Designing Approximations of Practice and Conceptualising Responsive and Practice-Focused Secondary Mathematics Teacher Education

Matthew P. Campbell  
West Virginia University  
Rebekah Elliott  
Oregon State University

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Opportunities for teacher candidates to investigate and enact the work of teaching in settings of reduced complexity—what are called “approximations of practice”—offer a promising path toward preparing more ambitious and equitable mathematics teachers. However, these approaches face the risk of not preparing individuals to continue ambitious and equitable practice within the socially- and culturally-defined work they are called to do in schools. In this paper, we discuss findings from ongoing design-based research in secondary mathematics teacher education around considerations for approximations of practice that are more “responsive” to school settings. We discuss analyses of methods courses and concurrent student teaching placements. Two main design considerations have emerged around the way in which approximations of practice are tied to the content and goals of school classrooms and how they specify and explicate the structure and complexity of teaching practice. From these findings, we propose future design and research opportunities.

Keywords · secondary mathematics teacher education · practice-focused teacher education · approximations of practice · teacher education design · design-based research

Introduction

There is a pressing need and call to prepare new mathematics teachers skilled at ensuring each student has access to rigorous academic work to develop mathematical proficiency and meet the demands of an increasing mathematically, statistically, and technologically complex society (Darling-Hammond et al., 2009; Education Workforce Advisory Group, 2010; Kilpatrick, Swafford, & Findell, 2001; National Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010). Teachers must draw upon students’ diverse cultural and linguistic resources in the mathematics classroom and position mathematics as a human practice and a tool for social change (Ministry of Education, 2011, 2013; R. Gutiérrez, 2011). These “ambitious and equitable” goals (Jackson & Cobb, 2010) have charged teacher educators with the task to prepare teacher candidates (TCs) with the knowledge and skills to teach in ways that are not common in school classrooms, while also preparing them to quickly step into classrooms and meet demands of accountability around student achievement on high stakes tests (Lampert et al., 2013). The effectiveness and adaptability of teacher education programmes
in taking on that task has been questioned and criticised, thus giving rise to alternatives that do not necessarily resolve the prevalent issues of teacher preparedness (Darling-Hammond, 2010; Ell & Grudnoff, 2012; Kumashiro, 2010; Louden, 2010; Wiseman, 2012). Often evoked in discussions on factors affecting the potential impact of university teacher education is the challenge of navigating what Feiman-Nemser and Buchmann (1985) identified as the “two worlds” of the university and the school classroom and the role of each setting in the development of teachers.

Addressing this divide has given rise to approaches to teacher education that are more practice-based (Forzani, 2014; Zeichner, 2012)—both in terms of field-based teacher development experiences, as well as the use of artefacts of practice (such as video and student work) in teacher development contexts. An emerging body of work also advocates for approaches that are “practice-focused” by identifying and developing the components of the work that teachers are called to do in their support of student development (Anthony, Hunter, Hunter, Rawlins, Averill, Drake, et al., 2015; Grossman & McDonald, 2008; McDonald, Kazemi, & Kavanagh, 2013). These efforts build on research that has contributed to a better understanding of what teachers need to do to realize ambitious and equitable goals in the classroom (Anthony & Walshaw, 2007; Ball & Forzani, 2009; National Council of Teachers of Mathematics, 2014). Supporting a practice-focused approach are purposefully crafted approximations of teaching (Grossman, Hammerness, & McDonald, 2009) that reduce the complexity of teaching while providing TCs opportunity to investigate, enact, and develop skill with a core set of ambitious and equitable teaching practices. While these approaches to teacher education serve as an emerging and promising trend, we contend that there has not been a focus on the outcomes across the settings of the university and the school classroom nor on what those outcomes tell us about these innovative teacher education pedagogies. Research is needed on how these pedagogies shape (and are shaped by) the cross-setting development of instructional practice among TCs.

In our work, we investigate and further conceptualise the responsiveness of the design and implementation of practice-focused teacher education approaches. By “responsive”, we mean attending to the goals, expectations, and common ways of working in schools while at the same time making progress on ambitious and equitable mathematical goals. This, in part, serves as a reinterpretation of the two worlds at play in the development of teachers. Specifically, instead of viewing the two worlds of the university and the school classroom as a divide that needs to be bridged or resolved, we consider how the differences of the two worlds need to be acknowledged and embraced. The cross setting work of teacher education is something to be leveraged, not considered as a barrier. In this paper, we explore this idea of responsiveness in teacher education by sharing analyses and findings from ongoing design-based research around the development and use of approximations of practice in secondary mathematics teacher education. We examine the evolution of the design and use of these approximations to detail an emerging set of design considerations as well as contributions to further conceptualisation and specification of responsive and practice-focused teacher education. We close with thoughts on further innovation and research opportunities in this arena.

**Focusing Teacher Development on Practice**

In this work, we draw on data and analyses from our efforts across secondary mathematics teacher education programmes in the U.S. focused on the development of skilled practice among TCs and effort toward more responsive designs. In order to further specify and articulate these emerging approaches to teacher education, we ask the question: What are
considerations for the development and use of approximations of practice of secondary mathematics teaching that are more responsive to the work of teaching in school settings? In this section, we draw on literature to further define these ideas and to establish the theoretical and conceptual foundations for this work.

**Activity theory as a theoretical lens**

Our work is rooted in sociocultural perspectives of teaching and teacher development, which motivate our attention to responsiveness as well as pedagogies of teacher education that focus on the development of skilled practice. We see teaching as a cultural practice (Stigler & Hiebert, 1999) informed by the histories, goals, and available tools of settings in which the practice occurs (Rogoff, 2003). Drawing on activity theory (Leont’ev, 1981; Wertsch et al., 1984) as a more specific theoretical lens, what a teacher does and how they develop as a practitioner is contingent on the work they are called to do as a teacher in a particular school setting and how they engage in tool-mediated action toward particular goals (Wertsch, 1991). The actions of a teacher—of all teachers—are not separate from the context in and for which those actions occur—the norms, rules, and expectations; the roles and division of labour; and the multiple communities that have a stake in the object of the actor’s actions. We contend that conceptualising and researching teaching and teacher development—and designing teacher education—demands coordinating these aspects in order to be consistent with our theoretical stance.

