

IDENTIFYING FRACTIONS



ON A NUMBER LINE



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Monica Wong discusses students' understanding of a fraction as a quantity through the use of number lines. Examples of student work are provided along with a classroom activity.

Fractions are generally introduced to students using the part-whole model. Yet the number line is another important representation which can be used to build fraction concepts (Australian Curriculum Assessment and Reporting Authority [ACARA], 2012). Number lines are recognised as key in students' number development not only of fractions, but whole numbers, decimals, and equivalence (Clarke, Roche & Mitchell, 2008). Number lines afford students the opportunity to compare numbers and promote students' understanding of a fraction as a quantity instead of being considered "one number over another number" (Van de Walle, Karp & Bay-Williams, 2013, p. 294), as often occurs with the part-whole model. The ability to use fractions as numbers without concrete referents is critical for later mathematical development. However, because the focus on fractions as numbers makes the number line a more abstract representation, difficulties inherent with dealing with more abstract mathematics can surface.

Considering the learning demands and conventions of number lines with which students need to become familiar, Wong (2009) investigated students' understanding of the number line model for fractions. First, the conventions of number lines, along with a task which can be used to gauge students' understanding, are presented. This is followed by a description of the strategies students used to identify fractions on a number line and a second task which assists teachers in identifying students' thinking

and reasoning. Finally, a classroom activity designed to enhance students' understanding when identifying fractions is presented.

The structured number line

A structured number line is used to represent mathematical information by the location of a mark or dot on a line. The line is a *linear scale*, a line marked into equal-sized divisions. The distance between zero and one, which represents one or the base *unit*, "is repeated over and over again to form the number

line" (Van de Walle et al., 2013). Variations in number lines make their interpretation complex and difficult at times. Structured number lines (Figure 1) often start at zero, but not always, and the numbers indicated do not have to be consecutive. They typically contain vertical lines or tick marks, which may be above, below or cross over the number line, and delineate sections on the number line. Other differences include the continuation of the number line past the last whole number, with or without an arrowhead at the beginning or end of the line.

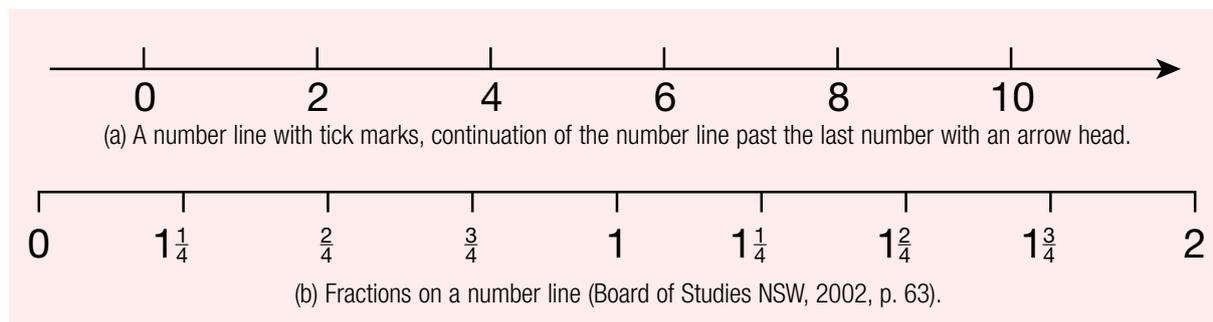


Figure 1. Examples of number lines and their variations.

Fractions on a number line

Fractional quantities extend our number system beyond whole numbers. These quantities are derived when the base *unit* of measure is inadequate and the unit needs to be subdivided into smaller equal-sized divisions. These fractional quantities in combination with whole units provide an

accurate means of measuring (Skemp, 1986). Fractions represent quantities which can be represented by a mark, dot or arrow on a number line (Figure 2). The distance from zero to the mark, dot or arrow represents a quantity which is represented numerically as a *fraction*.

