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Lung Disease, Indigestion, and Two-way Tables

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Abstract

This paper considers the responses of 115 school students to two problems based on information provided in two-way tables. In each case the question asks if one of the variables involved depends on the other. Contextual knowledge might suggest a dependent relationship in both but in one problem the data show independence while in the other the data imply an inverse relationship. A wide range of solution strategies illustrates the cognitive complexity dealing with information in two-way tables.

Introduction

Two-way tables have an identity crisis when it comes to their placement in curriculum documents. Are they part of statistics or part of probability? In the *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2013) they fall under the Chance sub-strand for Year 8 where students "represent … events in two-way tables … and solve related problems" (p. 54). The elaborations for this outcome throw no further light on the types of problems to be solved, except that they involve probability. The *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*, endorsed by the American Statistical Association (Franklin, Kader, Mewborn, Moreno, Peck, Perry, & Scheaffer, 2007), uses an example involving a two-way table to illustrate the recognition of an "association between two categorical variables" (p. 62). This description is a statistical embodiment. The interpretation of two-way tables depends very much on the contexts within which they are placed and the ways in which questions are posed. Gigerenzer (2002) provides a comprehensive contextual background on the importance of being able to interpret conditional information to guide decision-making with regard to risk, particularly with regard to medical testing for disease. He illustrates the link between probability and statistics (e.g., p. 108) by stating a testing outcome first in terms of conditional probabilities in percentages and second in terms of statistical natural frequencies.

In this study a two-way table is defined as a bivariate frequency table with two categorical variables each taking two values. One variable is arranged vertically and the other horizontally. Internally the table consists of four cells containing the frequency count of data satisfying each of the combinations of two values for the two variables. The row and column totals on the two boundaries of the table are sums of the counts for the two values of the individual variables. A grand total of the data count is in the lower right cell. If the values of the two variables are A and not A for one variable, and B and not B for the other variable, with internal frequencies of a, b, c, d, a two-way table has the form shown in Figure 1.

Variable 2		Variable 1: A	not A	Total
	B	a	b	a + b
	not B	c	d	c + d
	Total	a + c	b + d	a + b + c + d

Figure 1. Format for the two-way tables in this study.

Although a few studies had been carried out based on interpreting two-way tables by psychologists and economists as early as the 1960s (e.g., Smedslund, 1963) and Gigerenzer drew attention to issues in the 1990s (e.g., Gigerenzer, 1996), it was the work of Batanero, Estepa, Godino and Green (1996) that brought the topic to the attention of mathematics educators. They presented two-way tables with frequency data displaying three types of relationship: a "positive" direct relationship, a "negative" inverse relationship, or independence. Analysis of problems presented in this way may avoid engaging with Bayes' Theorem but requires a deep understanding and application of proportional reasoning and hence can be a challenge to middle school students. Although the phrase proportional reasoning is not used in Year 8 of *The Australian Curriculum: Mathematics* (ACARA, 2013), students are expected to solve problems with percentages, rates, and ratios (p. 52).

Two of the problems from Batanero et al. (1996) are the basis of this study and are presented in Figure 2. The Lung Disease problem asks if, based on the data presented, Lung Disease is dependent on smoking. Although the expectation from students' experience is likely to be "yes," the data in the table show independence. The Indigestion problem asks if, based on the data, elderly people's indigestion depends on taking a certain drug. Although this context is not likely to be as familiar to students as lung disease and smoking, students are likely to have encountered media stories about drugs causing the side effect of indigestion. The data in this table, however, point to the inverse conclusion, that in fact the drug may result in no indigestion.

Lung Disease:

The following information is from a survey about smoking and lung disease among 250 people.

	Lung disease	No lung disease	Total
Smoking	90	60	150
No smoking	60	40	100
Total	150	100	250

Using this information, do you think that for this sample of people lung disease depended on smoking? Explain your answer.

Indigestion:

The following information is from a study to assess if a certain drug produces indigestion (stomach trouble) in elderly people.

	Indigestion	No Indigestion	Total
Drug taken	8	8	16
No drug taken	7	1	8
Total	15	9	24

Using this information, do you think that the elderly people's indigestion depends on taking the drug? Explain your answer.

Figure 2. Two problems from Batanero et al. (1996).