This lens necessitates a reframed perspective on the relationships between professional education and school settings—from one that is seen as a unidirectional flow of information and tools from the university (intended to “fix” the common workings of a school), to a more responsive and bidirectional relationship (Cobb et al., 2009; Kazemi & Hubbard, 2008). Our interpretation is that teacher education designs must be responsive to the settings in which teachers teach—the work they do in those settings, the tools and cultural artefacts they use, and the goals toward which they are oriented—in the midst of efforts to establish more ambitious and equitable mathematical goals and teaching practices. Teacher educators must consider what teachers are enabled to do in the settings in which they work, not as a concession to the status quo in that setting but as an acknowledgement of the need for pedagogical innovations to be seen as useful by teachers and actually be useful in accomplishing the work of teaching as it is defined in schools. While it is often the concern of teacher educators and researchers alike that examples of ambitious and equitable teaching in schools are scarce (Anthony et al., 2015), a responsive approach would involve attending to both the prevalent practices of a particular setting as well as the needed steps toward improved mathematical opportunities for students. For example, a common practice of secondary mathematics classrooms in the U.S. is the review of homework problems. Responsiveness between a teacher education programmes and school settings could involve honouring the expectations that homework be assigned at the end of each class and that time be spent reviewing those problems while also developing supports for teachers to discuss those problems with students so they have opportunities to leverage the structure of problems and develop valid generalisations and processes—both valuable mathematical proficiencies with which students should engage. Through the lens of activity theory and from a perspective of responsiveness to the practices, tools, and goals of a given setting, we review and evaluate two sets of ideas in teacher education—clinical practice and practice-focused pedagogies—to motivate the work we share in this paper.
Viewing recommendations and advances in teacher education using activity theory

Emphasis on clinical practice. The push for situating teacher preparation in the setting of schools—physically and conceptually—has been a trend in teacher education for some time (Forzani, 2014; Zeichner, 2012). Accreditation committees and governmental policy in the U.S. and in New Zealand have called for increased accountability that assures TCs demonstrate instructional capacity to support diverse learners—measured, in part, by student performance on high stakes tests (Education Workforce Advisory Group, 2010; Ell & Grudnoff, 2012; National Council of Teacher Education [NCATE], 2010; U.S. Department of Education, 2015). Policy consistently advances the benefits of and need for extensive classroom experience in teacher education, meeting a call for curricular coherence and rigorous connections to the work of teaching (Council for the Accreditation of Educator Preparation [CAEP], 2013).

Amidst these recommendations and assertions, research continually shows a lack of consistent findings to suggest TCs’ clinical experiences are universally beneficial (Anderson & Stillman, 2013; Clift & Brady, 2005). Valencia, Martin, Place, and Grossman (2009) attribute this to the lack of specificity on what is entailed in teacher development within clinical settings, which divert candidates away from gaining skill and sensibilities for supporting diverse students meeting ambitious content goals. The notion that TCs will develop as professionals in the midst of the complex and challenging work without direct attention paid to specifying what is important to develop—or how its developed—leaves too much to chance rather than purposeful design (Ball & Forzani, 2011). Anderson and Stillman (2013) raise concern regarding the promotion of clinical teacher development as such claims are based on a primary focus on candidate self-report and cognitive constructs (such as beliefs) rather than TC practice and its connection to K-12 student performance. They and others (e.g., Clift & Brady, 2005; Valencia et al., 2009) suggest attention be paid in studies to theoretical and analytic approaches that considers the complexity of the settings of teaching and teacher development and contextual factors mediating practice.

Practice-focused teacher education and the use of approximations of practice. Aside from merely situating teacher development in settings of teaching, teacher education research and development has recently looked to truly put the work of teaching at the centre of teacher education by identifying and focusing on what have been called “high leverage” or “core” practices (Anthony et al., 2015; Ball & Forzani, 2009; McDonald, Kazemi, & Kavanagh, 2013; Windschitl, Thompson, Braaten, & Stroupe, 2012). Core practices are things that teachers do in high frequency and have been shown to be linked to improvements in student achievement. For example, Jackson and colleagues have identified ways that highly effective teachers open or launch lessons in routine ways (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013). Other practices that have gained traction within teacher development literature include building explanations, (Leinhardt & Steele, 2005), orchestrating discussion through the use of discursive moves and effective patterns and strategies of questioning (Chapin, O’Connor, & Anderson, 2009; Franke, Kazemi & Battey, 2007;) purposeful decisions about the mathematical ideas to elicit during a lesson (Stein, Engle, Smith, & Hughes, 2008), and orienting students to one another’s ideas and to big ideas in mathematics (Hunter, 2008). A focus on core practices allows teacher educators to address the complexity of teaching with integrity, yet do so in a way that it can be taken up with TCs in the limited time available in teacher education.

In our work, and from an activity theory perspective, we consider these core practices and the teacher strategies that carry them out in the classroom as a set of pedagogical tools for the work of teaching. With a view of tools and the way in which they enable the action of an actor toward particular goals in particular activity settings, we are careful to think about the extent to
which the pedagogical tools put forth in teacher education at the university serve as tools that enable individuals to realise what is expected of them as teachers in schools. Our concern, then, is that a focus on core practices runs the risk of not truly supporting the development of teachers for their work in schools and, thus, maintaining the perception of a divide between the two worlds of the university and the school classroom that is to be bridged by ideas from the university moving into the classroom by way of teacher educators and TCs.

