

Experiencing a mathematical problem-solving teaching approach: Opportunities to identify ambitious teaching practices

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Learning to teach is a complex matter, and many different models of pre-service teacher education have been used to support novice teachers' preparation for the classroom. More recently there have been calls for a focus on core high-leverage teaching practices and for novice teachers to engage in representations, decompositions, and approximations of practice. This study focuses on novice teachers' learning about core high-leverage teaching practices through engaging in a problem-solving approach to explore the learning and teaching of mathematics. Findings indicate experiencing this approach is an important first step towards novice teachers' learning about practices congruent with an ambitious teaching agenda such as justifying mathematical reasoning, emphasising conceptual understanding, and catering for all learners. Novice teachers also began to envisage how such ambitious mathematics pedagogies could be enacted in their future practice.

Keywords: practice-based pre-service teacher education; ambitious teaching; core high-leverage practices; problem-solving

Introduction and Literature review

Preparing to become a primary school teacher of mathematics poses significant challenges (Grootenboer & Jorgenson, 2009; Guillaume & Kirtman, 2005; Young-Loveridge, 2010). It has been argued that initial teacher preparation must move beyond an examination of student teachers' beliefs and knowledge, orientations and commitment to an emphasis on practice-based teacher education (Ball & Forzani, 2009). While there is little consensus about what the phrase "practice-based teacher education" means (Forzani, 2014), it is recognised that this places teaching as central to learning-to-teach (Anthony et al., 2015; Ball & Forzani, 2009; Hlas & Hlas, 2012). There is recognition that novice teachers' learning can take place in a variety of settings including university as well as school classroom environments (Forzani, 2014). Academic study remains highly relevant to an initial teacher education program (Rogers, 2007).

Practice-based teacher education is identified as having two threads. The first centres around core practices of ambitious teaching, and the second focuses on a range of pedagogical practices whereby novice teachers are engaged in representations, decompositions, and approximations of practice (Anthony et al., 2015). This paper concentrates on the first of these threads.



The defining feature of a core practices model for teacher education is a focus on specific pedagogical practices associated with ambitious mathematics teaching (Forzani, 2014; Ball, Sleep, Boerst, & Bass, 2009). Forzani (2014) explains that these core practices are centred around three key ideas. The first is that teaching should be aimed at “ambitious learning goals that are grounded in the expectation that all students will develop high-level thinking, reasoning, and problem-solving skills” (Forzani, 2014, p. 359). Secondly, teaching is perceived as a partially improvisational practice closely connected to the ideas children bring to the classroom. It is also recognised that ambitious teaching is partly predictable, and novice teachers need to be able to manage the uncertainty, and understand and develop skill with predictable aspects (Forzani, 2014). Lastly, the subject matter being taught is a “critical component of the goals and activities that constitute the professional curriculum” (Forzani, 2014, p. 359).

“Ambitious teaching” in mathematics refers to supporting all learners to develop conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning to solve authentic problems (Lampert & Graziani, 2009; Lampert, Beasley, Ghouseini, Kazemi, & Franke, 2010). The focus on deep, connected, and flexible understandings is consistent with recent mathematics education reforms that emphasise the development of a conceptual understanding as well as procedural fluency (Boaler, 2002, 2008; Lampert et al., 2010; Young-Loveridge, 2010). Indeed, teachers’ deep understanding of content being taught is regarded as a fundamental aspect of managing the uncertainty within teaching (Forzani, 2014; Hlas & Hlas, 2012) and being able to support the development of learners’ conceptual understanding (Frankcom, 2009). Ambitious teaching in mathematics is recognised as challenging yet vital for novice teachers to learn (Anthony & Hunter, 2013; Lampert et al., 2010).

Given the focus of ambitious teaching in mathematics and its recognised contingent nature, some core practices that are viewed as important for novices to learn include eliciting and interpreting student thinking, leading class discussions, facilitating small group work, anticipating student errors and misconceptions, making connections between multiple representations, and teaching through problem-solving (Forzani, 2014; Hlas & Hlas, 2012). Core practices are sometimes referred to as high-leverage teaching practices (Anthony et al., 2015). High-leverage teaching practices when executed proficiently promote significant enhancement in student learning (Forzani, 2014; Hlas & Hlas, 2012; Ball et al., 2009). It is proposed that focussing on a core set of practices that have the greatest impact on student learning may require an approach where “less may indeed be more” (Hlas & Hlas, 2012, p. S78).

It is contended that teaching needs to be broken down into parts that are central to practice and can be improved through specific instruction (Ball & Forzani, 2009; Grossman et al., 2009). In a similar vein, to help make teaching learnable by novices, high-leverage teaching practices are parsed into micro-practices. For example, launching a discussion and eliciting and following up students’ contributions are two (of twelve) micro-practices within the high-leverage teaching practice of leading a discussion.