Although in the early 1990s, *A National Statement on Mathematics for Australian Schools* (Australian Education Council [AEC], 1991) mentioned associations for bivariate data being "weak, strong, positive or negative," there has not been a great deal of research with school students on problems based on two-way tables. From a teaching perspective of considering conditional probability, Gigerenzer and Hoffrage (1995), Pfannkuch, Seber, and Wild (2002) and Watson (1995) suggested two-way tables be used as an easier method of solving Bayes' Theorem problems than the traditional formula approach. More recently, Watson (2011) employed them in untangling some complex claims found in the media. From a research perspective, Watson and Callingham (2005) used the Lung Disease problem as a survey item in a study confirming their statistical literacy hierarchy (Watson & Callingham, 2003), as did Watson, Callingham, and Donne (2008) in documenting students' responses and teachers' attempts to remediate incorrect answers. The problem was also used in in-depth interviews with teachers related to various aspects of pedagogical content knowledge (Watson & Nathan, 2010). To the authors' knowledge, the Indigestion problem has not been used in published research since the work of Batanero et al. (1996).

Although Estrada, Roca, and Batanero (2006) considered two-way tables, they were investigating conditional probability not the dependence or independence of variables.

Several issues arise in relation to these two problems for middle school students. Although often in assessment situations, correct final answers are rewarded with full marks, for these two problems it is possible to give the correct answer without complete analysis of all of the data provided in order to eliminate alternate solutions. In a two-way table, for example, a student might focus on one large number to claim a positive relationship, without checking to see if there are other numbers in the table that would challenge this conclusion. Hence it is important to provide a structural framework for responses that rewards justifications that employ the necessary elements of the problem for a valid conclusion, providing a set of levels of response of increasing quality. For problems of the type used here, it is also important to document the interference that occurs from the context set and the beliefs held about it by the person answering. Further, because there is a difference in the type of association in the two problems (no relationship and an inverse relationship) it is of interest to explore the association in levels of response between the two types of relationship.

Because of the presence of the underlying concepts of conditional and proportional reasoning associated with the two problems in the middle school curriculum, the question arose as to whether the performance of students improved consistently across the years of schooling.

These issues led to the following research questions.

1. To what extent can a hierarchical structure explain understanding displayed in middle-school students' solutions to the problems?
2. What degree of interference occurs from students' previous beliefs about the contexts of the problems?
3. What is the degree of association between levels of response to the two problems although they display different types of relationship?
4. Does performance at the cohort level change over the middle school years on these problems?

Method

Sample

The students in this project were students of the teachers participating in a 3-year professional development project in statistics education for school mathematics teachers (StatSmart) (Callingham & Watson, 2008, 2011). The students had been part of the project with their teachers for at least two years and had previously completed three designed longitudinal surveys. At the end of the project the final survey for this particular group of students included the Indigestion problem, not included in previous surveys, as well as the Lung Disease problem, which had been a linking item on all previous surveys. There was no requirement of the project that the teachers teach the content associated specifically with these problems, and there was no evidence from other teacher sources that teaching of the topic had occurred. The numbers of students in each year level are given in Table 1. For analysis purposes, data were combined in pairs of years for Years 6/7 and Years 8/9. Year 7 was in the elementary school for students in one Australian state, and in secondary school for the other two and there were low numbers in Year 6. The students were from three Australian states (Tasmania, 69; Victoria, 23; South Australia, 23). Fifty-one percent were male and 49% female.

Year	6	7	8	9	10
N	8	20	21	23	42

Analysis

Repeated reading of the responses to the Indigestion problem led to a hierarchical coding by the first author, which was then applied to the Lung Disease problem responses. The second author independently confirmed the coding and any discrepancies were discussed and agreement reached. This approach led to a slightly expanded coding scheme compared with that used in previous studies (Watson & Callingham, 2005). The coding scheme, shown in Table 3 has six levels of response. These followed the hierarchy identified by Watson and Callingham (2003). This Statistical Literacy hierarchy is shown in Table 2. It is characterised by both engagement with context and the application of statistical skills, both of increasing complexity.