These risks continue with the practice-focused teacher education pedagogies that have emerged in light of the recommendations of Grossman and colleagues (2009). In an attempt to develop skilled practitioners among TCs with a set of core practices, teacher educators and researchers have considered the use of approximations of practice (Grossman et al., 2009) as an opportunity for TCs to enact the core components of the work of teaching in contexts and situations of reduced complexity (Anthony & Hunter, 2012; Kazemi et al., 2009; McDonald et al., 2013). One form of approximation of practice is what Lampert & Graziani (2009) call “instructional activities”, or IAs, which serve as containers of core practices, pedagogical tools, and principles of high-quality teaching (Kazemi et al., 2009). Toward this end, IAs are designed to structure the relationship between the teacher, students, and the content in order to put a teacher in position to engage in and develop skill with interactive practices around facilitating rich discussions about mathematics. Concrete tools—what we will call a “planning protocol”—can further establish that structure by specifying the beginning, middle, and end of an IA, how materials should be used, and how space should be organised (McDonald et al., 2014). In the case of elementary mathematics, short activities such as number talks, choral counting, and strings of computational problems have been used as IAs (Kazemi, Lampert, & Franke, 2009; McDonald et al., 2014). All of these activities lend themselves to a focus on number and operation at the elementary level, and have the potential to become routine ways for a teacher to structure their interaction with students and particular types of content.

IAs and the practices that they contain serve as a focus of a set of activities in teacher education that provide TCs the opportunity to both investigate and enact the work of teaching (Anthony et al., 2015; Lampert et al., 2013; McDonald et al., 2013). This consists of observing, decomposing, and planning the IA; rehearsing the IA in the teacher education setting and in school classrooms, potentially with in-the-moment coaching from a teacher educator; and using artefacts of practice such as video and student work to analyse instruction and make connections between teaching practices, student learning, and a broader vision of ambitious and equitable mathematics teaching. Taken together, these opportunities for the investigation and enactment of practice are intended to make progress toward the main aspects of teacher development, as outlined by Hammerness and colleagues (2005): a vision of ambitious and equitable practice; knowledge of students and content; dispositions regarding students, content, and teaching; and a repertoire of practices and tools (Ghousseini & Herbst, 2014).

Our concern is that, through these innovative, practice-focused pedagogies of teacher education, TCs can ultimately become skilled with practices and pedagogical tools that enable work toward goals that are not representative of or reconcilable with those in school settings. Lampert and colleagues (2013), who have been at the forefront of articulating and enacting practice-focused pedagogies in elementary mathematics teacher education, concede:

We need to know whether these principles, practices, and knowledge carry over into novices’ classrooms, whether or not they are doing particular IAs. But as we conceive of the commitment to enact this kind of teaching as socially constructed, we need to understand what impact the schools and districts in which these classrooms are situated have on novices maintaining the capacity to do what they have learned (p. 15).
In this reflection, Lampert and her colleagues acknowledge that the work of teacher education—no matter how innovative or focused on ambitious and equitable teaching—does not exist in a vacuum. Due to the situated nature of the work of teaching and teacher development and also due to issues of accountability, others are also taking seriously the need to consider the way TCs are supported for their work in school contexts beyond the teacher education setting (Anthony et al., 2015). To extend this stream of ideas, we operate under the assertion that teacher educators must use an understanding of school settings and the work done in those settings to inform what they hope TCs develop through teacher education. Our hypothesis is that as practice-focused designs are better tied to and derived from the activity of teaching in schools, while also looking to make progress toward more ambitious and equitable goals, TCs would be better supported in developing skill as practitioners in ways that are viable and enabled in the school settings in which they teach.

We see the design of approximations of practice (such as the cycles of investigation and enactment around IAs), the principles and practices they are designed to contain, and the way in which they are used across settings in teacher education playing an integral role in an effort for responsiveness in practice-focused pedagogies of teacher education. This motivates an ongoing process of design-based research that we conduct across multiple institutions on the development and use of approximations of practice in secondary mathematics teacher education. In this paper, we share data, analyses, and findings from that ongoing work from which a set of design considerations for the development and use of approximations of practice in responsive secondary mathematics teacher education has emerged.

**Methods**

Design-based research (e.g., Edelson, 2002; K. Gutiérrez & Peniel, 2014) provides us the methodological considerations and tools to systematically examine and build broader ideas from our own design context around approximations of practice and responsive secondary mathematics teacher education. Design-based research is an iterative process of design, implementation, analysis, and redesign (Cobb et al., 2003; Design-Based Research Collective [DBRC], 2003). Not only has design-based research served in strengthening our own designs in secondary mathematics teacher education, but also in articulating a set of design considerations for the development and use of approximations of practice and in contributing broader implications for responsive and practice-focused teacher education pedagogies. We provide details on our research context, our data, and our analyses in the sections below.

**Context and participants**

Our work occurs across multiple institutions across the U.S. and continues to develop and evolve. The focus of this paper stems from work set within one of those sites—a small Master’s-level teacher licensure programme in secondary mathematics. The focus of the design is set in a sequence of two, ten-week secondary mathematics methods courses and in TCs’ field-based placements occurring during and after the methods courses. Within these methods courses there has been a focus on a specified set of core practices and the use of the investigations and enactment of an emerging set of IAs as a central pedagogical feature. The core practices identified as a focus were adapted from others in the field and centred on the work of eliciting students’ reasoning, orchestrating mathematical discourse, orienting students to one another and to the mathematics, and teaching toward mathematics goals. One concern of ours, which we discuss more in our findings, was the issue of “grain size”—that these practices are
component parts of the larger work of teaching and cannot be considered in isolation, though carrying out these practices involves the use of smaller-grained teacher moves (Boerst, Sleep, Ball, & Bass, 2011). Like Kazemi and colleagues (2009) we saw the role of an IA as being a container for these nested “sizes” of teaching practices.