High-leverage teaching practices vary with content and context (Ball et al., 2009). Teaching through problem-solving is considered to be one high-leverage teaching practice (Hlas & Hlas, 2012) and is also an expectation for the teaching of mathematics in past and current New Zealand curriculum documents. For example, the *Mathematics in the New Zealand Curriculum* document (Ministry of Education, 1992) advocated the use of a problem-solving approach, and the latest *New Zealand Curriculum* (Ministry of Education, 2007) also explicitly refers to mathematics as a problem-solving endeavour required for all levels of schooling. Moreover, understanding and responding to complex problems is integral to ambitious teaching (Forzani, 2014). Although curriculum documents have signalled a problem-solving focus for more than

twenty years, this has often been an aspect of mathematics teaching that has been overlooked (Holton, 2009; Quinnell, 2010) and not always enacted with success (Jorgensen & Dole, 2011).

Problem-solving and mathematics are intertwined. Holton (2009) states, “all mathematics comes from a problem” (p. 47). Problem-solving in mathematics often incorporates the use of tasks that enable a variety of mathematical avenues to be explored. A sustained period of time is required for solving worthwhile problems (Breyfogle & Williams, 2009), and there is an emphasis on the development of a conceptual mathematical understanding. Polya’s (1945, cited in Frankcom, 2009) model for problem-solving offers a four-stage process that underpins teaching through problem-solving. The four steps include understanding the problem; devising a plan for solving it; carrying out the plan and looking back and reflecting on the solution obtained (Ministry of Education, 1999).

The social norms and what it means to ‘do mathematics’ in a problem-solving classroom includes learners being expected to be creators of mathematical knowledge. Learners also support, communicate with, and help others in their learning (Boaler, 2002; 2008). Choice and autonomy characterise these classrooms with learners able to make decisions about which of a variety of tasks they undertake. Embedded in a problem-solving teaching approach for mathematics are various micro-practices including targeted scaffolding through routine questioning. Other embedded core or high-leverage teaching practices include small group work, eliciting learner thinking (Rigelman, 2007), justification and reasoning (Hunter & Anthony, 2010), and encouraging higher order thinking.

If mathematics teaching and learning is to reflect current curriculum requirements, novice teachers may need to be provided with experiences, such as a problem-solving approach, that support them to construct alternative understandings of what it means to teach and learn mathematics. Novice teachers who hold traditional views of mathematics or have experienced transmissive modes of teaching and learning this subject may encounter ideas that are incongruent with their own beliefs and opinions. Support from teacher educators may be needed for alternative ideas and pedagogies to be considered, and for possible conceptual changes to occur (Coburn, 2005; Kane, 2007; Korthagen, 2004; Labaree, 2000; Russell, 2007). While there is no consensus about both practice-based teacher education and core or high-leverage teaching practices in mathematics (Ball & Forzani, 2009) it is nevertheless important to explore how novice teachers might be best supported to develop their learning of what it means to be an effective mathematics teacher. The expectation that all students are supported to reach ambitious learning goals (Ball & Forzani, 2009; Forzani, 2014; Lampert, Beasley, Ghousseini, Kazemi & Franke, 2010) requires us to do so.

The research question in this paper investigates: ‘does experiencing and reflecting on a problem-solving approach in mathematics support novice teachers to learn about core high-leverage teaching practices?’

Context and research participants

In this study, small groups of up to four novice teachers participated in a mathematical problem-solving teaching approach in a New Zealand university setting. Their initial role was that of a mathematical problem-solver, and later as novice teachers considering and critiquing the approach for their future teaching. Data is drawn from the teacher educator’s planning and two audio-recorded focus group interviews with nine student teachers.

The context for this group of novice teachers was a compulsory 12-week mathematics education course within a one-year graduate diploma of teaching (primary). One intention of



this 48-contact-hour course was to support novice teachers to experience mathematics as a creative, constructive, and highly inclusive activity. The teaching was underpinned by a sociocultural perspective of learning in line with Cobb and Yackel's (1996) contention that an individual's mathematical thinking is influenced by participation in surrounding cultural practices. The course was also based on the premise that personal content knowledge, attitudes, and beliefs about mathematics influence teaching and learning. Novice teachers had regular opportunities to work on mathematical problems in small groups, and also taught mathematics with small groups of children in schools. The novice teachers were expected to reflect on university-based and associated school experiences and discuss these with reference to mathematics research literature and curriculum materials.