Table 3 shows the final codings used for both problems. At Code 1 idiosyncratic responses were likely to have made sense to the student but were incorrect. At Code 2 knowledge about the context in which the problem was set from the student's perspective was presented in a sequence to support a belief in a conclusion. Code 3 responses related together knowledge about study design and questioned issues such as sample size, data collection, and lurking variables, but provided no final justified answer. Such an open-ended argument demonstrated bringing relevant knowledge of the context together with some qualitative application of statistical skills. At Code 4 only a single number or one variable was used to suggest a response but there was clear understanding of the context. Code 5 responses used two numbers from different cells to draw a conclusion but without using enough information from elsewhere in the table to eliminate other possibilities. There was no use of any form of proportional representation. At Code 6, there was clear evidence of proportional reasoning, using data from four of

Table 2. *Statistical Literacy Hierarchy (from Watson and Callingham, 2003)*

	Level	Characterization
Level 6	Critical mathematical	Critical questioning engagement with context; use of proportional reasoning; appreciation of uncertainty in making predictions; interpretation of subtle language.
Level 5	Critical	Critical questioning engagement with context without use of proportional reasoning; appropriate use of terminology; qualitative use of chance and appreciation of variation.
Level 4	Consistent non-critical	Appropriate but non-critical engagement with context; multiple aspects of terminology use; limited appreciation of variation only in chance settings; some statistical skills such as mean, simple probabilities; and use of graphs.
Level 3	Inconsistent	Selective engagement with context; appropriate recognition of conclusions with little justification; qualitative use of statistical ideas.
Level 2	Informal	Colloquial or informal engagement with context often reflecting intuitive beliefs; single elements of complex terminology; basic one-step calculations.
Level 1	Idiosyncratic	Idiosyncratic engagement with context; tautological use of terminology; simple mathematical skills such as reading a single value in a table.

the cells related together with the variables to justify a correct solution. The percentage of Code 2 responses reflected the degree to which the contexts influenced the answers to the questions.

All students supplied assessable answers to both questions and scores were assigned to each according to the coding in Table 3. Indicative correlation coefficients were used to explore the relationship between the responses to the two questions. Using the means and standard deviations for each year group, 95% confidence intervals were calculated for the differences in means for each question. These were used to gauge development and sustainability across the middle school years.

Results

Research Questions 1 (Hierarchical structure) and 2 (Interference of context)

The percentages of response levels for each of the three year groups are given in Table 4. These are then exemplified with quotes from students across the years, with note taken of Code 2, reflecting students' prior beliefs about the contexts of the problems.

Table 3. *Coding for student responses to two-way table problems.*

	Category A	Not-Category A	
Category B	Internal Cell	Internal Cell	Row Total
Not-Category B	Internal Cell	Internal Cell	Row Total
	Column Total	Column Total	Grand total

Code	Description (with reference to table at the right)	
6	Provides evidence using all 4 internal cells or a pair of inside cells with the corresponding row or column totals.	*Lung Disease*: to conclude no association of smoking and lung disease. *Indigestion:* to conclude the inverse effect, less indigestion with the drug.
5	Provides evidence from two internal cells or one internal cell and a corresponding row or column total.	*Lung Disease*: usually to conclude dependence of disease on smoking without considering if other cells would show the same relationship (some additive comparison of the two cells). *Indigestion:* to conclude any of three possibilities (positive, negative, or no relationship).
4	Provides evidence explicitly from one internal cell or from one of the variables represented in either row or column totals (that is, recognizing only one variable).	*Lung Disease*: May use "most" or "more" but without reference to other possibilities. *Indigestion:* Often focuses on the single person in one cell.
3	Critical analysis of potential survey methods, the limitations of the methods used to collect the data, and/or the sample size.	
2	Conclusion based on knowledge of the context or opinion, but not on the data provided.	
1	Idiosyncratic attempt: numbers in contradiction to own claim or impossible to decipher.	
0	No response or no justification for a "yes" or "no."	

Table 4. *Percentage of levels of response across your levels*

Year	Code 0	1	2	3	4	5	6	N	Mean	Std Dev
	Lung Disease									
6/7	14%	4%	43%	7%	11%	18%	4%	28	2.64	1.73
8/9	5%	7%	18%	16%	7%	20%	27%	44	3.84	1.89
10	2%	5%	24%	5%	7%	26%	33%	42	4.21	1.83
Total All Years	6%	5%	26%	10%	8%	22%	24%	114	3.66	1.92
	Indigestion									
6/7	32%	0%	14%	0%	18%	14%	21%	28	3.00	2.43
8/9	7%	5%	0%	16%	5%	39%	30%	44	4.41	1.77
10	10%	7%	2%	10%	14%	26%	13%	42	4.29	1.89
Total All Years	14%	4%	4%	10%	11%	28%	29%	114	4.02	2.06

Responses coded 0 were either blank or gave a response with no justification.

