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Assessing Teacher Change in Facilitating Mathematizing in Urban Middle Schools: Results of an Effective Professional Development Program

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This study documents the change in teaching practices of a group of mathematics teachers in urban middle schools as they participated in a program of professional development to promote standards-based learning environments. The teachers made a shift in their classroom practice from a traditional, didactic lecture approach towards a role of facilitating conditions suitable for students' explorations and mathematizing. The stages of development the teachers experienced are described with regard to three critical domains: Pedagogy, Use of Mathematical Tasks, and Focus on Mathematics.

Reports from the National Council of Teachers of Mathematics (NCTM, 1980, 1989, 2000) and other influential organizations (National Commission on Excellence in Education, 1983; National Research Council, 1989; United States Department of Education, 1999), recommend that teachers rethink their goals and practice of teaching mathematics. Mathematics education has shifted away from characterizing mathematics as a discipline of facts, procedures, and formal proofs that can be transmitted by articulate teachers to diligent learners, towards one based on the constructive activity of learners, as suggested by NCTM (1989, 2000) standards. However, prior research indicates that a lack of success in some efforts to implement standards-based mathematics learning environments may be due to the fact that teachers generally have not been prepared for this endeavor (Hiebert, 2003). How might we educate teachers for this paradigmatic shift? Mewborn (2003) asserted, "Teachers' thinking needs to be at the center of professional development sessions just as children's thinking needs to be at the center of mathematics instruction" (p. 49). Therefore, the more providers of professional development know regarding teachers' thinking about mathematics teaching and learning, the better they can provide programs that facilitate teachers' standards-based practices in mathematics classrooms.

The purpose of this study is to contribute to a research base relating to effective teacher development by documenting teacher change within a teacher-development program. Of key concern when assessing the impact of a teacher-development program is whether the teachers have actually become better mathematics teachers, and if so, how did their teaching practices change? What pedagogical practices have evolved to support students' learning? Do the teachers select and make use of appropriate tasks to support students' constructions? Are they able to recognize the mathematics in students' thinking and use that understanding to provoke students' further growth of mathematical understanding and ideas? In order to understand this change, teachers' lessons were observed, studied, and assessed within three significant domains: Pedagogy, Use of Mathematical Tasks, and Focus on Mathematics.

Theoretical Framework

This research is based on the view that building mental representations is the foundation of doing mathematics. Rather than a discipline or body of knowledge (concepts, skills) to be transmitted, mathematics is defined from the perspective of "mathematizing" – the activity of interpreting, organizing, and constructing meaning of situations with mathematical modeling (Freudenthal, 1991). Mental images formed by individuals are used in building representations of mathematical ideas (Davis, 1984). To represent an idea, an individual may create a model or present a notation. Although the internal, cognitive representations are not available to us, certain features of them are made public as ideas are explained, justified, and shared with others.

The representations that students build are constantly reexamined and modified as they participate in mathematics activities (Tarlow, 2004, 2008, 2010; Warner & Schorr, 2004). According to Davis (1984), thinking about a mathematical situation involves cycling through a series of steps. The first step involves building a representation of the input data. This representation may be a concrete representation, such as building a model with physical objects. This view is consistent with that of Papert (1980) who described the metaphoric use of previous concrete experiences as a basis for building abstract ideas. From the data representation, a memory search takes place to construct a representation of relevant knowledge that can be used in trying to solve the problem. A mapping between the data representation and a knowledge representation is constructed. During this process, checks are made, and some representations may be modified or rejected.

After students have built their own representations for a problem task, they seem ready to listen to the ideas of other students (Maher & Martino, 1996). In doing so, their ideas may be challenged or supported. The resulting interactions may lead students to reject, modify, consolidate, or strengthen an original argument

(Maher & Martino, 1991). As learners cycle between representations in building justifications for their ideas, new knowledge is constructed. Personal explorations combined with social interaction support the modification and refinement of the students' theories.

The opportunity for students to test their ideas and hear the ideas of other students also provides a setting for the teacher to listen to and assess the thinking of the students involved in discussion. As the teachers monitor the thinking of their students, they are better able to pose timely questions that encourage students to build a deeper mathematical understanding (Maher & Martino, 1996). Davis and Maher (1997) asserted that it is essential for teachers to be aware of students' thinking about a mathematics problem and to continuously strive to estimate the nature of children's representations. Teachers' knowledge of students' thinking makes it possible for them to challenge and extend students' mathematical understanding. The role of the teacher becomes one of promoting conditions suitable for students' exploration. In this setting, the teacher's role shifts from conveyor of information to one of observer and moderator of children's thinking (Burns, 1985). Opportunities to develop mathematical reasoning and ideas and to build convincing arguments as justifications are supported in carefully crafted classrooms with learning environments designed to invite students to revisit, review, modify, and/or extend earlier ideas (Maher, 1998).