Of particular relevance to this research was the fourth module (of a total of five modules) of the mathematics education course. The module consisted of eight hours (four two-hour sessions taught over a period of two weeks) of face-to-face teaching focused on supporting novice teachers' learning about unit planning, teaching, and evaluating mathematics. The mathematical topic for the module was algebraic thinking and modelled the teaching of algebra using a problem-solving approach. In the first session a sequence of problems (connected by a theme) were modelled by the lecturer supporting novice teachers to recognise patterns within a physical context and working from finding recursive rules through to generalised rules/formulae for the relationships $y=2x$; $y=x+1$; and $y=2x+1$. In the second and third sessions students were invited to choose and work on a selection of algebraic problems (set in a variety of contexts (eg. calculating how much money would be earned at the rate of one cent on the first day and charging twice as much as the previous day for each subsequent day) ranging from relationships such as $y=4x$; $y=2x+6$ through to the more challenging $y=2^{x-1}$ and $y=x^2+2x$. The intention of the second and third sessions was for novice teachers to consolidate and/or refresh their understandings of algebra and experience this within a problem-solving approach. The novice teachers could choose to work independently or in small groups, and from time to time were invited by the lecturer (one of the researchers) to meet with others working on the same problem, and discuss their thinking and progress in solving the problem(s). These small group sessions were also an opportunity for questions to be asked and answered. The lecturer-researcher played an active role in guiding the novice teachers and asking pertinent questions as they worked on the problems (eg. what might come next? How could you record your findings? Is there any pattern that you have noticed? What else might you try?). Some novice teachers completed several problems during the second and third sessions, while others worked on the same one for three to four hours. The fourth and final session of this module, in addition to some teaching about unit planning and linking to the New Zealand curriculum document, provided an opportunity for the novice teachers to reflect on and discuss their experiences of the problem-solving approach including making links to literature (twelve readings about the teaching of algebra, problem-solving, and children thinking and talking mathematically were available and expected to be read). The lecturer-researcher also took this opportunity to foreground problem-solving as one approach novice teachers could use in their future teaching practice. High-leverage teaching practices made explicit, via discussion and questioning, included small group work, eliciting and interpreting learner's thinking by questioning, and using a range of representations as a strategy for the solving of algebraic problems.

This module prepared the novice teachers for their third assignment where they were expected to diagnostically assess a small group of children's understandings of algebra and then plan, teach, and evaluate a unit focused on algebra. The use of a problem-solving approach



was encouraged but not required. Their teaching occurred after the eight hours of university-based teaching and learning, and took place in a neighbouring year 7-8 intermediate school.

The lecturer-researcher kept written records of the planning for the module. Seven aims were identified for the module, and six key algebra concepts for the novice teachers to learn (or refresh their understanding) were identified and listed. Two of the aims were for the novice teachers to:

- 1) develop an appreciation of the power of generalising (eg. as a problem-solving strategy; and as the key to algebraic thinking); and
- 2) consider and experience pedagogical issues including children talking to learn in mathematics and using a problem-solving approach.

The six key algebra concepts were:

- 1) patterns - repeating, sequential; making, describing, continuing; number, and spatial;
- 2) patterns defined by rules - words then symbols; recursive rules, general rules;
- 3) patterns and rules illustrated by graphs;
- 4) equations and expressions - symbols (+, <, >, =, -, x, /) and letters (the use of variables);
- 5) equality; and
- 6) solving equations with unknowns.

At the conclusion of the course there was an opportunity for novice teachers to participate in audio-taped focus group conversations with both researchers. This research methodology was chosen as it is known to be useful for gathering data on attitudes, opinions and values, and empowering participants to share their thinking in their own words (Cohen, Manion & Morrison, 2007). During focus group conversations inquiries were made about mathematical teaching and learning experiences, including those experienced during the course and micro-teaching opportunities. Nine (of the class of thirty-five) novice teachers chose to participate (one group of three, and one group of six, with both researchers present for each conversation). The focus groups were informal but the novice teachers were asked whether there had been a particular activity, experience, discussion or reading that had impacted on them during the twelve-week mathematics education course. The novice teachers were also asked if they envisaged an impact on their teaching in the future from any identified course activity, experience, discussion or reading.

Ethical permission for this research was sought and gained from the university's ethics committee. Particular attention was given to the potential for a conflict of interest, given one researcher's role included teaching and assessing students' work as well as gathering data for this research. We were aware that simultaneously researching and teaching can compound issues of power imbalances between student teachers and the lecturer. This situation was ameliorated by sending out letters about the research a week before semester began, thereby providing novice teachers with time to read about and consider their willingness to be involved in the research. They had a second opportunity to consider their decision regarding involvement during the first week of semester. An additional consideration to avoid undue power imbalances was that the data gathered for the research was not linked to any formal assessment.



Data analysis and discussion

In this section we draw on an analysis of the lecturer-researcher's planning for the algebra problem-solving module and focus-group transcripts to consider what it might mean to design initial teacher education experiences that engage novice teachers with the nature of ambitious mathematics teaching. Data analysis has occurred in two ways. An emergent analytical approach (Strauss & Corbin, 1994; Borko, Liston & Whitcomb, 2007) was employed to analyse the transcripts of the focus group conversations. The transcripts were read and re-read with notes taken as issues relevant to ambitious teaching and core high-leverage practices emerged. Both researchers read the transcripts independently to improve reliability. While particular themes emerged, points of difference were also noted. Ongoing collegial discussions between the two researchers, reading and re-reading of the transcripts and reading the theoretical literature about practice-based teacher education, ambitious teaching, and high-leverage teaching practices also provided data. This process reflects the thinking of St. Pierre (2011) who writes, "if we don't read the theoretical and philosophical literature, we have nothing much to think with during analysis except normalized discourses that seldom explain the way things are" (p. 614).