- [Lung] Don't smoke. [Year 7]
- [Indig] I'd say yes. [Year 10]

Code 1 responses made idiosyncratic claims or used numbers that could not be interpreted.

- [Lung] It doesn't depend on smoking. But from smoking and non smoking. The difference is greater with 90 getting lung disease and 60 not that's 30 people difference and non smoking is 20 people difference. [Year 10]
- [Lung] 10 more people with L/D were smoking. Maybe? [Year 9]
- [Indig] No because the numbers for indigestion and no indigestion are the same and no drug taken with indigestion is the same as drug taken with digestion. [Year 9]
- [Indig] It depends if the drug is taken because there were fairly even numbers for having indigestion. There were 8 people who took the drug and had indigestion and 7 who didn't take the drug and still had indigestion. So the drug is really the cause. It's just chance. [Year 10]

Responses that were based on beliefs about the context, not considering the data presented were coded 2. With reference to Research Question 2, for the Lung Disease problem this was marginally the most popular response, whereas there was virtually no influence on the Indigestion problem.

- [Lung] No you could get from car smoke so anyone could get it. [Year 6]
- [Lung] Smoking does cause lung disease I <u>think</u> it [is] now a fact. [Year 7]
- [Lung] Yes and No. Because people can still get lung disease without being a smoker. [Year 8]
- [Indig] No not really, anything can cause stomach trouble, you don't have to take something for it to happen. But in some cases it does and has affected stomach trouble, I guess it's just how well your stomach can take to things. [Year 10]
- [Indig] No. About half the people that took the drug got indigestion so scientists could look at why that is. But out of 7 people that did not take the drug only 1 person didn't get indigestion. [Year 7]

Some responses criticised the design of the study (Code 3).

- [Lung] No. The reason why I think this is because the sample size is much too small to make a decision. [Year 6]
- [Lung] You cannot make an accurate assumption as the same amount of people weren't tested. [Year 9]
- [Indig] No, they could have got more people to even it out and then it would be the same. [Year 8]

- [Indig] You can't make an accurate assumption. It's not a clear representation as the amount surveyed n [who took] the drug taken [*sic*] is different from that of those who didn't take the drug. Also, this is only a small amount of people from the total population and you don't know any other factors such as whether they all ate the same food etc. [Year 9]

The responses that used data from one cell or only one variable (e.g., row or column totals) to reach a conclusion were coded 4.

- [Lung] Yes it depends on smoking because 90 people with lung disease is smoking and that is most of the people out of this graph.

	Lung Dis	NoLgDis	Total
Smoking	**90**	**60**	**150**
NoSmkng	**60**	**40**	**100**
Total	**150**	**100**	**250**

[Year 6]

- [Lung] No smoking has less total.

	Lung Dis	NoLgDis	Total
Smoking	**90**	**60**	**150**
NoSmkng	**60**	**40**	**100**
Total	**150**	**100**	**250**

[Year 10]

- [Indig] Yes because with no drug taken there is only 1 person with no indigestion.

	Indigest'n	No Indig	Total
Drug	**8**	**8**	**16**
No drug	**7**	**1**	**8**
Total	**15**	**9**	**24**

[Year 7]

- [Indig] More people take the drug and less people take nothing. The elderly are more dependent on drugs.

	Indigest'n	No Indig	Total
Drug	**8**	**8**	**16**
No drug	**7**	**1**	**8**
Total	**15**	**9**	**24**

[Year 9]

Code 5 responses employed information from two cells to reach a conclusion of "Yes," "No," or "no relationship." Although the conclusion may have been technically correct the response did not consider other cells that might have contradicted the conclusion.

- [Lung] Yes because 90 smokers had lung disease but 60 non-smokers had it.

	Lung Dis	NoLgDis	Total
Smoking	**90**	**60**	**150**
NoSmkng	**60**	**40**	**100**
Total	**150**	**100**	**250**

[Year 7]

- [Lung] The majority of people that smoke had lung cancer, so yes!