Davis (1993) asserted that a teacher cannot effectively tell children how to construct mathematical ideas or why they are important. They could tell them some sequence of words, but the words would not elicit any meaningful representation of anything at all. "'Telling' might give them some words, but it would not help them to build up the metaphoric mental imagery that is the basis for true understanding" (Davis, 1993, p. 299). Therefore, according to Davis (1994):

> If one takes seriously the various new suggestions about the teaching and learning of mathematics--if, for example, one takes seriously the NCTM Standards (NCTM, 1989)--then one is faced with asking teachers to play a quite new role… It will not be easy for teachers to shift to the new role—working alongside students, trying to be aware of the student's thinking, working to help the student modify that thinking in an appropriate way—instead of standing in front of the class and giving a lecture. (p. 17)

The importance of building teaching practices based upon knowledge of student thinking requires that research guide us in understanding how teacher development programs can facilitate teachers' development in three critical domains--pedagogy, use of mathematical tasks, and focus on mathematics--in order to create learning environments that support students' mathematizing and construction of mathematical ideas. Just as children's learning in mathematics can be characterized by the process of construction, so too is the learning process with teachers. In order to build new ideas about mathematics teaching and learning, the teachers in this program were engaged in experiences that involved on-site teaching, reflection, and conversations in an environment where learning is seen as constructing and mathematics is taught as mathematizing.

Method

Professional Development Activities

Professional development activities took place within a model designed to deepen teachers' content and pedagogical knowledge and to create and sustain collaborative learning communities. The model integrates: (1) on-site staff development in schools by the researcher, which includes planning, co-teaching, and "debriefing" lessons in the teachers' classrooms; and (2) guided inter-visitations to classrooms with teachers of a similar grade level, using a form of lesson study, in which teachers collaboratively plan, observe, and analyze actual classroom lessons, drawing out implications for teaching and learning. The inter-visitations are designed to build knowledge and community. To develop a truly collaborative community, teachers need to see each other in action. Teachers are asked to empower students in their classrooms, and in these professional development activities this approach is modeled by empowering teachers to ask questions and to expand their thinking about best practices.

Data and Analysis

The research design[1] is a case study, which took place during three consecutive years. Thirty-six middle-grade (6-8) teachers, in two urban schools with diverse student populations, participated in the teacher-development activities, although not all of the teachers were in the program for all of the three years due to faculty changes. Careful field notes were taken during all professional development activities and participating teachers kept a journal of their observations and reflections. Data were coded and interpreted in order to draw conclusions and make inferences about teacher development with regard to three domains: Pedagogy, Use of Mathematical Tasks, and Focus on Mathematics.

Results

Pedagogy

Stage 1. In the initial stages of the program, the teachers' pedagogical practices were based upon a transmission model. A typical lesson involved the teacher telling and showing the students how to perform a procedure, and afterwards the students practiced using the procedure with the goal of obtaining the correct answer, which was previously calculated by the teacher. Questions that teachers asked the class were primarily to obtain the correct answer or to ask if anyone had any questions. Students were not required to explain their reasoning or to provide justification for their thinking. During the occasions that the teachers asked students to explain how they "got" their answers, the sought-after response involved a student stating the steps followed when performing the procedure. Homework generally consisted of a page of additional problems to practice. The following day, "Do Now" activities were typically a set of problems to provide students with additional practice, using the previous day's procedure or reviewing procedures from previous units for review.

Stage 2. Teachers at stage two of development began to move away from telling and explaining towards supporting students' constructions of concepts, skills, and ideas. However, at this stage, the teachers were using pedagogical strategies in a routine manner, rather than in response to assessment of where students were in their development of reasoning about the particular mathematics being investigated. For example, teachers posed questions that asked for students' reasoning, but then did not follow-up the response to make the most of the opportunity to build upon the

[1]The research design was motivated by a collaboration with Catherine Fosnot and Antonia Cameron, with whom I co-directed a teacher-development initiative in New York City K-8 schools for two years.

student's mathematical thinking. At this stage, the typical teacher response to the student's reasoning was, "Does anyone disagree?" or "Does everyone agree?" or "Does everyone understand?" or "Does anyone not understand?" In addition, although the teachers allowed more "wait time" following questions posed, the amount and occasions of wait time were not always appropriate to the degree of complexity of the question posed. Furthermore, when teachers planned for students to share their solution strategies, the end of the lesson was just that: sharing, by teachers selecting random volunteers to come to the board to show their work. The teachers did not plan and orchestrate a carefully scaffolded discussion of strategies, models, and big ideas.