In this paper we focus on three design cycles that took place during the 2012-2013 academic year. With “design cycles”, we refer to the work of the teacher educator, including the selection or construction of an IA (including the production of planning protocols for each IA) and support of TCs across the various phases of the cycle of investigation and enactment, including the coached peer rehearsals and the arrangement of “designed settings” (McDonald et al., 2013) in which TCs could enact an IA with classroom students. The design process involved the two authors (with one serving as the instructor of a given methods course), a third mathematics teacher educator, and consultation with a middle or high school mathematics teacher serving as a host for the rehearsals of an IA. During the year of this study, we partnered with two teachers—a middle school mathematics teacher, Ms. Calhoun, during the first design cycle and a high school mathematics teacher, Mr. Ellison, during the second and third cycle (all names of partner teachers and TCs are pseudonyms). While these two partner teachers were also hosts for TCs more traditional student teaching practicums (and their existing relationship with the programme contributed to their involvement as partners in this design process), the aspect of their role we describe here is their hosting of student rehearsals in a designed setting involving the teacher educators and TCs collectively visiting the partner teacher’s classroom for discussions and enactments of an IA and their embedded practices.

Three TCs—Casey, Georgia, and Susan—served as the focus of our development and analyses. During the 2012-2013 academic year, these three TCs constituted the entirety of the secondary mathematics cohort in this one programme. Two of the TCs (Casey and Georgia) entered the programme immediately after completing an undergraduate degree in mathematics at the same institution. The third TC (Susan) came to the programme after experience working in non-traditional schools and needed to complete a set of mathematics content courses to make up for course requirements not covered by her initial undergraduate degree. The TCs were not involved in the design or research analyses of a given IA but were participants in the various phases of work around each of the three IAs. Each of the three TCs had two student teaching experiences separate from the work of the methods courses that we saw as a site for further understanding their practice and the potential impact and design needs of our use of IAs and various pedagogies of investigation and enactment.

In our initial thinking about the design of IAs for secondary mathematics teaching and our effort to be responsive, we established the need for IAs that were viable in the partner teacher’s classroom, leaving us considering the length of an IA (preferring to design relatively short activities) as well as the mathematical content (namely the way the content was relevant or novel to the students in a given classroom, while also not conflicting or interfering with the goals of the partner teacher). We saw the work in elementary mathematics—with IAs focused on the domain of number and operation—as a baseline from which to conceptualise approximations of practice and to build our design. Through our design research process our original IA design framework evolved, leading to IAs that departed from the examples seen in the work of Lampert, Kazemi, and colleagues along multiple dimensions. An inventory of the structure of each of the three IAs, their specific content, and the settings in which TCs rehearsed the IA with classroom students is provided in Table 1. In all cases, TCs had the IA modelled for them, were given a planning protocol with which to plan their own rehearsals, rehearsed in the methods course with their peers and with coaching from the teacher educator, enacted the IA with classroom students, and collectively discussed and analysed the enactments. We will discuss more below about how these IAs, their underpinning design considerations, and their
use in a series of pedagogies of investigation and enactment evolved as the main takeaway of this paper. It is worth noting that the three IAs summarised in Table 1 were designed and used as part of this stage of our work and are not meant to be exemplars, nor do they necessarily suggest the form we think IAs should take for work with secondary mathematics TCs. They do provide the working examples through which our design research process evolved and from which the set of design considerations we outline in this paper emerged.

Table 1
Description of IAs from the Three Design Cycles

<table>
<thead>
<tr>
<th>IA</th>
<th>Summary of IA</th>
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<tbody>
<tr>
<td>1) String of Computational Problems – Middle School</td>
<td>A string is short activity consisting of specifically sequenced problems designed to highlight a particular mathematical idea, notably a computation strategy. In this string, a sequence of four multiplication problems was used to bring forth and motivate the use of a strategy for mental computation in which one factor can be halved and the other doubled to create an equivalent product.</td>
</tr>
<tr>
<td>2) Explaining a Concept through Connections across Representations – High School, Algebra II</td>
<td>This IA involved the purposeful design of a sequence of prompts and representations to construct an explanation. As a result, this IA had similar structure to a string, with a focus on a concept and connections across representations instead of a computational procedure. This specific instance had TCs supporting students’ reasoning of exponential change and how it can be visualised across graphs, tables, and functions. Students were shown, in sequence, the graphs of three exponential functions with the third graph providing an example that defines the boundaries of the explanation being constructed about exponential change and its relation to the closed form of the function.</td>
</tr>
<tr>
<td>3) Building a Definition from an Investigation – High School, Geometry</td>
<td>This IA considered the way in which a mathematical idea or relationship could be motivated and defined through posing a problematic mathematical situation and an ensuing investigation. This specific instance focused on the development of definitions for the basic right triangle trigonometric ratios of sine, cosine, and tangent. The TC facilitated the building of explanations of these ratios using the constant ratios across the similar triangles in order to define the three trigonometric ratios.</td>
</tr>
</tbody>
</table>

Data sources

We collected video of all events in the methods class and partner teachers’ classrooms around observing, decomposing, planning, rehearsing, analysing, and reflecting on the three IAs. A key component of the design-based research process and the maintenance of rigor and trustworthiness is the process of reflexivity and transparency (Auerbach & Silverstein, 2003), so the first author maintained a “reflexive journal” along with the video records to account for each aspect of the design, to serve as a form of field notes, and as a first-hand account of the decisions that were made throughout the year and the factors that influenced those decisions in the moment. This record of the ongoing design decisions supported the retrospective analyses we outline below. After the three design cycles, the three TCs were individually interviewed by
the first author to discuss the work across the methods courses as well as their work in their school placements.