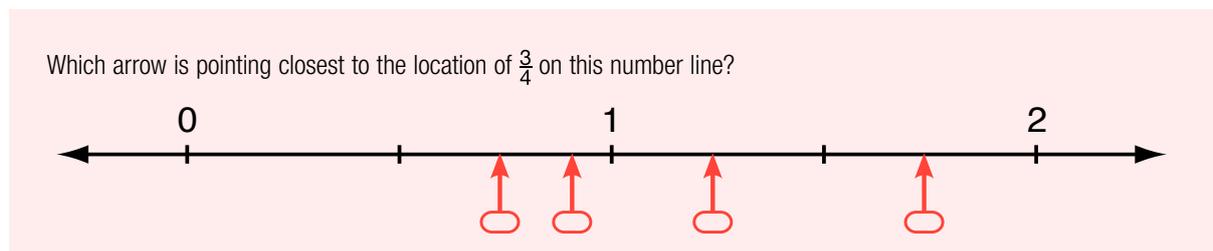


Figure 2. Fractions marked on a number line from a NAPLAN Year 7, non-calculator item. © Australian Curriculum, Assessment and Reporting Authority 2012¹.

Fractions represented on a number line require students to use proportional thinking. As shown in Figure 2, halfway between each consecutive number is marked. The mark between 0 and 1 represents the quantity $\frac{1}{2}$, while $1\frac{1}{2}$ is equidistant from 1 and 2. To find the fractions represented by the arrows A, C, D, students need to divide the half sections

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into two equal parts, thus quarters, and correctly identify the number of quarters that comprises the quantity. To find the fraction represented by the arrow B is more difficult as it requires the division of the section into five equal parts.

The identification of the important features of the number line and understanding the proportional nature of the number line are some of the problems encountered by students. Therefore tasks which can be used to assess students' understanding are needed. The tasks (see Figures 3 and 6) described in the following sections were part of a pencil and paper assessment, *Assessment of Fraction Understanding*, used to evaluate primary school students' understanding of fractions (Wong, 2009). Both tasks were presented to 297 students from Years 3 to 6, attending three primary schools in the Sydney region.

Assessing students' understanding of number lines

The ability to recognise the key features of a structured number line is necessary in order to understand fractions as a quantity. To assess students' understanding and identify potential misunderstandings, one possible task requires students to estimate the location of one on a number line which has zero and one-third marked (Figure 3). This task requires students to consider the proportional nature of the number line in order to identify the size of the unit or location of one.

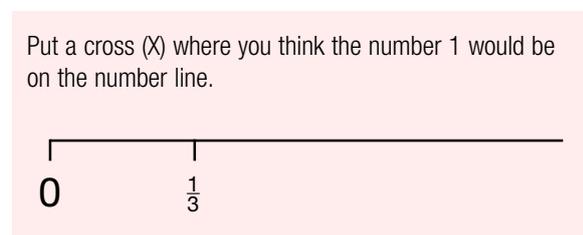


Figure 3. A task used to examine students' understanding of number lines.

Results from the 297 students tested (Wong, 2009) showed that 20.2% ($n = 60$) of students answered the item correctly. These students were able to approximate the unit by iterating one-third three times. In contrast, 22.6% ($n = 67$) of students located 1 at the end of the

number-line by disregarding the scale. Pearn and Stephens (2007) would contend that these students exhibit some understanding of the quantitative aspect of fractions, but not proportionality. Other responses included:

- 17.8% ($n = 53$) of students located 1 at one-third of the distance from zero to $\frac{1}{3}$
- 11.1% of students ($n = 33$) marked somewhere between 0 and $\frac{1}{3}$
- 3.7% ($n = 11$) located 1 at $\frac{1}{3}$.

For these students, the results suggest that one-third is not considered a quantity as the location of one is less than or equal to one-third. For those students that marked one-third of the third, research suggests that these students may view fractions as an action, hence finding one-third of something (Wong, 2009). A further 11.1% ($n = 33$) marked 1 at $\frac{2}{3}$ of the unit, while the remaining 13.5% ($n = 40$) responses showed no discernible pattern. The thinking by these 24.6% of students is unclear. Overall, this task enables teachers to identify some of the potential areas of difficulty that students' encounter when the proportional nature of the number line and the concept that a fraction represents a quantity is not understood.