Stage 3. Teachers who progressed to stage three were teaching to support students' constructions of mathematical ideas. The operative behaviors of these teachers were: genuine questioning to reveal and build upon students' thinking; appropriate use and amount of wait time; and supporting class discussions in relation to students' construction of strategies, models, and mathematical ideas.

Use of Mathematical Tasks

Stage 1. At stage one, teachers did not use open-ended problem-solving tasks or the tasks chosen required predictable procedures that the students had been practicing. This behavior was observed during both instruction and assessment. Activities required students to state memorized facts or to perform procedures. When "word problems" were used, the task was like that of previously practiced problems in class, such as finding the sales price of a discounted item.

Stage 2. Teachers at stage 2 of development used mathematical tasks, but the tasks chosen did not support students' mathematizing. For example, proportion tasks were not realistic: a runner's rate on a treadmill was given with the assumption that that rate remained constant. In other cases, the context of the tasks did not connect to the mathematics of the problem. For example, a problem gave the age of a child and an algebraic relation to the age of the father, but determining the age of the father had no connection to the context of ages. In addition, tasks used at this stage often sparked no student interest, such as a problem that required students to use the Pythagorean Theorem to find the distance from second base to home plate on a baseball diamond. Overheard was the comment, "Just Google it."

Stage 3. In stage 3, teachers used realistic tasks with genuinely problematic situations that supported the students' mathematical development. Tasks were selected or designed so that students would have several entry points to approach the problem; there was more than one possible strategy to find a solution, and there was no predictable path to find the solution. In addition, the tasks could be modified to meet the needs of all learners. For example, centimeter graph paper was available for students who needed support for drawing grids to find the solution to the Border Problem, which requires students to predict, find, and then generalize the number of shaded squares on an n x n grid. The tasks provoked discourse, justification, and connections.

Focus on Mathematics

Stage 1. For teachers in stage one, the teachers' mathematical focus was on students' accuracy in the use of teacher-modeled procedures and students' production of teacher-anticipated answers. At this stage, the teachers were not able to "see" the mathematical ideas, misconceptions, or alternate strategies in the students' work, in order to support the students' development of a higher level of mathematical understanding.

Stage 2. Teachers who progressed to this stage became more aware of the mathematics in the students' work and began to build upon those ideas. They were sometimes able to "seize the mathematical moment" during class discussions in order to support students' development of strategies and big ideas, but not consistently. When questioning about concepts, the focus was on understanding the particular mathematical concept of the day, rather than fostering students' connections among strategies; extending the students' broader understanding of the mathematical big ideas; and of supporting the development of students' mathematical cognitive ability.

Stage 3. At this level, teachers took advantage of most of the "math moments" during class discussions, thus actively facilitating the students' construction of conceptual understanding and mathematical ideas. This was most often observed during the "sharing" portion of the lesson; teachers at this level scaffolded the discussions from less efficient to more efficient use of strategies or they used students' presented models to provoke students' justifications, connections, and generalizations.

Conclusion

Results of this study indicated that there were significant changes in the teaching practices of the teachers who participated in this teacher-development program in each of three domains: Pedagogy, Use of Mathematical Tasks, and Focus on Mathematics. Initially, the teachers primarily based their practice on teaching by telling and modeling; they seldom used appropriate mathematical tasks to support students' mathematizing; and they rarely capitalized on the "math moments." During the three years of the teacher-development program, the teachers' practice underwent change towards facilitating students' mathematical growth, and the teachers developed in all three domains. The teachers whose pedagogical practices initially involved teaching by telling (Stage 1) moved towards routinely supporting students' constructions (Stage 2). Teachers who participated in all or most of the three years of the teacher-development activities moved towards genuinely facilitating students' construction (Stage 3). The teachers' practice developed similarly in the other domains. Teachers' pedagogy changed; they were better able to select and use appropriate mathematical tasks; and, more often they noticed and used the "math moments" to support students' constructions. "Teacher turn-over" restricted some of the teachers' participation to one year or less, and none of those teachers exhibited stage three behaviors with consistency.

Implications

Although there are limits to conclusions and generalizations that can made on the basis of a case study, this research provides an opportunity to examine teacher change in a successful teacher-development program, with regard to Pedagogy, Use of Mathematical Tasks, and Focus on Mathematics. In doing so, we can gain insight into important characteristics that will be useful in creating effective teacher-development programs that empower teachers and support collaboration, in order to facilitate students' mathematizing and promote student learning. This has important implications for curricula and pedagogy for schools and teacher education programs, as well as for college and university curricula.

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