We also collected video of TCs’ instruction during their student teaching placements. Our attention to TCs’ teaching in classrooms was not for the purposes of determining the effectiveness of our established design but as a part of the design process. We see the collection of such data and the reinterpretation of its purpose as a key aspect of responsive teacher education. These video data consisted of two lessons from each TC in a part-time practicum during the first design cycle and a sequence of two or three lessons from each TC in a full-time practicum after the third design cycle. Field notes taken by the first author, who observed all of these recorded lessons, were also part of the data. TCs were interviewed before and after the sequence of lessons during the full-time student teaching placement to provide more information on the content of the sequence, intended instructional features, and factors that influenced the planning and/or enacted instruction.

Data analyses

A process of design-based research involves two levels of data analyses: (1) the ongoing analysis that informs the immediate and in-the-moment design decisions from cycle to cycle to support TC development, and (2) the retrospective analysis of the sequence of cycles that looks at the full corpus of data, including the reflexive journal, to consider the process of design and development. The ongoing analyses took place during and in between design cycles as a product of continual meeting and debriefing with the design team (specifically the teacher educators/researchers) and served as the basis of subsequent design decisions. These design decisions were based on a number of in-the-moment factors, including the variety of needs and demands (Edelson, 2002)—from our partner teacher and his or her school setting, from the university setting, and from our informed sense of the progress and needs for improvement in TCs’ practice and overall experience. The ongoing design decisions and their rationale were accounted for through reflexive journaling and represented in subsequent planning protocols, course materials, and videos of investigations and enactments.

Making sense of a set of ongoing analyses and decisions in retrospect is an important part of the design research process. In the work we share here, we were interested in developing a set of considerations for the development and use of approximations of practice of secondary mathematics teaching that are more responsive to the work of teaching in school settings. We looked at the full corpus of data from across the three cycles, as well as data (e.g., video and field notes) from interviews and TCs’ classroom placements after the third design cycle as part of a process of retrospective analysis. In addition to framing our work theoretically, activity theory served as an analytic framework in our retrospective analyses to make progress on the stated research questions. Specifically, we were able to organise and ask questions of our data to infer the goals, expectations, and pedagogical tools of the work of teaching and then make connections to what that told us about the work of approximating practice in more responsive ways. Along those lines, we had two analytic questions:

- What needs and goals of the partner teacher, the TCs, and/or the teacher educators are being addressed by the design?
- What are the pedagogical tools being discussed and used across the three IAs and how do those pedagogical tools play a role in TCs’ practice in student teaching?

For the first analytic question, we drew upon data from which we could identify the needs and goals being addressed within and across the three cycles and make sense of how and why those needs and goals emerged and evolved. This was one way to think about important design considerations. We were also interested in any dilemmas that were presented over the course of
our design process that required the negotiation (not necessarily resolution) of potentially competing needs. The reflexive journal served as a primary source for identifying these needs and goals. We looked to video of the methods course sessions to note any explicitly stated needs or goals. We also crafted memos with inferences about other needs or goals being addressed through the enacted design, such as how time was spent and the topics of focus. Finally, we used responses from TCs in the interview settings that focused on their perceptions of the focus of the teacher education work and on any further needs that they noted as not being addressed through the design as they moved into full-time student teaching or into their careers as mathematics teachers.

For the second analytic question focused on pedagogical tools discussed and used across settings, we again looked to our reflective journal as a way to revisit explicit decisions made in our design process—that time regarding the design of the IAs themselves and the way in which smaller grained pedagogical tools were embedded within those structures. We looked to protocols for and enactments of the individual IAs to identify the pedagogical tools being discussed and used and looked for themes and trends across the three IAs. Similarly, we analysed video of TCs’ enactment of lessons in their classroom placements to identify themes and trends in pedagogical tools used in those enactments and mapped back to our analyses from the investigation and enactment of the three IAs. Finally, we used the interviews as a way to get additional insight into the pedagogical tools TCs saw themselves engaging with, the ones they felt enabled to use in their classroom placements, and the ones they did use in their placements. Through the retrospective analysis through these two analytic lenses, we looked for ways to capture and further articulate additional design considerations for the work of developing and using approximations of practice in secondary mathematics teacher education.

Emerging Design Considerations for Secondary Mathematics Approximations of Practice

We group our findings into two sets of design considerations. First, we have re-evaluated the way in which an IA needs to be aligned to the content, structure, and goals of the secondary mathematics classroom, especially in terms of the in-the-moment work occurring in partner teachers’ classrooms. This has implications for the design of an IA as well as the way in which TCs must engage with it. Second, we have realised the challenge of attending to the complexity of ambitious and equitable teaching through the development and use of an IA. This is done, in part, through identifying and making sense of the nested pedagogical tools within an IA—both for their individual purposes and routine structures as well as their connection to a broader view of ambitious and equitable mathematics teaching and to instruction that makes progress toward specified mathematical goals for students. We use examples from our evolving design and analyses to identify and define these considerations and discuss their importance in responsive and practice-focused designs.

Identifying mathematical content and goals for approximations of practice

Throughout our design process, we confronted a need to maintain integrity to the mathematical development—both content and practice—of the students in the secondary classrooms in which we situated our pedagogy of practice. This had implications on how we worked to support TCs’ development of the mathematical resources to reason about and enact a given IA. Negotiating these two emerging needs, in addition to a focus on the development of TCs’ instruction, was a key consideration for us as teacher educators. This “dilemma” is not one for teacher educators
to look to resolve outright, but to acknowledge as part of the work of managing and negotiating multiple (and sometimes competing) goals in the work of responsive and practice-focused teacher education. We feel as though these considerations are not just a product of our own context, but indicative of the nature of secondary mathematics teaching, learning, and teacher development.