Identifying fractions on a number line

Using number lines to explore fractions is confusing for many students as their knowledge is typically grounded in the part-whole or area model. This can often limit students' understanding of a fraction, $\frac{x}{y}$, as x out of y parts, rather than considering a fraction as a quantity. This type of thinking promotes the use of a *double counting* strategy to identify a fraction represented on a number line (Ni, 2001). For example, a student may count the total number of divisions on the number line, which represents the denominator, then count the number of divisions from zero to the marked fraction (dot or arrow), which represents the numerator (Ni, 2001). Although the fraction $\frac{3}{4}$ is represented on each of the number lines in Figure 4, variations in counting strategies result in the identification of different quantities. Figure 4(a) depicts the count of equal sections to the dot representing the

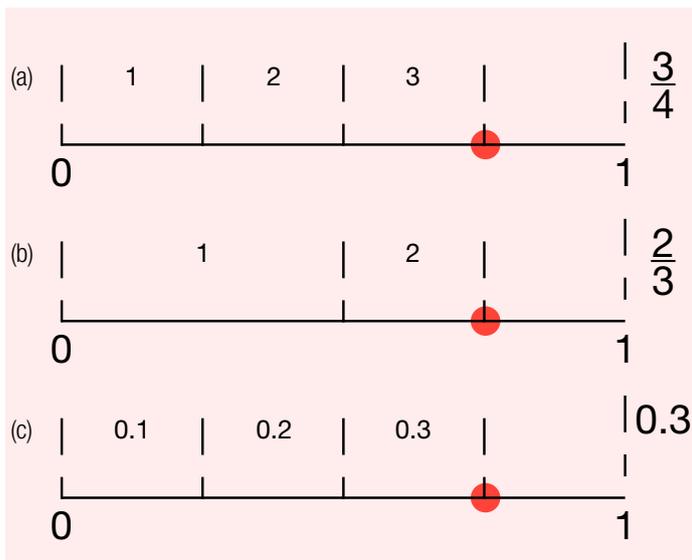


Figure 4. Counting the number of divisions strategy variations used to identify a fraction.

numerator, resulting in the identification of the fraction $\frac{3}{4}$; whereas Figure 4(b) depicts the count but the requirement for equal-sized divisions is ignored (Mitchell & Horne, 2008; Pearn & Stephens, 2007) and therefore the fraction is identified as $\frac{2}{3}$. Another practice exhibited by students is the decimalisation of the number line, thus creating confusion between fractions and decimals (Mitchell & Horne, 2008) with 0.3 identified as the fraction quantity shown in Figure 4(c).

Rather than counting the number of divisions in the double counting strategy, some students count the tick marks (Drake, 2007; Pearn & Stephens, 2007), possibly disregarding fractions as a quantity altogether. Counting tick marks lacks robustness, as the first tick mark defines the start of the first partition, which usually represents zero. Thus five tick marks are needed to create four equal sections or quarters on a number line (Figure 5(a)). To identify the quantity represented by the dot in Figure 5(a), counting does not include the first tick mark, as the first mark represents zero. The remaining tick marks are counted and represents the denominator. The number

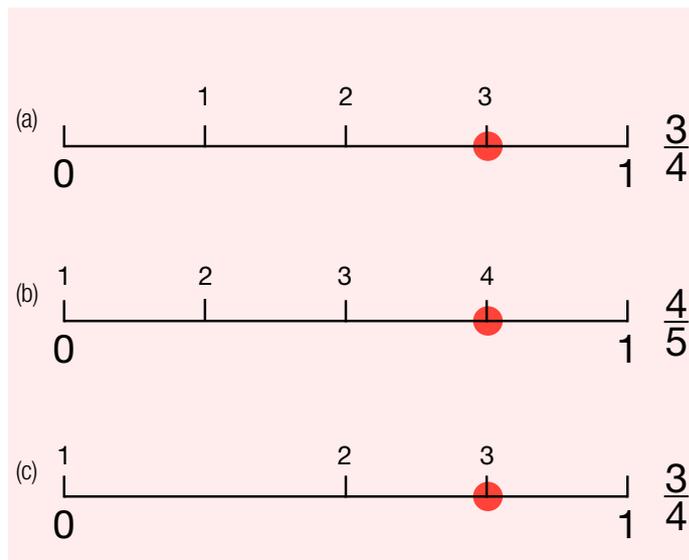


Figure 5. Counting the tick mark strategy variations used to identify a fraction.

