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Virtual vs. Concrete Manipulatives in Mathematics Teacher Education: Is One Type More Effective Than the Other?

Dr. Annita W. Hunt, Ed.D.
Dr. Kelli L. Nipper, Ph.D.
Dr. Linda E. Nash, Ph.D.
Clayton State University

Are virtual manipulatives as effective as concrete (hands-on) manipulatives in building conceptual understanding of number concepts and relationships in pre-service middle grades teachers? In the past, the use of concrete manipulatives in mathematics courses for Clayton State University's pre-service middle grades teachers has been effective in building conceptual understanding of a variety of mathematical topics. This paper presents the results of a three-year study in which 78 middle grades mathematics teacher candidates used various concrete and virtual manipulatives to study fractions, integers and non-decimal-based numbers. They then compared each type of manipulative for ease of use and helpfulness in understanding the concepts addressed.

Since the inception of Clayton State's University's Middle Level Teacher Education program in 1993, concrete manipulatives have been used successfully in the junior-level Number Concepts & Relationships course. In addition to a textbook, students were required to purchase a manipulative kit that included Cuisenaire Rods, Pattern Blocks, Fraction Circles, and Two-color Counters. Students worked collaboratively to build conceptual understanding of the arithmetic of fractions and integers, using those manipulatives. Students' understanding was assessed by small group and whole class discussions and by performance tests. Certain results occurred consistently, year after year, class after class. Invariably, students declared, "I wish I had been taught this way. This makes so much more sense." And "I always hated *(fill in different topics—fractions, negative integers, etc.)*, but now I understand it."

Rationale

Although successful in the past, are concrete manipulatives the best resource for today's students? The ubiquitous use of various forms of technology by today's youth provides more incentive to investigate the usefulness of virtual manipulatives. Might they be more motivating to twenty-first century students than the more toy-like concrete manipulatives? Considering the meaning of "conceptual understanding" and the often omitted component of bridging from the concrete to the abstract, is there a difference between concrete and virtual manipulatives in the ease of that transition? Is there an advantage to using both types; and if so, does order matter? It is important for teacher educators to investigate these questions, and the teacher education classroom provides the ideal setting in which to conduct this research.

Research Questions

What do teachers perceive as the advantages and disadvantages of each manipulative format? Specifically,

- What roles should each type play in mathematics teacher education?
- Is there a difference in the effectiveness of the two formats to build pre-service teachers' conceptual understanding?
- Is one type easier to use or more readily available than the other?
- Is there an advantage to incorporating both types of manipulatives, and
- If so, does the order in which they are used impact the development of conceptual understanding or students' ability to transition to the abstract algorithms?
- Does one format facilitate bridging to the abstract better than the other?

To answer these questions, we began with a review of current research reports.

Current Research

Conceptual Understanding

Because we are concerned with the enhancement of *conceptual understanding*, it is imperative that we define the expression. Although definitions differ, we will adopt the statement from Hiebert and Lefevre (1986, pp. 3-4) that conceptual knowledge is "knowledge that is rich in relationships… Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network." The relationships formed by the use of manipulatives incorporate visual, tactile, and kinesthetic experiences. Adding cooperative learning and reflective discussion further enhances the depth of understanding and the likelihood of retention (Daniels et al., 1993; Garrity, 1998).

Pedagogical Content Knowledge

Broadening our net to include a focus on pre-service teachers' construction of conceptual understanding in a course that combines content and pedagogy, it is also important to understand the expression "pedagogical content knowledge." In *The Middle Path in Math Instruction: Solutions for Improving Math Education* (2004), Shuhua An examines mathematics teachers ability for addressing and correcting students misconceptions based on the criteria of pedagogical content knowledge. Kilpatrick notes that teachers in An's study have limited success "bridging from manipulative materials to mathematical ideas" (Kilpatrick, 2005, p. 258). Consequently, pedagogical content knowledge plays a vital role in teaching and learning. For this study, the useful implications of An's findings are (1) the importance of having teachers build their own conceptual understanding to enable them to identify and correct their students' misconceptions, and (2) the need to incorporate "bridging construction" into the course.

Research Involving Concrete Manipulatives

Regarding the pedagogical impact of *concrete manipulatives*, there is a plethora of available information. Based on their own experiences, authors of the NCTM S*tandards* from 1989 and 2000 recommended giving students "experiences in using a wide range of visual representations" to solve mathematics problems (2000, p. 284). Many researchers (e.g., Van de Walle, 1973; Grouwns, 1992; Vinson et al., 1997) have found a connection between the use of manipulatives and a decrease in students' math anxiety levels. In their study of pre-service mathematics teachers, Vinson et al. (1997) reported that the use of manipulatives served a "two-fold" purpose:

> First, the concrete experiences aided in pre-service teachers having a better understanding of the mathematical concepts and purposes for procedures. Secondly, the use of manipulatives assisted the pre-service teachers in learning how to teach with more than just modeling a procedure on the chalkboard (p. 8).