	Lung Dis	NoLgDis	Total
Smoking	90	60	150
NoSmkng	60	40	100
Total	150	100	250

[Year 10]

- [Indig] Yes I do. When the drugs are taken they have more stomach problems than when they don't take the drugs.

	Indigest'n	No Indig	Total
Drug	8	8	16
No drug	7	1	8
Total	15	9	24

[Year 8]

- [Indig] With the people which haven't taken the drug most of them have indigestion so I think it's wise to take the drug.

	Indigest'n	No Indig	Total
Drug	8	8	16
No drug	7	1	8
Total	15	9	24

[Year 7]

At the highest level (Code 6), responses used all four internal cells or two internal cells and the associated row/column totals to reach the appropriate conclusion. For the Indigestion problem, some students said "No," not meaning "no relationship" but meaning "no the opposite to what would be expected."

- [Lung] No, they have the same ratio. [Year 8]
- [Lung] 90/150 = 180/300. 60/100 = 180/300. For this sample of people, I don't think lung disease depended on smoking. [Year 9]
- [Lung] No, because the same percentage of people had lung disease in both tests. [Year 9]
- [Lung] No. In both, 3/5 people will have lung disease [Year 10]
- [Indig] If you don't take drugs it is rare to get no indigestion but if you do it is a 50-50 chance. [Year 7]
- [Indig] I think that it actually would help prevent indigestion as 1/2 of the people who had taken the drug got indigestion and 7/8 of the people who hadn't taken the drug got indigestion. 1/2 = 4/8 7/8 > 4/8 [Year 9]
- [Indig] When the drug is taken it's a 50-50 chance of getting indigestion but when there is no drug is a 1 in 8 chance of not getting it. [Year 9]
- [Indig] No. People who take the drug have a lower ratio to people who don't. [Year 10]

Research Question 3 (Association of responses)

For each year level grouping, the correlation of response levels between the two questions is given in Table 5. Only for Year 10 was the relationship statistically significant. The consistency of performance across the two problems increases over the year groupings. In Year 10, although the average performances were not significantly different from those in Years 8/9, it was much more consistent across the two problems.

Table 5. *Correlation between problems for year groups*

Year	6/7 (n = 28)	8/9 (n = 44)	10 (n = 42)
Correlation	– .009 (*ns*)	.145 (*ns*)	.553 ($p < 0.001$)

Research Question 4 (Cohort change)

The 95% confidence intervals for the differences in means between the three pairs of year group for each question are shown in Table 6. For both questions the differences between Yr 6/7 and Yr 8/9 and between Yr 6/7 and Yr 10 were statistically significant, whereas the differences between Yr 8/9 and Yr 10 were not.

Table 6. *Confidence intervals for mean differences between each pair of year groups for each question*

Question	Comparison	Mean Difference	95% Confidence Interval
Lung Disease	Yr 8/9 – Yr 6/7	1.20	(0.32, 2.08)
Lung Disease	Yr 10 – Yr 8/9	0.37	(–0.43, 1.17)
Lung Disease	Yr 10 – Yr 6/7	1.57	(0.70, 2.44)
Indigestion	Yr 8/9 – Yr 6/7	1.41	(0.42, 2.40)
Indigestion	Yr 10 – Yr 8/9	–0.12	(–0.91, 0.66)
Indigestion	Yr 10 – Yr 6/7	1.29	(0.25, 2.32)

Discussion

The structure used for analyzing these two problems recognized two types of engagement with them: using the data provided in the problem or using other information known to the student. These elements—the engagement with the social context using information known to the student and the application of statistical skills using the data provided—are characteristic of increasing levels of statistical literacy (Watson & Callingham, 2003).

The necessity of context before statistics can make sense has long been recognized (e.g., Rao, 1975) but in this study there is the question of whether it can interfere with the statistical reasoning that takes place. Hence, it is interesting to note that Code 2 responses represented 26% of responses to the Lung Disease problem but only 4% for the Indigestion problem, probably reflecting the more common understanding in society about issues of lung disease and smoking than about drug usage and indigestion. The difference in the percentages supports the view that it is genuine interference of beliefs, rather than a desire to avoid engaging with the mathematics that accounts for the high value for the Lung Disease problem; otherwise it would be expected that the percentage of Code 2 responses would be the same for both problems. Code 3 arguments were the basis for 10% of the responses to both problems, indicating the consistency of the concern for methodological issues across the two problems. Except for Code 2, the percentages for the codes were quite similar.