In order to represent the perspectives of all research participants, findings are presented in the form of quotes from each of the nine research participants. Consistent with Clandinin and Rosiek's (2007) call for researchers to listen to people's stories about everyday experiences, the presenting of the novice teachers' 'voices' is a deliberate attempt to present and acknowledge their experiences and perspectives.

Four key points emerged during the analysis. These are:

- the power of novice teachers experiencing a problem-solving approach;
- problem-solving as a way for novice teachers to learn about catering for diversity in mathematical learning needs with links made to their potential future teaching practice;
- problem-solving as a way for novice teachers to experience and learn about high-leverage teaching practices; and
- the continuing relevance and importance of attending to the affective domain in teacher education.

The power of novice teachers experiencing a problem-solving approach

An analysis of the focus group transcripts revealed that eight of the nine novice teachers had not previously experienced a problem-solving approach. Comments regarding their previous experiences mentioned the importance that was placed on rote-learning, personal struggles and a lack of understanding, and a reliance on text-books with little teaching support. Typical of the responses was Jack who said:

... what I do remember about maths, and it's similar to what's been said, was in Intermediate, that's probably the earliest I could remember. And it was always, 'turn to page whatever and work through problems 1 to 10'. And the answers are always in the back. And I would have went through the whole of intermediate slipping through the ... No one, I don't remember anyone following me up or checking off. I'll just rule off and move on. And how it's impacted on me - that's kind of messed me up.

Tony mentioned his perception of mathematics as impenetrable. He said:



I found mathematics to be or to seem to be quite an impenetrable sort of a thing. You just kind of couldn't access it. Or I couldn't access it. And um and so because of that I ended up I s'pose with a fairly negative outlook on mathematics.

In contrast Neil was the only novice teacher in either discussion to remember solving problems. He said:

I remember the maths challenges and things. That was probably my earliest memories of actually doing the maths. And it was all about doing the real problems. They'd give you the problem-solving stuff. It was always like a real-life application... . So it was the challenge, and it was fun, and also a competition. But nearly all the problems were related to real life so you could see the actual benefit of doing it.

The novice teachers were positive about their experiences with a problem-solving approach. Charlotte referred to being less fearful about mathematics. When asked about her thoughts at the end of the mathematics education course, she referred to change, and said, "... taking that fear out of maths. It doesn't have to be a daunting scary thing. Like you can be empowered if you're given the right material". Similarly, Damian commented:

You can make math a lot of fun. And there are so many different approaches to it, as well. All that problem-solving stuff we did too, just really sort of impacted me. And made me think, hey, you can actually do things in these ways, and even if you just give a kid a problem and a little bit of guidance, it's still math. You're letting them do it themselves and it might be more meaningful that way, rather than force feeding them on rote teaching.

Tony referred to the problem-solving module has having the biggest impact on him within the course, and becoming almost excited about doing mathematics. He said, "... to say I'm excited about it is probably going a bit too far (laughter) - don't want to exaggerate, but it's enjoyable now. Which it wasn't before". These comments were representative of the thinking expressed by all nine of the novice teachers within both focus group discussions. The interview question had not specifically mentioned the problem-solving module. Rather, it was an open question asking whether there had been a particular activity, experience, discussion or reading that had had an impact on them during the twelve-week mathematics education course. Given the openness of the question it is interesting that a significant portion of each focus group discussion was spent discussing that particular module.

Considering the paucity of the novice teachers' previous mathematical problem-solving involvement, and their enthusiasm for this module, we argue that experiencing a problem-solving approach, with the novice teacher situated as a learner/mathematical problem-solver, is an important first step towards novice teachers learning about ambitious mathematics teaching. Following this experience, problem-solving could be a potential 'pedagogy of practice' whereby novice teachers have an opportunity to enact problem-solving in practice with their colleagues and/or children. This contention requires further research.

The learning of key algebra ideas was an embedded and integral part of the problem-solving module, providing an opportunity for the development of "robust skill in and knowledge of the subject under study" (Ball & Forzani, 2009, p. 503). Charlotte's comment, "That's the first time algebra's made sense. Ever." indicates her learning of at least some algebra concepts. The problem-solving module also alerted some of the novice teachers to the importance of their personal content knowledge. Jack insightfully commented, "The other dilemma I've got is the importance of your content knowledge. And you can't draw water from an empty well". Tony reflected on the depth of his learning, saying, "That whole process driven sort of stuff seems to be really, for me anyway, has a lot of a much deeper impact on my



learning. And my personal learning". These comments about mathematical knowledge, an important feature of a core practices model for teacher education (Forzani, 2014), provides further justification for the inclusion of a problem-solving experience before engaging in problem solving as a potential pedagogy of practice.