**Considering goals for classroom students’ mathematical development.** Our first IA—the string of computational problems focused on the mental multiplication strategy of making a potentially easier problem by halving one factor and doubling the other—was conducted in a middle school classroom. Deriving and working with multiplication or other computation strategies was not a focus in this classroom, despite the potential novelty of this “halving and doubling” method. However, we saw the IA as an opportunity to build on existing work from teacher educators in elementary mathematics (e.g., Kazemi et al., 2009), to present and work on an interesting IA structure (i.e., the string of computational problems), and to work with content that was not too complex (for TCs or for the classroom students, though we came to realise that was not the case). We did this knowing that, while number and operation as a content focus accounts for a significant portion of the focus in the elementary grades, secondary mathematics teachers (and teacher educators) make decisions about a more diverse set of mathematical domains and topics. We found that this way of “reducing the complexity of teaching” ran counter to needs and expectations that were present within the first cycle and in preparation for the second and third cycle. While the mathematical focus of the first IA did not prevent us from the opportunity to rehearse in Ms. Calhoun’s middle school classroom, aligning to the mathematical content of Mr. Ellison’s high school classrooms was a requirement. This resulted in a focus on exponential growth and representations of exponential functions for the second IA and a focus on the basic trigonometric ratios for the third IA, both of which required different IA structures from the ones with which we were originally familiar. This also highlights an important challenge in this work for secondary mathematics teacher educators who, regardless of pedagogy, are often making decisions about the content on which to focus with TCs and practicing teachers.

The tension between the content of an IA and the content in a given classroom was also salient among the TCs, who seemed to struggle with the idea of teaching multiplication to middle school students. In her interview after the third cycle, Susan distinguished the first IA from the others because of the way in which the content focus and goal was not “authentic” for those students. On the other hand, all three TCs noted the benefit of the second and third IA, with Georgia commenting on the ways in which the IAs had general qualities that informed her work as a teacher. We found these types of responses surprising given the way in which the second and third IAs took on less of a routine and compact form in the way that the string did. However, all three TCs commented on the way in which the second and third IA engaged them more authentically and meaningfully in planning for and teaching toward a clear and meaningful mathematical goal. When asked if she saw a difference between the first IA and the second and third IAs, Casey said, “I think [in the second and third IA] we focused on specific content in order to try to work toward a clear learning goal” and that, through such IAs, teacher candidates were “more aware of the process of … how to plan a lesson that has a mathematical storyline and reaches a goal”. Susan shared that the work of responding to students’ ideas is something that, “you can’t learn unless you are working with actual students putting out actual ideas,” which she saw the second and third IAs—with their more authentic goals for students—providing the opportunity to do. Finally, Georgia shared that the second and third IAs put them in a position to more authentically build mathematical ideas and explanations with students.

The idea of authentically teaching toward clearly defined goals was also a problem of practice that TCs identified at the start of the second methods course, prior to the work around
the second IA and after their part-time student teaching experience that ran concurrently with the first design cycle. From observations of early classroom work, TCs were confronted with students’ contributions and the need to make sense of those ideas and make decisions about how to use those ideas toward a determined goal for a lesson. Casey and Georgia both showed struggle with interpreting students’ ideas, specifically incorrect ones, in the midst of whole class discussion and Susan’s instruction failed to provide openings for students to share their reasoning. This resulted in negative impact on the progress made toward the stated goals for the lesson as ideas and discussions were bypassed, and getting through the planned set of activities for the lesson was privileged. In sum, the logistical needs of our collaboration with a partner teacher as well as the instructional needs of the TCs motivated the alignment of the content focus of an IA with that of a particular school classroom.

We also found that this design consideration was not simply a matter of identifying a topic. The process of designing the second IA involved negotiations and revisions regarding the specific mathematical focus and goals within that topic. When establishing a focus on exponential functions, our design team began to conceptualise possible foci for the IA. For example, we considered ways for students to compare the nature of growth in an exponential function to that of a quadratic given some of their general similarities, but notable and important differences. However, after further conversations with the partner teacher, the focus became more about the connections across various representations of exponential functions. During the enactment of the second IA, we saw additional ways in which the teacher educator and partner teacher must collaborate to establish the content focus of an IA. Core to the IA was the idea of a “growth factor” in an exponential function, and how that growth factor can be seen as the base in the symbolic representation of the function. TCs were supported in thinking about this connection themselves and in understanding the way in which to engage students in building that connection. However, students in Mr. Ellison’s classroom had not been talking about growth factors (e.g., “the output doubled with an increase of one to the input”) and, instead, used an idea of per cent growth to talk about change (e.g., “the output increased 100 per cent with an increase of one to the input”). This caused an unexpected tension in the moment for the TCs that was not anticipated in the planning of the IA. With both of these considerations—the articulation of specific goals and the mathematical “history” of a classroom—we better understand the need for a process of close discussion and revision between a teacher educator and a partner teacher. This might be an uncommon relationship or process in many teacher education programmes and one that can further constrain the limited time in teacher education programmes. Furthermore, TCs should be involved more actively in these discussions, either with the partner teacher or through a more focused examination of curricular materials, student work, or other artefacts.

Considering TCs’ mathematical development and understanding of goals for students. While the specific content and goals of an IA were decided in collaboration with a partner teacher (thus taking that responsibility away from solely the teacher educator) and also served in addressing a need regarding the work of “teaching toward clear mathematical goals”, this foregrounded other needs. In the context of particular content and specified goals, TCs needed to be supported in developing the mathematical resources for investigating and enacting an IA—for interpreting and evaluating students’ ideas and for making instructional decisions toward particular ends. With content and goals being articulated that were contingent to a particular classroom, though, we were not in the position where we followed a theme or thread of mathematical ideas across the IAs. This is not to say that a teacher educator could not make such thematic decisions regarding content in advance. In our context, we were left resolving a tension in which TCs (and, to an extent, teacher educators) were engaging with “new” content and goals.
Our analyses of the early work with TCs around the second and third IA showed several instances where we put into place design features meant to prepare TCs for the mathematical aspects of their work in the IA, such as the use of mathematics problems given to TCs in the second cycle that brought forward the relevant mathematics of exponential change. A primary artefact for foregrounding TCs’ mathematical development was developed in the midst of the second and third design cycle to further explicate the specialised mathematical ideas at play in an IA. This “instructional explanation tool” — based on the work from Leinhardt (2001) around criteria for instructional explanations — served as a resource to support TCs’ understanding of the mathematics of an IA. This tool provided TCs with ideas about as well as prompts to further uncover how to problematise an idea, draw upon students’ prior understanding, exemplify an idea, and consider the boundaries of an idea. While it is common to think about teachers’ content knowledge and the ways to prepare teachers for instruction, we were dissatisfied with using only approaches that had TCs completing the mathematics task (or related problems) themselves. The instructional explanation tool provided a framework through which TCs and teacher educators could think about mathematics in a way that supported enactments.