Research has also shown the use of manipulatives in the mathematics classroom to be motivating. In her study of high school geometry students, Garrity (1998) found that using manipulatives and cooperative groups motivated her students. She concluded, "In order to give meaning to math teaching, students are best served by learning concepts by actual manipulation of physical materials. Motivation is best accomplished when there is an active involvement with physical objects" (p. 21).

Other researchers also support the "manipulation of physical objects" as a deterrent to "pseudo-learning" (Carin & Sund, 1975, p. 338) and recommend the inclusion of "opportunities for reflection" to balance effective learning practices (Daniels et al., 1993, p. 9). In a study using concrete manipulatives in two 8th-grade pre-algebra classes, Hinzman (1997) reports that students' mathematics performance was enhanced and their attitudes were significantly more positive than those of students from previous years.

Research Involving Virtual Manipulatives

Because the advent of *virtual manipulatives* is relatively recent, the research regarding them is less prevalent than that of concrete manipulatives. Moyer et al. (2002) define a *virtual manipulative* as "an interactive, Web-based, visual representation of a dynamic object that provides opportunities for constructing mathematical knowledge"(p. 185). In addition, the authors identify one's interaction with virtual manipulatives as an example of the process of representing mathematics recommended by the NCTM *Standards*. "Because it is advantageous for students to internalize their own representations of mathematics concepts, interacting with a dynamic tool during mathematics experiences may be much more powerful for internalizing those abstractions" (Moyer et al., 2002, p. 187).

Noting that "students learn in different ways", Schackow (2006-2007, p. 10) describes several activities for which mathematics teachers can use virtual manipulatives to teach fraction concepts to middle school students. She recommends the National Library of Virtual Manipulatives (NLVM) website (http://nlvm.usu.edu/en/nav/vlibrary.html) as one that contains many worthwhile activities. Schackow, however, has expanded the list of concepts for which certain NLVM manipulatives can be used. For example, she illustrates how to use NLVM's virtual Color Chips, Geoboards, and Pattern Blocks to model computations with fractions. In addition, she lists several advantages of virtual manipulatives. They are

- Available—"Teachers may be limited in the quantities and types of concrete manipulatives available to them" (p. 10).
- Time-saving—"Teachers may not have time to make their own manipulatives" (p. 10).
- Motivating—"(Middle school) students may find working on a computer with virtual manipulatives more desirable than using concrete manipulatives that they might view as childish" (p. 10).

Schackow concludes,

> Using the virtual manipulative activities discussed in this article can help students deepen their conceptual understanding of fraction computations and avoid such struggles and frustrations…(and) may lead to student exploration and classroom discussion that will enable students to make sense of fraction computations (p. 10).

Several studies compared the use of concrete and virtual manipulatives to teach mathematics. Brown (2007) conducted an experiment to determine if students who used virtual manipulatives would out-perform students who used concrete manipulatives. The subjects in her study were 48 6th-grade students in an urban public school. Her results indicated that students who received instruction with concrete manipulatives out performed students who used virtual manipulatives, but that both types of manipulatives enhanced the learning environment. Brown's results are suspect, however, since there were differences in the academic ability of the two groups. In addition, the types of virtual manipulatives used (Fraction Bars) were different from the types of concrete manipulatives used (Pattern Blocks). Both differences could have influenced the results.

In their classroom study, Reimer and Moyer (2005) investigated virtual manipulatives, reporting increased success teaching fractions with virtual manipulatives over paper-and-pencil instruction. They also indicate that an advantage of virtual manipulatives is that they provide a connection between dynamic images and abstract symbols (pp. 10-11). Brown (2007) noted the advantage that virtual manipulatives take less time to manipulate.

Other studies, such as those done by Olkun (2003) and Dorward & Heal (1999), indicate that virtual manipulatives are as engaging and provide equally as strong an effect on mathematical understanding as do concrete ones. These mixed findings led us to question what preservice teachers perceive as the advantages and disadvantages of each format of manipulative in teacher education.

Implications for Teacher Education

A survey conducted in Australia by Howard et al. (1997) to determine the use of manipulatives among primary and secondary mathematics teachers raised questions regarding the issue of whether teachers' acceptance of the usefulness of manipulatives has "a solid conceptual base" (p.9). The researchers also indicated that "there is a clearly expressed need…for further training in the use of manipulatives in mathematics teaching", a fact that "has implications for both pre-service teacher education programs and teacher development sessions" (p. 9). Surely the same need exists in the United States. The NCATE/NCTM Program Standards (2003) for Middle Level Mathematics Teachers support investigation of the role of virtual manipulatives in the teaching and learning of mathematics.