Problem-solving to cater for diversity, and making connections to future teaching practice

The novice teachers noticed and were appreciative of the way a problem-solving approach catered for a diversity of mathematical learning needs, and provided a possible alternative to the practice of streaming children according to perceived ability. Some novice teachers related this aspect of the problem-solving teaching approach to their future teaching while others focused on their current experience in the university setting. Charlotte said:

... I think the way that problem-solving method that you do in class where everyone is able to start, is brilliant. And there are different finishing points. It shows me how you can implement a maths group room that can cater to everyone but doesn't scare the pants off people.

Ngāwae supported Charlotte's comment, adding:

...And they still feel... like they're doing the same maths. It's not like one's doing dumbie maths and one's doing amazing maths. If it's the same theme. You feel like you're doing the same maths.

Neil and Andrea explained how they appreciated having a choice about which problem to start with. Andrea said:

Yeah, that was really good. Automatically I always start right at the bottom and see where I am. This is a bit easy. And then you can move yourself. Whereas being told that you have to do something makes it seem harder than it is. You can figure it out yourself. You think, yeah I can do that. Move myself along. But being told where to go, I reckon that's more scary. Like oh I think I can do it. I can't do it.

Andrea further commented that the approach was different to streaming. She said, "it wasn't like streamed. It wasn't like 'the circles come to the mat'. Anyone working on this problem, you can come up. You could be in between or wherever...". Juliet commented in a similar way, saying:

But this allows everyone to step in. It allows everyone to be included. It has that safety of - is that just age, I don't know - I found this year, I can say 'I don't get it, I don't understand that'. But I think it does cater for that happening within a classroom too. That inclusive classroom where someone does feel safe to say, 'I don't get it. I don't understand'. Being so, caters for diversity. People coming on at their own time. At their own level.

Participating in this problem-solving module has alerted novice teachers to one way of creating an inclusive classroom: one that would heed Forzani's (2014) and Ball and Forzani's (2009) call for the learning of *all* children as a central goal of teaching. This small group of novice teachers were also able to make connections between their experience in the problem-solving module, and their future teaching. During the focus group discussion Ngāwae realised that because she was able to "do the math" she had missed the opportunity to simultaneously observe how the approach was being modelled. She remarked, "That's what I'm sort of regretting now. I should have gone down to the mat to learn from a teacher's perspective as to



how you should come across". It appeared to be challenging for Ngāwae to simultaneously pay attention to the mathematics and possible pedagogies. In contrast, some of the novice teachers were more aware of the modelling that was taking place. Charlotte said:

It was all very good how ...(lecturer-researcher's name) asked us when dealing with that problem solving, you went through it, and you said how could that work? Instead of streaming, and you know, you just did point out all the different things that were going on.

Another novice teacher, Damian, queried whether the modelling of the problem-solving approach was a deliberate act on the part of the lecturer. Debriefing, commenting, critiquing and discussion are important aspects of focussing teacher education on practice (Ball & Forzani, 2009) and it seems that teacher educators need to make an ongoing and deliberate effort to make such 'practice' transparent to novice teachers.

The novice teachers had begun to make connections with their future teaching practice. Jack commented that he doubted he would have "... enough grounding to stick to the problem-solving method and what we've been talking about. So yes, it's going to be hard to stick to". And Charlotte expressed her concerns about the open-ness of a problem-solving approach. She said:

That's what so hard about the problem-solving thing is that you don't know where it's going to lead. Like you don't have a textbook where you can say ok, that's the answer. Because you have no idea about what's going to come up and if you're going to know how to deal with it.

These novice teachers' insights provide an argument for a problem-solving experience to be followed with opportunities to practice the enactment of this approach. This would support their learning about anticipating and managing the uncertainty that is an inherent aspect of teaching mathematics (Forzani, 2014). Juliet, Tony, Odette and Neil expressed some excitement about the possibilities of enacting this approach in the future teaching. Juliet said:

It opens it all up and it's exciting and it's not as daunting. Still is a bit but not as daunting because it's a problem solving nature - we can talk about this, we can discuss processes.

This variety in thinking points to the need for further research to ascertain how the novice teachers translate their experiences into an enactment of core high-leverage practices.

Problem-solving as a way to learn about high-leverage teaching practices

In this section we describe practices that the novice teachers identified as important to their experience of the high-leverage teaching practice of problem-solving. Firstly, embedded in the novice teachers' comments was the realisation that children are expected to be problem-solvers and capable of critical thinking. Given the lack of personal experience the novice teachers had had with problem-solving, realising that mathematics could be taught using this approach appeared to be quite an 'eye-opener'. Ngāwae put it this way:

...that is something that's just really changed me. I thought we just have to explicitly model it for them, and then they do it, whereas it should be like a co-constructed modelling, really.

