 2 is a very important skill for students to know. This 'swapping' or 'reversing' or 'spin-around' is known as the commutative law. Once students know this, then the number of number facts required to be learnt is halved. Also, if given 8 + 16, the younger student can re-think it as 16 + 8 which makes the computation easier, regardless of the method used.
- Double facts of 1 to 10. Leutzinger (1999) suggests that an important strategy for children is using number facts they know, such as doubles facts, to assist them in finding answers to related facts such as near doubles. For example: 6 + 7: 6 + 6 = 12, 12 + 1 = 13.

Dole and McIntosh's (2004) Module 2 includes a teaching sequence and excellent resources for teaching all of the above skills. Students need to be encouraged by teachers to use alternative and invented strategies which are efficient and are transferable to other situations; teachers need to value them. One approach is to use games such as those recommended by Tickle and Burnett (2007).

Implications

By early high school, all participants had moved on from simple counting strategies to more sophisticated ones. Teachers need to ask and watch students attempting mental computations, so any errors in procedures can be discovered and remediated. Students need to be encouraged to attempt different methods. The diversity of invented strategies in such a small number of participants totally surprised me, and I am still asking people I meet, "How do you solve 16 + 8?"

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