of tick marks up to and including the dot is counted, which represents the numerator, hence the fraction $\frac{3}{4}$. Figure 5(b) depicts the count when the first tick mark is counted in the process, providing an incorrect quantity: $\frac{4}{5}$. Further, discarding the need for equal-sized divisions and counting the first tick mark, as shown in Figure 5(c), can result in the correct answer. This may lead to thinking the student understands fractions and number lines but such an assumption would be incorrect. Hence, as educators, we need to consider tasks that are reliable and can alert us to these possible misunderstandings.

Assessing students' strategies for identifying fractions on a number line

When number lines extend beyond 1, students frequently view the entire number line as a single unit, ignoring the scale. Tasks which incorporate a number line extended past 1 can allow the identification of potential misunderstandings. A second task (Figure 6) can be used to identify strategies employed by students when identifying a fraction on a number line. This task requires students to

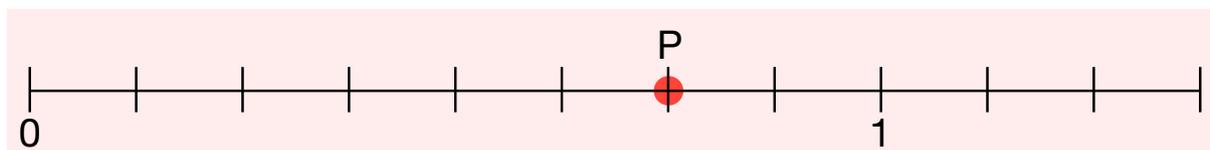


Figure 6. Extending the number line helps identify student errors.

recognise the size of the unit and apply an appropriate strategy to identify the quantity, hence the fraction represented by point P.

The students who attempted the first task (Figure 3) also attempted this task as part of their pencil and paper assessment (Wong, 2009). Results from the 297 students showed that 26.6% ($n = 79$) of students answered the question correctly giving $\frac{3}{4}$ or $\frac{6}{8}$. Other responses included:

- 7.1% ($n = 21$) students gave the response $\frac{6}{10}$, suggesting the entire number line is considered the unit;
- 2.7% ($n = 8$) students gave the response $\frac{7}{11}$, suggesting the counting of tick marks commencing from zero as described in Figure 5(b) along the entire number line;
- 4.7% ($n = 14$) students gave the response 6, suggesting whole number counting of tick marks;
- 24.2% ($n = 72$) students did not attempt the question; and
- 34.7% ($n = 103$) students gave other non-classifiable responses.

These responses suggest that students need to review the features of the number line, which can be undertaken by creating a ruler using informal units.

A classroom activity for students

For students to construct and understand the features of a number line/scale, they need to incorporate zero, recognise the distance between zero and one as a unit of measure, and understand that the unit can be sub-

divided into fractional parts. Students in a Year 4 class were guided through an activity which explored these aspects through the making of a ruler/number line calibrated using an informal unit. The task steps were as follows:

1. Choose an informal unit of measure.
2. Replicate the unit of measure.
3. Create fractional parts (halves and quarters).
4. Construct the number line by placing units end to end without gaps and mark the whole numbers on the number line.
5. Locate and label the halves on the number line by placing the half units end to end from zero. Repeat for quarters.