TCs expressed that the instructional explanation tool would be something that would support them moving forward as a way to think about the mathematics that they are teaching. Casey expressed that the tool would be useful in the future because of the way it helps, decompose a mathematical idea … and try to plan a lesson in a way that first problematises an idea for students, see what their prior knowledge is and how you can build off of it. So all of these things can be used to plan a lesson around anything. That’s how [the second and third IA] were more generalisable and helpful because, I mean, that’s what we’re going to need to do as teachers (Casey, Post-Methods Interview, 3/21/2013).

The planning protocols for each IA also included notes with specialised considerations about the content and its link to instruction. For example, we noted for TCs that sine, cosine, and tangent are defined using right triangles, however those ratios become tools that can be used to talk about relationships in all triangles (using the law of sines or cosines) and also have a connection to trigonometric functions. While those considerations are beyond the scope of the third IA, they are important components of what a teacher would need to have available in defining the ratios initially. In addition to these written artefacts, an increased amount of time during the planning and analysis of an IA with TCs was devoted to focusing on these ideas. Examples include discussion of possible student responses (and sample teacher responses to those ideas) and review of what students in the partner teachers’ class have already done (and specifics including the language used around certain concepts). We, as teacher educators, also provided insight to decisions made about the problem used, how it is displayed, and the language to be used in the IA.

We found, though, that this added attention to the mathematical content of an IA — both in terms of the focus for classroom students and supporting the development of the resources needed for teaching — presented a tension with how we could attend to the development of instructional skill. Ultimately, this tension arose because of the limits of time in teacher education and because our IAs, themselves, became increasingly complex as instructional structures. The latter, considered in tandem with TCs’ opportunities to teach in classroom placements, gave rise to a second set of considerations around the way in which an IA — and our work with TCs around an IA — attend to the complex work of ambitious and equitable mathematics teaching and the ability to do that work in schools.
Specifying and attending to structure of ambitious and equitable teaching

In our initial design approach, we took up the definition of an IA as a routine classroom activity structure that was both connected to broader principles of ambitious and equitable teaching as well as a container of instructional practices and teacher moves, such as eliciting students’ reasoning, orchestrating mathematical discourse, orienting students to one another and to the mathematics, and teaching toward ambitious mathematics goals. We considered all teacher actions—the IA, the component parts of an IA (such as launching a task, allowing students to work individually or in pairs, and constructing explanations as a whole class), and the embedded practices and moves—as pedagogical tools, albeit ones of multiple and nested grain sizes. While the different “sizes” of teacher action taken on in an approximation of practice and the complexity of their interconnectedness was an initial concern of ours, it was through our design process that we articulated that concern and developed considerations to address it.

Our initial thought was that an IA would both serve as a vehicle for the investigation and enactment of practice in teacher education, as well as a replicable activity structure that could be used in school classrooms. However, our TCs did not replicate any of the three IA structures in their student teaching after the third design cycle. While Casey and Georgia said in their interviews after the third cycle that they would consider the use of the IA structures in their student teaching, Susan pointedly avoided those structures and highlighted that she was overwhelmed by their complexity. Despite that reluctance to consider the entirety of the IAs, Susan made mention of the way that the planning protocols for the IAs “broke the [IA] up into different parts and we related the different parts of the [IA] to what it was going to do for students and their learning”. This finding from the observations of TCs’ teaching and this insight from Susan moved us to considering the component parts of each IA, how those parts related to the work that TCs were doing in their classroom placements, and how that would inform our design and use of approximations of practice.

In our analyses across the three IAs and the TCs’ enactment of them, we began to identify multiple levels at which to discuss the component parts of an IA and also identified some frequently highlighted pedagogical tools within each of those levels. Within an IA, we noted that there is a sequence of events—what we call “episodes”. These episodes—such as launching an activity or task, monitoring student individual or small group work, eliciting solutions and ideas, and constructing explanations with the whole class—commonly occurred across the three IAs and, themselves, have appeal as things that teachers would regularly do in a classroom. It is important to note that an individual episode does not carry its own meaning but, in the context of a whole IA or lesson, it does different work for a teacher in making progress on mathematical goals and it requires different work of the teacher to carry it out.

One example of this is the way launching mathematical work served as a consistent episode across the three IAs. The work of launching and assigning was tied to a principle of providing students access and orientation to the mathematics. Built within these launches and introductions were moves to accomplish the work of highlighting expectations and objectives, ways to consider making a mathematical idea problematic, providing narrative to focus students’ work and discussions, and eliciting and making use of students’ prior understanding of an idea. Even a move as fine-grained as the TC simply giving students time to reason about a problem or prompt was a regular structure built into the IAs. However, despite an implicit focus on the launching of mathematical work, its purposes, and its component practices and moves, this constellation of practice contained within an IA was not identified by TCs as a distinct set of pedagogical tools. That is not to say that TCs did not begin lessons or assign problems in their classroom, but they tended to do that work in a way that did not provide
students’ access and an orientation to the mathematics in the same way that was built into the IAs.