Andrea commented:

I think you can get really caught up in being constructional[sic] in Maths and say: 'This is what you do. This is how you do it.' But then, taking on that approach and stepping back and letting the kids do it, they're going to learn better from that if they're doing it themselves.



Another aspect noticed by the novice teachers was the use of multiple strategies to solve problems. Andrea commented, "I think it's really good, acknowledging different ways to get an answer". Neil agreed, saying:

It's just moved away from the algorithms and 'this is how you do it', 'this is right and wrong'. To a more kind of flexible approach to Maths. There's not one right or wrong way".

Charlotte appreciated "seeing other people's way of working it out".

The novice teachers also learned about the value of children being expected to explain and justify their thinking, and make sense of the mathematics. Juliet referred to the "valu[ing] of understanding and process", and Damian commented that teachers could "let the kids go away, work things out, put it in their own words, and then report back, say how they did it". Odette realised, "It's not ok for kids to just give the answer. But they should be justifying how we got there, not just in equations, but in words we fully understand". Odette and Tony also explained how justifying thinking had become the usual practice for their whole group of novice teachers. Odette explained:

And in our class, when someone, when you get it, someone's almost guaranteed to say, 'how did you get that'? So you do have to be able to explain it... Yeah, when we're working through the problems, say like they're, they're the Algebra ones. Someone will get it first and then we'll be like, 'well how did you figure that out?' so you'll have to backtrack.

Tony agreed saying, "It's almost the second question".

Having enough time to think and process, and solve problems was something that five of the novice teachers referred to in the focus group discussions. Their comments have led us to wonder if the provision of adequate time is an important micro-practice within the high-leverage-teaching-practice of problem-solving. Jack reflected back to his school days and commented that he, "could have processed it if I was kind of given the chance. That's what I'm finding out now". Damian said, "There was actually plenty of time to go through them at your own pace, and you had time to work things out, as well. Have discussions about things". Neil indicated taking the time to solve problems supported the sense-making process. He said:

Time is the key to learning it all properly and learning it all well. Rather than being rushed through it. And kind of just skimming the surface. And if you've made sense of it for yourself, it made it a whole lot more meaning.

Providing sufficient time to genuinely engage in the solving of meaningful problems has supported the novice teachers' learning of problem-solving as a high-leverage teaching practice. We contend this finding may add to the suggestion made by Hlas and Hlas' (2012) that rather than attempting to prepare novice teachers for all aspects of teaching, "less may indeed be more" (Hlas & Hlas, 2012, p. S78). That is, taking the time to provide novice teachers with a genuine problem-solving experience, including reflecting on, discussing and linking the approach to relevant literature, at the expense of covering all possible topics, may be worthwhile.

The continuing relevance and importance of attending to the affective dimension

In analysing the transcripts it became clear that the novice teachers' past experiences, beliefs and attitudes towards mathematics and attendant emotional reactions were an integral part of their teacher education experience. Tony recalled his struggles with mathematics and said:



A big part of it was coming to the realisation that I was never taught. I was just given the question, given the answer, kind of thing. You either knew how to do it or you didn't, but you were never taught any strategies and ways to approach it, that sort of thing. And so seeing, like, you modelling different teaching approaches and strategies and stuff like that. If I'd had that back then my whole outlook would have been very different.

Neil and Andrea identified that providing the novice teachers with a variety of problems (from which they could self-select) created a comfortable space. Neil said, "The way it was set up so you know, go to where you feel comfortable. So we could all kind of choose what we get". Juliet said, "we felt respected and honoured even though we're coming in at the bottom. That our experience has mattered. Our experience is valid. And I think everyone's appreciated that". Much has been written about affective dimension of teacher education (see for example, Lomas, Grootenboer, & Attard, 2011; Beswick, 2007; 2008). While Ball and Forzani (2009, p. 597) argue for "making practice the core of teachers' professional preparation", rather than factors such as beliefs, knowledge, orientations and commitment, we suggest it is as important to provide novice teachers with experiences that simultaneously support their learning about the craft of teaching and provide an emotionally safe and comfortable environment. The problem-solving module provided at least this small group of novice teachers with a safe and nurturing space where they could experience an alternative pedagogical approach for the learning and teaching of mathematics. This enabled them to move their thinking to a more positive frame, and one that is more closely aligned with ambitious teaching.

Final remarks

Data from this study revealed that for the majority of this small group of novice teachers problem-solving had not been part of their personal past mathematical experiences. Engaging in a module of algebraic problem-solving was positively received. It led to shifts in their thinking about what it means to 'do' mathematics and their envisaging of how they might support children's learning of mathematics through ambitious teaching. We argue that experiencing and reflecting upon a problem-solving approach is an important step towards learning about ambitious mathematics teaching. Problem-solving could then be a potential 'pedagogy of practice' whereby novice teachers practice and reflect on problem-solving with their colleagues and/or children. This contention requires ongoing research.