Perhaps the most compelling charge regarding the role of virtual manipulatives in mathematics education comes from two Turkish educators, Durmus & Karakirik (2006). They define a *concrete experience in a mathematics context*

> not by its physical or real-world characteristics but rather by how meaningful (are the) connections it could make with other mathematical ideas and situations…Hence, it is very important to encourage learners to reflect on actions they make in order to be able to perceive mathematical processes as objects. (p. 3)

They further advocate, "every student should be given an opportunity to play with manipulatives. Just a demonstration by a teacher is not sufficient to realize their full potential and not in line with the theoretical rationale of their usage since they are meaningful to the extent they involve interactive activities" (p. 4). Durmus & Karakirik concur that "manipulative materials should be used in conjunction with exploratory and inductive approaches" (p. 4) and conclude that

> "Most manipulatives in mathematics simply implement the 'learning ***with*** model' approach. However, educators also need to consider the possibility of designing manipulatives employing the 'learning ***to*** model' approach since full potential of any technological device could be achieved through its usage as a communication tool to model the concepts and relations at hand." (p. 6) (Italics and bolding added.)

Our Study

Theoretical Framework

In general, incorporating manipulatives in teacher education has a two-fold purpose:

- To aid in " teachers having a better understanding of the mathematical concepts and purposes for procedures" and
- To assist them in "learning how to teach with more than just modeling a procedure on the chalkboard" (Vinson et al., 1997).

Our study goes a step further to include a third purpose:

- To engage pre-service teachers in an investigation of the differences in effectiveness of concrete and virtual manipulatives to build understanding of mathematical concepts.

This paper presents the results related to our third objective.

The Participants

Fall semester 2008, 2009, and 2010, a total of seventy-eight aspiring middle grades mathematics teachers were enrolled in Clayton State's junior-level Number Concepts course. Eleven were male and 67 were female. There were 39 African Americans, 31 Caucasians, 4 Asians and 4 Hispanics.

The Method

The course met for 75 minutes twice a week. In 2008, demonstrations by the instructor with concrete manipulatives were followed by students working collaboratively to perform one of the following exercises: (1) find equivalent fractions; (2) add, subtract, multiply and divide fractions; (3) add, subtract, and multiply integers; and (4) add and subtract non-decimal-based numbers. Concrete manipulatives used included Pattern Blocks, Fraction Circles, Cuisenaire Rods, Two-color Counters, and Color Tiles with a (paper) Chip Abacus. Following the use of concrete manipulatives, homework assignments sent students to specific websites to investigate those same concepts using virtual manipulatives. After each concept was studied using both concrete and virtual manipulatives, students completed a survey in which they compared the two types of manipulatives for *ease of use*, *helpfulness to build conceptual understanding*, and *other advantages and disadvantages* of each.

In an attempt to compensate for possible confounding of results, the instructor attempted to alternate the order of concrete and virtual formats with the 2009 and 2010 classes. In both semesters, the attempts proved to be problematic for several reasons. Although Clayton State's students are required to have access to laptop computers, invariably several computers were inoperable. In addition, simultaneous Internet connection for multiple computers proved to be an issue, making the 75-minute class period insufficient for both instruction and practice to occur. However, when these issues occurred each semester, students were regrouped so that every student had computer access in order take advantage of the available resources. Ultimately, by fall 2010 it was becoming more evident that students found concrete manipulatives to be both easier to use and to understand and that using virtual manipulatives second seemed to work well as a bridge between concrete and abstract thinking.

Means of Assessment

After each concept was studied using both concrete and virtual manipulatives, students completed a survey in which they compared the two types of manipulatives for *ease of use*, *helpfulness to build conceptual understanding*, and *other advantages and disadvantages* of each. (Homework assignments and surveys may be found at http://cims.clayton.edu/ahunt/RESEARCH%20HOMEPAGE.htm.

Results

The qualitative data resulting from the surveys were quite interesting and informative. A comparison of participants' responses regarding e*ase of use* and *helpfulness for understanding* by manipulative format and across concepts yielded the following results:

- ≈76% found concrete manipulatives *easier to use* than virtual manipulatives
- ≈ 82% found concrete *more helpful for understanding* than virtual manipulatives.

Table 1 below lists the participants' perceptions of the advantages and disadvantages of each manipulative format.

