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Principles and Guidelines for Equitable Mathematics Teaching Practices and Materials for English Language Learners[1]

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In this essay, the author describes principles for equitable mathematics teaching practices for English Language Learners (ELLs) and outlines guidelines for materials to support such practices. Although research cannot provide a recipe for equitable teaching practices for ELLs, teachers, educators, and administrators can use this set of research-based principles and guidelines to design equitable mathematics instruction, developing their own approaches to supporting equitable practices in mathematics classrooms. The recommendations presented use a complex view of mathematical language as not only specialized vocabulary but also as extended discourse that includes syntax, organization, the mathematics register, and discourse practices. The principles and guidelines stress the importance of creating learning environments that support all students (but specifically those learning English) in engaging in rich mathematical activity and discussions.

KEYWORDS: English language learners, mathematics education

The purpose of this essay is to describe principles for equitable mathematics teaching practices for English Language Learners (ELLs) and outline guidelines for materials to support such practices. The approach to equity used here is based on Gutiérrez's (2009, 2012) discussion of four dimensions of equity: access, achievement, identity, and power. Using these dimensions, I contend that ELLs need *access* to curricula, classroom practices, and teachers shown to be effective in supporting the mathematical academic *achievement*, *identities*, and *practices* of these students. I define equitable teaching practices for students who are learning English in mathematics classrooms as those that (a) support mathematical reasoning, conceptual understanding, and discourse—because we know such practices lead to learning important mathematics, and (b) broaden participa-

[1] The principles and guidelines described and outlined here are informed by a sociocultural and situated perspective on mathematical thinking, on language, and on bilingual mathematics learners; for details of this framework see Moschkovich, 2002, 2007b, 2010.

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tion for students who are learning English—because we know that participation is connected to opportunities to learn.

To support mathematical reasoning, conceptual understanding, and discourse, classroom practices need to provide all students with opportunities to participate in mathematical activities that use multiple resources to do and learn mathematics. To broaden participation, classroom practices need to provide all students with opportunities to use multiple ways of engaging in classroom discourse. Equitable classroom practices, then, are fundamentally focused on honoring student resources, in particular, the "repertoires of practices" (Gutiérrez & Rogoff, 2003) that students bring to the classroom. Equitable mathematics classroom practices for ELLs should be informed by knowledge of students' experiences with mathematics instruction, language history, and educational background (Moschkovich, 2010). Teachers need to know details of a student's history with formal schooling, for example, which grades they attended, where, and in what language (or languages). They should have some information about their language history, for example, are they literate in their home language, what is their reading and writing competence in the home language. Some students may not have had any formal instruction in the language spoken at home. Another important piece of information is the students' history with school mathematics instruction: when they had mathematics classes, in what language, and for which topics.[2]

We often hear that "academic language" is important for English Language Learners, but this phrase can have multiple meanings. Interpretations of this phrase often reduce the meaning of "academic language in mathematics" to single words or technical vocabulary. In contrast, the recommendations for teaching practices and materials described here are based on research and a view of language that run counter to commonsense notions of language. These principles and guidelines use a more complex view of mathematical language as not only specialized vocabulary but also as extended discourse that includes syntax, organization, the mathematics register (Halliday, 1978), and discourse practices (Moschkovich, 2007c). The phrase "the language of mathematics" is used here not to mean a list of vocabulary or technical words with precise meanings, but rather the communicative competence necessary and sufficient for competent participation in mathematical discourse practices (Moschkovich, 2012).

While learning vocabulary is necessary, it is not sufficient. In other words, learning to communicate mathematically and participate in mathematical discussions is not a matter of merely learning vocabulary. During discussions in mathematics classrooms, students are learning to describe patterns, make generalizations, and use representations to support their claims. The question is not whether students who are ELLs should learn vocabulary, but rather how instruction can

[2] For more details on equitable practices see Moschkovich, in press.

best support students to learn vocabulary as they actively engage in mathematical reasoning about important mathematical topics. Therefore, the principles and guidelines presented here stress the importance of creating learning environments that support all students (but specifically those learning English) in engaging in rich mathematical activity and discussions.