The Codes 4 to 6 recognize the use of an increasing number of data values in the solution. The need to account explicitly for all data to eliminate alternate interpretations reduced the number of appropriate decisions given credit compared to the analysis of Batanero et al. (1996). It is important for teachers to be aware of this necessity and to ask students for a complete justification rather than a single answer, perhaps based on one or two cells in the table. The move to proportional reasoning, such as the use of percents to make comparisons, is an important component of the middle years' curriculum. The fact that nearly 30% of all students reached this level for Indigestion, compared with nearly one-quarter for Lung Disease, suggests that mathematically the Indigestion problem was slightly easier, even though the context appeared to be less familiar. The implication for teachers is that they need to develop generic statistical skills in their students that can be applied regardless of context.

On one hand it might be suggested because each problem was presented with data in a two-way table that students would use the same method of solution and there would be a high correlation of scores. On the other hand, the existence of different types of association represented and the different contexts in the data of the two problems might suggest otherwise. There was no association of levels of response for Years 6/7 students suggesting there was little recognition of similarity by these students (Table 4). Although the correlation of levels of response was positive for Years 8/9, it was not significant, whereas the correlation for Year 10 was significant ($p < .005$). This may represent an increasing recognition of the similarity of the problems with more years of experience in mathematics classrooms.

This extra experience in mathematics classrooms by Year 10, however, does not appear to have resulted in a continued increase in level of performance on the two problems that was observed from Years 6/7 to Years 8/9 (Table 5). Given the presence of topics associated with percentages, rates, ratios, and two-way tables in about Year 8 of middle school curricula (e.g., ACARA, 2013; AEC, 1991; Franklin et al., 2007; National Council of Teachers of Mathematics, 1989), it is perhaps not surprising that performance increases significantly from Years 6/7 but then levels off. At least it does not regress to a large degree. It should be noted, however, that continued review of the topics associated with these problems is needed in the senior years of secondary school, otherwise the skills will be lost. Considering complex conditional problems as suggested by Watson (2011) is exceedingly difficult without the basic skills and awareness of the need to consider all information presented in cells of two-way tables.

Implications and Conclusions

Although this study took place in Australia, the topic and the results are also relevant in other countries. The broad issues related to risk, raised for example by Gigerenzer (2002), are relevant to all statistically literate adults across the world. He provides many cases of misunderstandings, which could hopefully be reduced in number with attention paid to two-way tables in the middle and high school years. In the United States this is recognized in the *Common Core State Standards for Mathematics* (Common Core State Standards Initiative, 2010) in Year 8 under "Investigate patterns of association in bivariate data."

> Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables (p. 56).

Further at the high school level the *Common Core* recommends using two-way tables to "decide if events are independent and to estimate conditional probabilities" (p. 82). These two extracts show that the *Common Core* appreciates the important role two-way tables have to play in the meeting of statistics and probability. Along with stressing that data are "numbers in context" (p. 79) the *Common Core* also recognizes the importance of proportional reasoning at every year from Year 6 (e.g., pp. 41, 46, 52, 67).

This study has highlighted the complexity of analyzing two-way table problems and emphasized the importance of asking students to justify their answers. It has shown that the context of the problem may interfere with decisions due to pre-conceived ideas and that at times students can reach a correct conclusion without eliminating alternative possibilities. It is important for teachers to be aware of the large variation in possible answers and explanations in order to assist students and assess appropriately. Although this study was not about teachers' pedagogical content knowledge (PCK) (Shulman, 1987), the understandings explored here should be part of the knowledge teachers incorporate into their classroom PCK.

It is rather surprising that more research has not been carried out in relation to two-way table problems, particularly in comparing or contrasting associations of variables that are positively or negatively related, or independent. Further studies could look at the overall magnitude and relative size of the numbers in the cells of tables and whether this contributes to the difficulty. Also of interest is the relationship of the context chosen for the data presented and the likely pre-conceived ideas that students may have about it. Additionally, a future study investigating the levels of understanding of their teachers as was done by Watson and Nathan (2010) for the Lung Disease problem based on interviews with the teachers might be fruitful. There may be a need for professional learning for teachers, particularly in relation to inverse relationships.

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