Table 1

Virtual vs. Concrete: Perceived Advantages and Disadvantages

Advantages

Concrete	*Virtual*
o Simpler, more moveable o Tactile experience adds a dimension of learning o Allows student to be more creative selecting pieces o Student has more control o Process is traceable o Allows trial and error o Units are easier to distinguish, make the whole easier to see o Easier to relate to real-world applications o Less expensive than technology o Allows me to be more cognitive of the operations I am performing o Requires more thinking o Do not make me feel rushed o I was able to think about what it actually means to multiply and divide fractions o Students can be more creative o Allows teacher to involve the whole class in an interactive lesson o It broke the concepts down for me in a way that I will never forget. o Makes the concept stick better o Allow information to be received visually and kinesthetically o Helps me understand the concepts better o "Clarifies misconceptions and builds connections between mathematical concepts and representations, encouraging more precise and richer understandings"	o Immediate feedback—you know when it's right or wrong o Easier to maneuver and keep together o A lot quicker to grasp the concept o Offer a larger variety of experiences o Allow more complex operations to be learned o Catches the attention of the "technology generation" o Actually have to relate (what you're doing with the manipulatives) to the numbers o More accessible at home than the concrete ones o Gives step-by-step instruction, making me see what I was really doing o Makes integer subtraction a lot more clear o Gives hints if you get the answers wrong o Would keep students' attention o Often provides explicit connections between visual and symbolic representations

Table 1 continued

Virtual vs. Concrete: Perceived Advantages and Disadvantages

Disadvantages	
Concrete	*Virtual*
o Limited in the fractions that can be used (i.e., you have only a few denominators) o Can't actually see the numbers on the manipulatives so you may miss the concept o Requires internal affirmation rather than external***** o No feedback on whether you are right or wrong****** o Not very challenging o Doesn't allow you to add or subtract fractions in your head	o Can't actually touch them o No instructions on how to enter a problem o Models for some content not yet available o Sometimes forces you to think abstractly o More suitable for use after a student who's already mastered the concept o Some make it too easy o Computers do the work for the students so they are able to guess the correct answer o May limit the teacher's ability to follow the students' thought processes o I sometimes take the technology for granted and don't obtain a full understanding as to why a problem is solved a certain way o It sometimes forces you to think abstractly o Takes away for the "hands and mind to work together" o Might feel like "do" vs. "learn/explore" o Doesn't allow you to layer pieces to see equivalences o Doesn't really make you find the answer on your own

***** Depending on your perspective, this could be an *advantage*, especially for prospective teachers.

****** One student said, "You have to figure it out for yourself!"

Conclusions

The results of this study clearly indicate that the pre-service teachers who participated found concrete manipulatives to be easier to use and more helpful in building conceptual understanding. Each group of students, however, made comments regarding the advantages of using both types of manipulatives. For example, one said, "I think that both virtual and concrete manipulatives are beneficial for learning the material." Another noted, "I feel confident that if I use both methods, concrete and virtual, (students) will gain a better understanding of fractions overall." Still another stated her belief that "using (both) concrete and virtual manipulatives will help me to differentiate instruction."

Students also expressed the opinion that the order in which the manipulatives are used is important. Most thought experience with the concrete should precede experience with the virtual. One student plans on "using concrete manipulatives first in my classroom and then virtual manipulatives for mastering the math information." Others commented, "the concrete manipulatives won me over as a good method for learning the concept and the virtual manipulatives are a good method for enforcing the concept" and certain virtual manipulatives "would be more suitable for a student that has already mastered the concept instead of for a student who is just beginning to learn the concept." Another noted that "using virtual before concrete can be counterproductive. Computers will do the work for the students so they are able to guess the correct answer and then receive immediate feedback."

In addition to the reasoning about order, an interesting phenomenon occurred. Virtual manipulatives seemed to create a natural bridge from the concrete to the abstract. Students were able to see the connection between the virtual processes and the paper-and-pencil algorithms they had memorized in elementary middle school. On at least one occasion when instructions on how to enter a particular problem virtually were not available, one student worked backwards from the algorithm to figure it out, commenting that using virtual manipulatives "sometimes forces you to think abstractly." Another said, "There is no way I could have done this abstractly without using manipulatives first. It helped me to see the logic behind it all." In summary, our three-year study seems to indicate that:

- Incorporating both types of manipulatives into the instruction of prospective mathematics teachers not only helps them to build their own conceptual understanding but also provides them with sound pedagogical strategies for use with their future students.
- Concrete manipulatives appear to be more effective for building pre-service teachers' conceptual understanding, with virtual manipulatives used to reinforce those concepts.
- The majority of students found concrete manipulatives to be easier to use but not necessarily more readily available than virtual ones.
- There seems to be a definite advantage to incorporating both types of manipulatives.

- The order of manipulative use appears to impact the development of conceptual understanding and the students' ability to transition to abstract algorithms. Using concrete, followed by virtual, is recommended.
- Once conceptual understanding is effected with concrete manipulative, the subsequent use of virtual manipulatives seems to facilitate bridging to the abstract.

We encourage others to replicate our study with various participant groups and to share their results with us. This area of educational exploration is in its infancy, but its impact on mathematics understanding has the potential to become a very powerful tool, especially in the hands of mathematics teacher educators.

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