Enacting the recommended principles and guidelines requires that teachers develop skills and strategies for leading, supporting, and orchestrating mathematical discussions, whether these occur in small groups or with the whole class. A review of the research suggests that professional development that has an impact on student achievement provides "adequate time for professional development and ensures that the extended opportunities to learn emphasize observing and analyzing students' understanding of the subject matter" (American Educational Research Association [AERA], 2005). Two other characteristics of effective professional development include linking professional learning to teachers' real work and using actual curriculum materials. Therefore, professional development can support teachers in learning these skills and strategies through long-term work in the context of particular mathematics topics, for example, focusing on teacher questions to support student algebraic (Driscoll, 1999) or geometric thinking (Driscoll, DiMatteo, Nikula, & Egan, 2007). These skills also can be supported through long-term professional development that exposes teachers to examples of best practices for supporting mathematical discussions and engages teachers in reading about discourse in mathematics classrooms (e.g., Moschkovich, 1999, 2007c; O'Connor & Michaels, 1993; Sherin, 2002; Stein, Engle, Smith, & Hughes, 2008), watching classroom video (e.g., Chapin, O'Connor, & Anderson, 2003; Sherin & van Es, 2005), lesson study (e.g., Fernandez, 2005), and so on. These skills and strategies for teaching mathematics are fundamental to supporting students in the Common Core State Standards (CCSS), the Standards for Mathematical Practice, and teaching mathematics for understanding, and are essential for supporting ELLs.[3]

Principles for Equitable Mathematics Instruction for ELLs

The following sections summarize (briefly) research relevant to principles for equitable mathematics instruction for ELLs. The summary includes: (a) research-based recommendations for effective instruction for ELLs (in general, not

[3] There are materials available that specifically address teaching mathematics to ELLs. There are also materials that, although they do not target ELLs in particular, can be used to support teachers in learning to orchestrate mathematical discussions (e.g., *Five Practices for Orchestrating Productive Mathematics Discussions* [Stein & Smith, 2011] and *Classroom Discussions: Using Math Talk to Help Students Learn, Grades 1-6* [Chapin, O'Connor, & Anderson, 2003]). See http://www.corestandards.org/Math for the CCSS for Mathematical Practice.

specific to mathematics); (b) research-based recommendations for effective instruction in mathematics (for all students, not ELLs in particular); and (c) research-based recommendations for effective mathematics instruction specific to ELLs that is aligned with the CCSS. A principled approach to teaching mathematics to ELLs would include characteristics from each section.

What is Effective Instruction for ELLs?

Although it is difficult to make generalizations about the instructional needs of all students who are learning English, instruction should be informed by knowledge of students' experiences with mathematics instruction, language history, and educational background (Moschkovich, 2010). In addition, research suggests that high-quality instruction for ELLs that supports student achievement has two general characteristics: a view of language as a resource rather than a deficiency, and an emphasis on academic achievement, not only on learning English (Gándara & Contreras, 2009).

Research provides general guidelines for instruction for ELLs. Overall, students who are labeled as such are from non-dominant communities and need access to curricula, teachers, and instructional techniques proven to be effective in supporting the academic success of ELLs. The general characteristics of such environments are that curricula provide "abundant and diverse opportunities for speaking, listening, reading, and writing" and that instruction should "encourage students to take risks, construct meaning, and seek reinterpretations of knowledge within compatible social contexts" (Garcia & Gonzalez, 1995, p. 424). Teachers with documented success with students from non-dominant communities share some characteristics (Garcia & Gonzalez, 1995): (a) a high commitment to students' academic success and to student-home communication, (b) high expectations for all students, (c) the autonomy to change curriculum and instruction to meet the specific needs of students, and (d) a rejection of models of their students as intellectually disadvantaged. Curriculum policies for ELLs in mathematics should follow the guidelines for traditionally underserved students (AERA, 2006), such as instituting systems that broaden course-taking options and avoiding systems of tracking students that limit their opportunities to learn and delay their exposure to college-preparatory mathematics coursework.

What is Effective Mathematics Instruction?

According to a review of the research (see Hiebert & Grouws, 2007), mathematics teaching that makes a difference in student achievement and promotes conceptual development in mathematics has two central features. First, teachers and students attend explicitly to concepts; second, teachers should give students the time to wrestle with important mathematics. Mathematics instruction for ELLs should follow these general recommendations for high-quality mathematics instruction, for example, by encouraging students to explain their problem-solving and reasoning (AERA, 2006; Stein, Grover, & Henningsen 1996).

What is Effective Mathematics Instruction for ELLs Aligned with the CCSS?