A problem-solving experience enabled these novice teachers to learn about a range of high-leverage teaching practices congruent with those identified in the literature. Novice teachers speculated that children could be mathematical problem-solvers and critical thinkers, and that multiple strategies can be used to solve problems. The novice teachers also noticed and valued the emphasis on explaining and justifying one's thinking. Having sufficient time to fully engage in the problem-solving process was appreciated, and appeared to lead to deeper learning. We concur with Hlas and Hlas (2012) who suggest that an emphasis on less rather than more might be in the best interests for novice teachers' learning. Participating in mathematical problem-solving also enabled the novice teachers to experience and identify one way of catering for diversity in mathematical learning needs. Data suggests that teacher educators need to make an ongoing and deliberate effort to make practices explicit.

A limitation of this research is the small group of participants. It is recognised that those who volunteered to be a part of the study may have been more positive about their experiences within the mathematics education paper. It would be valuable to gather data from a larger number of students.



As well-documented in research, past experiences, beliefs, and attitudes affect teachers' thinking about mathematics and what it means to teach and learn this subject (Grootenboer, 2008; Wieman, 2011). Data from this small group of novice teachers confirmed this. The novice teachers appreciated how the algebraic problem-solving module provided a safe and comfortable environment that respected and honoured past experiences. While pedagogies of practice are vital aspects of pre-service teacher education (Ball & Forzani, 2009) we contend providing a safe and supportive environment for learning about the teaching of mathematics is also important.

Some of the novice teachers began to consider the enactment of the problem-solving approach for their future teaching. Responses varied from positive to some apprehension. Research suggests that teaching practice be decomposed into 'parts' that can be learned and practised (Ball & Forzani, 2009; Forzani, 2014). While problem-solving may generate feelings of unease, and uncertainty with regard to which pathways students may follow, we wonder if Polya's steps of problem-solving (1945, cited in Frankcom, 2009) could provide a framework for breaking down this approach into parts that are visible to and able to be learned by the novice teacher. A future pedagogy of practice could be the enactment of a problem-solving approach, using Polya's steps, to ease the uncertainty and contingent nature of such teaching.

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References

- Anthony, G., & Hunter, R. (2013). Learning the work of ambitious mathematics teaching. In V. Steinle, L. Ball & C. Bordini (Eds.), *Mathematics education: Yesterday, today and tomorrow. Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia* (pp. 699-702). Melbourne, Australia: MERGA.
- Anthony, G., Hunter, R., Hunter, J., Rawlins, P., Averill, R., Drake, M., & Anderson, D. (2015). Learning the work of ambitious mathematics teaching. Teaching and Learning Research Initiative. Retrieved June 15, 2015, from http://www.tlri.org.nz/sites/default/files/projects/TLRI_Anthony_Final%20Summary%20Report%20%28v3%29.pdf.
- Ball, D. L., & Forzani, F. M. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education*, 60(5), 497-511.
- Ball, D. L., Sleep, L., Boerst, T. A., & Bass, H. (2009). Combining the development of practice and the practice of development in teacher education. *The Elementary School Journal*, 109(5), 458-474.
- Boaler, J. (2002). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning*. Mahwah, New Jersey: Lawrence Erlbaum.
- Boaler, J. (2008). *What's math got to do with it?: helping children learn to love their least favourite subject - and why it's important for America*. New York: Viking.
- Beswick, K. (2007/8). Influencing teachers' beliefs about teaching mathematics for numeracy to students with mathematics learning difficulties. *Mathematics Teacher Education and Development*, 9, 3-20.