We saw another trend around the use of individual moves, such as the classroom talk move of the TC prompting a student to restate the idea of a peer. This was a move that was frequently specified across the IAs and identified as a pedagogical tool that gives students in the room access to an idea, promotes the work of mathematical argumentation and discourse, and allows the teacher to have the class dwell on important mathematical ideas. The promoted importance of such a move was more apparent to TCs based on their work in the methods courses and their responses in the interview during which they identified such talk moves as tools they would look to use in their teaching. While this was a move we saw TCs using in their classroom placements, its use seemed targeted toward different ends—either used at times in which the mathematical importance was unclear or used in a way the seemed to do the work of classroom management, such as engaging a student who was not paying attention.

We see these examples of the work of launching and the use of classroom talk moves as two illustrations of the complexities of addressing the multiple and interconnected levels of teacher action embedded within an IA. While our initial design focused on the way in which an IA served as an established and articulated activity structure, we now see the need for teacher educators to make explicit the nested episodes, practices, and moves within the IAs—motivating their purposes, situating them relative to one another, and supporting their realisation through concrete actions. Doing this would involve making these explicit connections on records such as the planning protocol, through discussion and decomposition of the IA, and through the in the moment and reflective coaching that is tied to the rehearsals. This is not to say that one could work on a launch or with talk moves in isolation—absent of a larger activity and/or purpose—but that their purpose and routine structure can be made explicit in the context of more authentic instructional work. We see this as important for the development of TCs, who may not always succeed in planning and using IA structures in their instruction but who can still put to use and have flexibility with other pedagogical tools in practice.

We also see these two examples as illustrations that reinforce the need for explicating tools that are responsive to the work of teaching in schools. We are interested in the ways in which TCs launched mathematical work and used talk moves are indicative of the work of teaching as it was defined in schools. For example, we speculate that the way that TCs introduce problems and prompts in the classroom, which was done in very abbreviated ways, is a sign of the privileging of time-efficient instruction in schools. It is these embedded conceptions of teaching that inform teacher actions that must be acknowledged in the design of IAs and the decisions about their explicated component parts. The pedagogical tools, across multiple levels, that are explicated in the midst of an approximation of practice are only tools if they have meaning and utility in the context of the work in a particular setting. If TCs are expected to teach mathematics in ways that are efficient and focused on procedures, then the pedagogical tools they develop in teacher education must look to meet these demands, while also looking toward more incremental progress toward ambitious and equitable aims.

**Implications and Future Directions**

We see these design considerations as a step in the evolution of the emerging body of work on responsive and practice-focused pedagogies in teacher education, specifically the development and use of approximations of practice. It is important to reiterate, though, that the IAs we discussed here as well as other aspects of our work with TCs are a part of the context of our design-based research work and are not to be considered a finished or recommended set of
products or practices. The ideas we share in this paper serve as a new set of working hypotheses and design considerations that are the focus on ongoing design and research efforts. As we move forward with our work in secondary mathematics teacher education, we have drawn our attention to two ideas for IAs that may have leverage in school classroom contexts. First is an IA centred on the work of “going over a mathematics problem”, as that is a common cluster of episodes emerging from TCs work in classrooms making it easy to see how the space could exist in classrooms to establish more productive routines. Second is an IA that foregrounds the work of making use of mathematical errors in discussion, given TCs’ efficient instructional work often leads to errors being quickly corrected or ignored instead of being used productively in mathematical discussion. We see these ideas as having potential in realising our emerging design considerations and in promoting sets of pedagogical tools that enable TCs to do the work they are expected to do in schools, albeit in ways that are slightly more ambitious and equitable. While the goal is often to shift away from prevalent, procedurally focused forms of instruction in classrooms, tools put forth in teacher education must look to accomplish the work of teaching as defined in schools, and at the same time, press on its boundaries.

An understanding of the work of teaching as it is defined in schools as well as the possible openings for innovation requires further research on the work that TCs and practicing teachers do in school settings in order to identify the pedagogical tools that are used and to infer how the activity of teaching is defined in school settings. Along with our continued design work, we plan for a closer examination of TCs’ practice in school classrooms in order to make progress on what to leverage and how to leverage it through responsive pedagogies of practice. We have started these efforts by exploring the themes that began to emerge about TCs’ practice in schools using more focused analyses. Activity theory has provided us the theoretical and analytic tools for those continuing efforts by allowing attention to the complexity of activity settings as well as the nested levels of teacher activity. Efforts to develop skilled teachers and approximate the work for the purposes of teacher development relies on an understanding of what the work entails, though it is an understanding that has been lacking, in part due to the common divisions between research on teacher education and research on teaching (Grossman & McDonald, 2008).

As we discussed from the outset, one possible critique of the idea of responsive teacher education that we put forth in this paper could be that our goals for mathematics classrooms are not ambitious or equitable enough and that we are simply advocating a replication of the type of instruction and opportunities to learn that trouble classrooms and their students. However, an approach of responsive teacher education, as we conceptualise it, does not establish as a fixed object the prevalent practices of classrooms and schools to which to be responsive, thus making it adaptable to a variety of settings and to incremental change over time. We see this as important for thinking about the way in which these designs can fit a range of classroom situations in which TCs are being prepared. We also see this approach as useful in thinking about practice-focused pedagogies not just for TCs but also for the professional development of practicing teachers. Ultimately we see a responsive approach to practice-focused teacher education pedagogies not only as one that has the potential to be highly effective but also as an approach that is adaptive and robust. Calls to prepare TCs with the knowledge and skills to teach in ambitious and equitable ways, while also meeting demands of accountability and of the culturally-defined work of schools requires such innovative and adaptive approaches to teacher education.
References


Designing Approximations of Practice


Authors

Matthew P. Campbell
West Virginia University, College of Education & Human Services, P.O. Box 6122, Morgantown, WV 26506
email: mpcampbell@mail.wvu.edu

Rebekah Elliott
Oregon State University, College of Education, 104 Furman Hall, Corvallis, OR, 97331
email: elliotttr@science.oregonstate.edu