First and foremost, mathematics instruction that is aligned with the CCSS means teaching mathematics for understanding (Hiebert, 1997). All students should use and connect multiple representations, share and refine their reasoning, and develop meaning for symbols. Mathematics instruction for ELLs should align with the CCSS, particularly in these four ways:

- **Balance conceptual understanding and procedural fluency.** Instruction should balance student activities that address important conceptual and procedural knowledge and connect the two types of knowledge (Hiebert, 1997; Hiebert & Grouws, 2007).

- **Maintain high cognitive demand.** Instruction should use high cognitive demand mathematical tasks and maintain the rigor of tasks throughout lessons and units (Stein, Grover, & Henningsen, 1996; Stein, Smith, Henningsen, & Silver, 2000*)*.

- **Develop beliefs.** Instruction should support students in developing beliefs that mathematics is sensible, worthwhile, and doable (Schoenfeld, 1992).

- **Engage students in mathematical practices.** Instruction should provide opportunities for students to engage in mathematical practices such as solving problems, making connections, understanding multiple representations of mathematical concepts, communicating their thinking, justifying their reasoning, and critiquing arguments (for the CCSS for Mathematical Practice see http://www.corestandards.org/Math).

Recommendations for Mathematics Instruction for ELLs

Effective instruction for ELLs should have the principles previously noted; these principles are important for mathematics instruction generally and mathematical instruction that is aligned with the CCSS specifically. In addition, there are several recommendations that are specific to mathematics instruction for ELLs. Instruction for ELLs should not emphasize low-level language skills over opportunities to actively communicate about mathematical ideas. Research on language and mathematics education provides general guidelines for instructional practices for teaching ELLs (Moschkovich, 2010). Mathematics instruction for ELLs should address more than vocabulary and support ELLs' participation in mathematical discussions as they learn English. Instruction should draw on multiple resources available in classrooms (objects, drawings, graphs, and gestures) as well as home languages and experiences outside of school. Below, I expand on these general guidelines by providing four recommendations to guide teaching practices.

- **Recommendation #1: Focus on students' mathematical reasoning, not accuracy in using language.** Instruction should focus on uncovering, hearing, and supporting students' mathematical reasoning, not on accuracy in using language (Moschkovich, 2010). Instruction should focus on recognizing students' emerging mathematical reasoning and focus on the mathematical meanings learners construct, not the mistakes they make or the obstacles they face. Instruction needs to first focus on assessing content knowledge as distinct from fluency of expression in English so that teachers can then build on, extend, and refine students' mathematical reasoning. If we focus only on language accuracy, we miss the mathematical reasoning.

- **Recommendation #2: Focus on mathematical practices, not language as single words or vocabulary.** Instruction should move away from simplified views of language and interpreting language as vocabulary, single words, grammar, or a list of definitions (Moschkovich, 2010). An overemphasis on correct vocabulary and formal language limits the linguistic resources teachers and students can use to learn mathematics with understanding. If we only focus on accurate vocabulary, we can miss how students are participating in mathematical practices. Instruction should provide opportunities for students to actively use mathematical language to communicate about and negotiate meaning for mathematical situations. Instruction should provide opportunities for students to actively engage in mathematical practices such as reasoning, constructing arguments, expressing structure and regularity, and so on.

- **Recommendation #3: Recognize the complexity of language in mathematics classrooms and support students in engaging in this complexity.** Language in mathematics classrooms is complex and includes multiple: representations (objects, pictures, words, symbols, tables, graphs); modes (oral, written, receptive, expressive); kinds of written texts (textbooks, word problems, student explanations, teacher explanations); kinds of talk (exploratory, expository); and audiences (presentations to teacher, peers, by teacher, by peers).

- **Recommendation #4: Treat everyday and home languages as resources, not obstacles.** Treating home or everyday language as obstacles limits the linguistic resources for communicating mathematical reasoning (Moschkovich, 2007d, 2009). Everyday language and academic language are interdependent and related—not mutually exclusive. Everyday language and experiences are not necessarily obstacles to developing academic ways of communicating in mathematics (Moschkovich, 2002, 2007a, 2007b, 2007c). All students, including ELLs, bring linguistic resources to the mathematics classroom that can be employed to engage with activities designed to meet the CCSS. As students continue to expand their linguistic repertoires in English, students can use a wide variety of linguistic resources—including home languages, everyday language, developing proficiency in English, and nonstandard varieties of English—to engage deeply with the kinds of instruction called for in the CCSS (Bunch, Kibler, & Pimentel, 2012).