- Borko, H., Liston, D., & Whitcomb, J. (2007). Genres of empirical research in teacher education. *Journal of Teacher Education*, 58(1), 3-11.
- Breyfogle, L., Williams, L. (2008). Designing and implementing worthwhile tasks. *Teaching Children Mathematics*, 15(5), 276-280.
- Clandinin, D.J., & Rosiek, J. (2007). Mapping a landscape of narrative inquiry: Borderland spaces and tensions. In D. J. Clandinin (Ed.), *Handbook of narrative inquiry: Mapping a methodology* (pp. 35-75). Thousand Oaks, CA: Sage.
- Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist*, 31(3-4), 175-190.
- Coburn, C. E. (2005). The role of nonsystem actors in the relationship between policy and practice: The case of reading instruction in California. *Educational Evaluation and Policy Analysis*, 27(1), 23-52.
- Cohen, L., Manion, L. & Morrison, K. (2007). *Research Methods in Education*. New York: Routledge.
- Frankcom, G. (2009). Investigative mathematics: Discovering the teacher mathematician inside. In R. Averill & R. Harvey (Eds.), *Teaching Secondary School Mathematics and Statistics: Evidence-based Practice* (pp. 23-36). Wellington: NZCER Press.
- Forzani, F. M. (2014). Understanding “core practices” and “practice-based” teacher education: Learning from the past. *Journal of Teacher Education*, 65(4), 357-368.
- Grootenboer, P. (2008). Mathematical belief change in prospective primary teachers. *Journal of Mathematics Teacher Education*, 11(6) 479-497.
- Grootenboer, P., & Jorgensen, R. (2009). Towards a theory of identity and agency in coming to learn mathematics. *Eurasia Journal of Mathematics, Science and Technology Education*, 5(3), 255-266.
- Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record*, 111(9). Retrieved February 26, 2015 from <http://www.tcrecord.org/content.asp?contentid=15018>
- Guillaume, A. M., & Kirtman, L. (2005). Learning lessons about lessons: Memories of mathematics instruction. *Teaching Children Mathematics*, 11(6), 302-309.
- Hlas, C. H., & Hlas, C. S. (2012). A review of high-leverage teaching practices: Making connections between mathematics and foreign languages. *Foreign Language Annals*, 45(S1), S76-S97.
- Holton, D. (2009). Problem-solving in the secondary school. In R. Averill & R. Harvey (Eds.), *Teaching Secondary School Mathematics and Statistics: Evidence-based Practice* (pp. 37-52). Wellington: NZCER Press.
- Hunter, R. & Anthony, G. (2010). Developing mathematical inquiry and argumentation. In R. Averill and R. Harvey (Eds.), *Teaching Primary School Mathematics and Statistics: Evidence-based practice*. Wellington: NZCER.
- Jorgensen, R., & Dole, S. (2011). *Teaching mathematics in primary schools* (2nd ed.). Crows Nest, NSW: Allen & Unwin.
- Kane, R. (2007). From naïve practitioner to teacher educator and researcher: constructing a personal pedagogy of teacher education. In T. Russell & J. Loughran (Eds.), *Enacting a pedagogy of teacher education: Values, relationships and practices* (pp. 60-76). London: Routledge.
- Korthagen, F. (2004). In search of the essence of a good teacher: towards a more holistic approach in teacher education. *Teaching and Teacher Education*, 20, 77-97.
- Labaree, D. (2000). On the nature of teaching and teacher education. *Journal of Teacher Education* 51(3), 228-233.
- Lampert, M., Beasley, H., Ghousseini, H., Kazemi, E., & Franke, M. (2010). Using designed instructional activities to enable novices to manage ambitious mathematics teaching. In M. K. Stein & L. Kucak (Eds.), *Instructional Explanations in the Disciplines* (pp. 129-141). New York: Springer.
- Lampert, M., & Graziani, F. (2009). Instructional activities as a tool for teachers' and teacher educators' learning. *The Elementary School Journal*, 109(5), 491-509.
- Lomas, G., Grootenboer, P., Attard, C. (2012). The affective domain and mathematics education. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Research in Mathematics Education in Australasia 2008-2011* (pp. 23-37). Melbourne: MERGA.
- Ministry of Education. (1992). *Mathematics in the New Zealand Curriculum*. Wellington: Learning Media.

- Ministry of Education. (1999). Teaching problem-solving in mathematics: Years 1-8. Wellington: Learning Media.
- Ministry of Education. (2007). The New Zealand Curriculum. Wellington: Learning Media.
- Quinnell, L. Why are mathematical investigations important? *Australian Mathematics Teacher*, 66(3), 35-40.
- Rigelman, N. (2007). Fostering mathematical thinking and problem solving: the teacher's role. *Teaching Children Mathematics*, 13(6), 308-314.
- Rogers, G. (2007). Learning-to-learn and learning-to-teach: Experiencing history in teacher education. Paper presented at the BERA Annual Conference, Institute of Education, University of London, Sept 5-8, 2007.
- Russell, T. (2007). How experience changed my values as a teacher educator? In T. Russell & J. Loughran (Eds.), *Enacting a pedagogy of teacher education* (pp. 182-191). London: Routledge.
- St. Pierre, E. A. (2011). Post qualitative research: the critique and the coming after. In N. K. Denzin & Y. S. Lincoln. (Eds.), *The Sage handbook of qualitative research* (pp. 611-625). Thousand Oaks: Sage.
- Strauss, A., & Corbin, J. (1994). Grounded theory methodology: an overview. In N. Denzin and Y. Lincoln (Eds.), *Handbook of Qualitative Research* (pp. 273-285). Thousand Oaks, CA: Sage.
- Wieman, R. (2011). Students' beliefs about mathematics: Lessons learned from teaching note-taking. *Mathematics Teacher*, 104(6), 406-407.
- Young-Loveridge, J. (2010). A decade of reform in mathematics education: Results for 2009 and earlier years. Findings from the New Zealand Numeracy Development Projects. Wellington: Learning Media.
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