The final product, shown in Figure 7, was created by Kay. First she chose a straw and trimmed it to her desired length, which become her reference unit. Kay then replicated the unit by measuring and cutting straws aligned against the reference straw. Half units were created by carefully folding the straw in two and cutting it. She then compared both parts to check they were equal in length. If they were not of equal length, they were discarded because she found trimming either or both parts to make them equal reduced the combined length and they would no longer equal the length of her referent unit. Kay then constructed the number line/ruler by taping straws of unit length to a piece of paper and whole units were marked commencing from zero. Using the half unit, Kay then marked halfway between each consecutive whole number. An

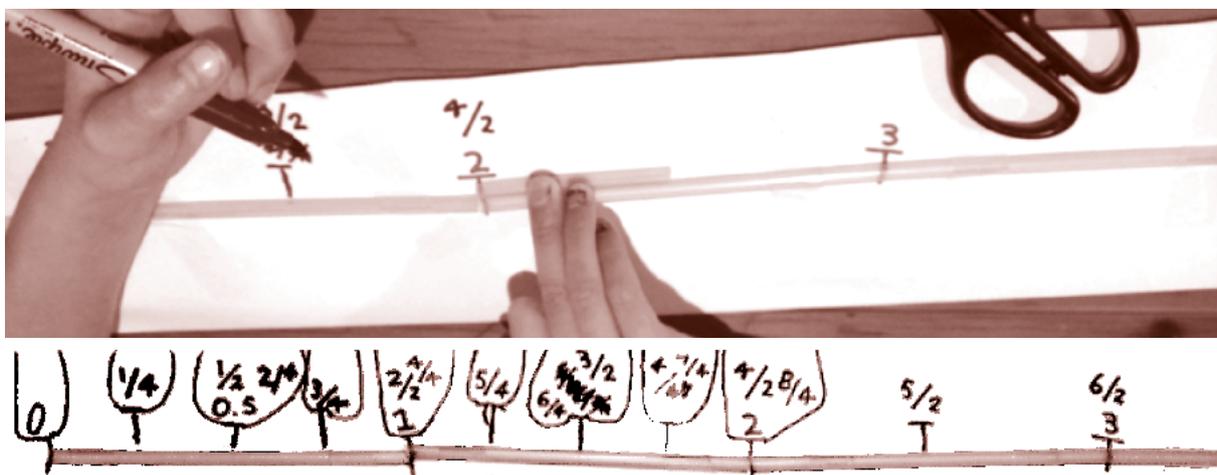


Figure 7. Creating a tape measure using straw units.

alternative method would have been to tape the half straws against the number line, end to end. She then counted the number of half units and labelled the tick marks using improper fractions on the number line. The labelling of quantities using improper fractions required students to interpret $\frac{1}{2}$ not as x out of y , or one out of two, as associated with the part-whole model, but as one-half, where the numerator tells us how many of the unit fraction quantities we have. So $\frac{2}{2}$ is conceptualised as two halves, $\frac{3}{2}$ as three halves—which equals the distance from zero measured by three half-units from zero. The same process was conducted for quarters. Similarly, $\frac{1}{4}$ can be conceptualised as one quarter, and $\frac{5}{4}$ as five quarters, etc. The ruler can then be used to measure the length of objects, and describe them in terms of whole units and fractional parts.

Conclusion

Number lines provide an alternative to the part-whole fraction model and x out of y thinking which becomes inadequate when contemplating improper fractions. Number lines promote students' understanding of fractions as quantities. Fractions can be represented as a dot or arrow on a number line and represent the distance from zero to its marked location. For students to determine the distance represented, they need to recognise the proportional nature of a number line as well as the unit of measure, and then select an appropriate strategy to calculate that distance. These factors make it difficult for some students to view fractions as numbers.

The tasks described in this paper can be used to assess students' knowledge and errors in understanding of number lines and fractions; assessment can provide teachers with valuable information which can be used to progress students' understanding as particular answers suggest specific ways of thinking.

To further promote understanding and rectify errors, the hands-on activity of creating a ruler using informal units and marking a scale using fractions can be used to explore the ideas of proportionality and unit

recognition. This activity enabled students to move on from the x out of y notion of fractions. The ruler activity enabled students to reconceptualise fractions as quantities which are comprised of multiples of unit fractions, e.g., four-halves or $\frac{4}{2}$ or five-quarters, $\frac{5}{4}$. This reconceptualisation of fractions as quantities and the use of the number line further allows the demonstration of fraction addition, subtraction and equivalence, comparison with whole number and decimals within a consistent framework.

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