Guidelines for Mathematics Practices and Materials for ELLs[4]

The guidelines described here are adapted from and based, in part, on work by the Understanding Language Mathematics Workgroup. That work, currently under development, aims to provide general guidelines and instructional principles that hold promise for maximizing alignment between mathematics instruction for ELLs and the CCSS for Mathematical Practice. The work by this discipline specific workgroup (which I am a member) has informed, and been informed by, efforts on the part of the more general Understanding Language (UL) Workgroup that is developing key principles for instruction intended to guide educators and administrators as they work to help ELLs meet standards in various content areas.

As the Mathematics Workgroup conducted our work, I developed the following Guidelines for Mathematics Instructional Materials. The purpose of these guidelines was to develop a shared understanding of how instructional materials and approaches for teaching ELLs in mathematics might be framed in ways that are aligned with the CCSS. These guidelines draw in part on papers prepared for the January 2012 Understanding Language conference at Stanford University (http://ell.stanford.edu/papers/practice) and were modeled after the Guidelines for English Language Arts (ELA) materials (Bunch, 2012). The guidelines described,

[4] These guidelines were developed using the Understanding Language project's English Language Arts Unit Guidelines as a model (see Bunch, 2012).

while developed to correspond with the UL project-wide Principles and parallel the ELA Guidelines, are distinct in that they specifically address the CCSS for mathematics and are intended to inform the adaptation of mathematics instructional materials to address the needs of ELLs.[5]

1. **Engage students in the eight CCSS for Mathematical Practice.** When designing instruction, consider how students will participate in the eight standards for Mathematical Practice across the various modes of communication (reading, writing, listening, speaking) that students might use during instruction. It is not necessary to include every practice in every lesson; the goal is to provide students opportunities to actively participate in these mathematical practices when possible and appropriate.

   CCSS for Mathematical Practice

   1. Make sense of problems and persevere in solving them
   2. Reason abstractly and quantitatively
   3. Construct viable arguments and critique the reasoning of others
   4. Model with mathematics
   5. Use appropriate tools strategically
   6. Attend to precision
   7. Look for and make use of structure
   8. Look for and express regularity in repeated reasoning

   When considering #6 during instruction for ELLs, it is important to remember that emerging language may sometimes be imperfect and that mathematically precise statements need not to be expressed in full sentences. It is also crucial to recognize that mathematical precision lies not only in using the precise word but also in making precise mathematical claims.

2. **Keep tasks focused on high cognitive demand, conceptual understanding, and connecting multiple representations.** Mathematics instruction for ELLs should follow the general recommendations for high-quality mathematics instruction: (a) focus on mathematical concepts and the connections among those concepts; and (b) use and maintain high cognitive demand mathematical tasks, for example, by encouraging students to explain their problem solving and reasoning (AERA, 2006; Stein et al.,

[5] Neither these guidelines nor the "Understanding Language Principles" should be confused with the Publisher's Criteria for the Common Core State Standards in Mathematics, a more extensive document intended for commercial textbook companies and curriculum developers that was prepared by the Council of Chief State Schools Officers and others independent from the work of Understanding Language and which does not focus explicitly on ELLs.

1996). Explanations and justifications need not always include words. Instruction should support students in learning to develop oral and written explanations, but students also can show conceptual understanding by using pictures (e.g., a rectangle as an area model to show that two fractions are equivalent or how multiplication by a positive fraction smaller than one makes the result smaller).

3. **Facilitate students' production of different kinds of reasoning.** Instruction and materials should provide opportunities for students to produce different types of mathematical reasoning (i.e., algebraic thinking, geometric thinking, statistical thinking, etc.) and to share and compare reasoning. Instruction needs to include different language functions (purposes) such as describing, comparing, explaining, and arguing. Although sentence frames can be useful scaffolds, these should be used flexibly and fluidly, more as sentence starters than rigid formulas for producing perfect sentences.

4. **Facilitate students' participation in different kinds of participation structures.** Students should have opportunities to participate in a spectrum of participation structures—from informal collaborative group interactions to formal presentations—in ways that allow them to use their linguistic resources (e.g., first language, everyday language) and cultural resources (e.g., alternative algorithms). Materials should provide structures that allow students to collaborate with others, articulate ideas, interpret information, share explanations, present their solutions, and defend claims. Teacher led discussions are only one setting for mathematical discussions and instruction should support student participation in classroom mathematical discussions in other settings such as in pairs or in small groups. When creating these different structures, consider student proficiencies not only in English but also in mathematics as well as literacy in their first language.

5. **Focus on language as a resource for reasoning, sense making, and communicating with different audiences for different purposes.** Activities calling students' attention to features of language (e.g., grammatical structures, vocabulary, and conventions of written and oral language) should only occur in conjunction with, and in the service of, engagement with the mathematical ideas, mathematical practices, and multiple representations at the heart of high cognitive demand mathematical tasks. There are many ways to address vocabulary, including introducing, using, and reviewing. The pre-teaching of vocabulary should be carefully considered.

Vocabulary should not be introduced in isolation, but instead be included in activities that involve high cognitive demand mathematical work: reasoning, sense making, explaining, comparing solutions, and so on. When introducing new vocabulary, it is useful for students to first have a successful and engaging experience discussing their mathematical reasoning and developing their conceptual understanding, then later label, discuss, and review the vocabulary, having first grounded meanings in actually doing mathematics.

6. **Prepare students to deal with typical texts in mathematics.** Typical written texts in mathematics include not only word problems and mathematics textbooks but also other students' written explanations that are shared in small groups and a teacher's or a student's solution written on the board. Typical written texts also include assessment problems and scenarios for modeling. Oral texts include explanations, descriptions of solutions, conjectures, and justifications. The goal of instruction should not necessarily be to "reduce the language demands" of a written text, but instead to provide support and scaffolding for ELLs to learn how to manage complex text in mathematics. There are several reasons to not adapt the language of a task: (a) changing the language of a task can change the mathematical sense of the task; (b) it is not yet clear which adaptations are best to make for which students, for which purposes, or at which times; (c) instruction should support students in understanding complex mathematical texts as they are likely to appear in curriculum and assessment materials; and (d) experiences that allow ELLs to engage with authentic language used in mathematics (with support) can provide opportunities for their continued language development.

Closing Thoughts

Equity and social justice considerations require that ELLs have access to high-quality and effective mathematics instruction. Currently, we do not have a set of empirical studies showing that a specific curriculum, teaching approach, or instructional practice is the cause for an effect on the learning, achievement, or motivation for ELLs. However, we have decades of research on effective teaching for students from non-dominant communities, even if not specifically in mathematics. We also have reviews of research pointing to the general characteristics of effective mathematics teaching, not specific to ELLs but still relevant. The recommendations summarized here are an attempt to collect what we already know while we continue to conduct more research relevant to mathematics teaching for ELLs.

When I attended the Privilege and Oppression in the Mathematics Preparation of Teacher Educators (PrOMPTE[6]) conference, I was involved in work with the Understanding Language Mathematics Workgroup. At that time, I had just completed the first phase of a project developing resources for teachers to address the needs of ELLs in their mathematics instruction. The goal of that project was to develop materials to illustrate how mathematical tasks aligned with the CCSS can be used to support mathematics instruction for ELLs.[7]

During the PrOMPTE conference, I decided to use that work to also develop a set of general principles for designing instruction and reviewing materials because I hoped these principles could provide resources for mathematics educators. I left PrOMPTE deeply committed to doing something that could inform practice. The set of principles outlined here is thus a result, not only of my work with the Understanding Language project but also of the discussions and conversations at PrOMPTE.

My intention in this essay was not to provide a perfect definition of equitable teaching practices for ELLs, but rather to establish some common ground using reviews of relevant empirical research. It is my sincere hope that the principles, recommendations, and guidelines provided prove useful for designing equitable mathematics instruction, reviewing curriculum materials, and supporting mathematics educators in preparing new teachers.

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[6] Privilege and Oppression in the Mathematics Preparation of Teacher Educators (PrOMPTE) conference (funded by CREATE for STEM Institute through the Lappan-Phillips-Fitzgerald CMP 2 Innovation Grant program), Michigan State University, Battle Creek, MI, October 2012. Any opinions, findings, and conclusions or recommendations expressed herein are those of the authors and do not necessarily reflect the views of the funding agency.

[7] The workgroup members used tasks from two publicly accessible curriculum projects: *Inside Mathematics* (see http://www.insidemathematics.org) and *Mathematics Assessment Project* (see http://map.mathshell.org/materials/index.php). Members of the workgroup developed the materials and a team of experts reviewed the materials; all materials developed will be available online at the Understanding Language website (see http://ell.stanford.edu/).

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