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Delegating Mathematical Authority as a Means to Strive Toward Equity

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In this article, the author provides insight into the pedagogical processes for delegating mathematical authority to students, through the use of specific classroom structures, as a means to strive toward equity. Employing qualitative methods, the author analyzes transcripts of classroom video, along with field notes and teacher and student interviews, collected during one semester of the participating teacher's Algebra I course. The author addresses how the teacher's practice was striving toward equity through the use of classroom structures that delegated mathematical authority to students. Analyses revealed that the teacher delegated mathematical authority through the use of student presentations, Shuffle Quizzes, and Participation Quizzes. Each instance featured was chosen to highlight a different facet of the ways in which delegating authority repositioned students as competent sense-makers.

KEYWORDS: delegating mathematical authority, equitable teaching and learning practices, mathematics education

Attention to complex practices involved in striving toward equity has become a growing priority in mathematics education (see, e.g., Bartell, 2013; Cohen & Lotan, 1997; Esmonde, 2009a; Gutiérrez, 2007; Martin, 2003; Nasir & Cobb, 2007; National Council of Teachers of Mathematics [NCTM], 2000; Zavala, 2014). With a few exceptions (e.g., Boaler & Staples, 2008; Jilk, 2007; Staples, 2008), the field does not yet have significant qualitative empirical evidence of equitable teaching and learning practices in secondary mathematics classrooms. In this article, I adopt Esmonde's (2009b) definition of equity as "a fair distribution of opportunities to learn" for all students (p. 1008). Teachers who strive toward equity intentionally pursue practices that help students to view everyone as capable of learning high-level content (Cohen & Lotan, 1997; Esmonde, 2009b; NCTM, 2000). This definition of equity, which assumes all students are competent to do high-level mathematics, also implies that equitable classrooms open up interactional space for a broad range of competent ideas.

Attention to equity suggests the need to evaluate enacted classroom structures and practices (Boaler, 2002). Previous research on classroom structures has attended to: teacher discourse through revoicing (O'Connor & Michaels, 1993); students'

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use of explanations as tools (Esmonde, 2009a); and teachers' discursive positioning moves facilitating English learners' opportunities to take on agentive problem-solving roles (Turner, Dominguez, Maldonado, & Empson, 2013). Because mathematics educators do not yet deeply understand pedagogies that move toward equitable learning opportunities (Esmonde, 2009b), this study is motivated by the need for qualitative documentation of equitable teaching and learning practices.

Given that cooperative learning practices such as complex instruction (e.g., Boaler & Staples, 2008; Cohen & Lotan, 1997, 2014; Featherstone et al., 2011) have been linked to equitable teaching and learning opportunities, here I share findings from a single case study on one teacher whose pedagogical practices centered on the use of complex instruction. My analysis of the collected data (transcripts of classroom video, classroom field notes, and teacher and student interviews) attends to the elements of the teacher's practice that strive toward equity. The purpose of the study was to contribute to filling the gap in the empirical research on equity to "uncover a range of solutions focused on what works, where, when, and why" (Martin, 2003, p. 18). Analysis of the teacher's pedagogical practices reveals empirical evidence of this teacher's process for striving toward equity by the use of specific classroom structures that served to *delegate mathematical authority* to her students.

Gresalfi and Cobb (2006) found that positioning students to have mathematical authority made students responsible for a higher level of mathematical argumentation. They described mathematical authority as

> the degree to which students are given opportunities to be involved in decision making and whether they have a say in establishing priorities in task completion, method, or pace of learning. Thus authority is not about "who's in charge" in terms of classroom management but "who's in charge" in terms of making mathematical contributions. (p. 51)

Gresalfi and Cobb asserted that *who* has mathematical authority in a particular classroom depends on *who* has been given the opportunity to verify that a given mathematics contribution is reasonable. Researchers have studied the delegation of authority as a way that teachers can make students responsible for their own and their classmates' learning (Bianchini, 1999; Lotan, 1997); support students to use one another as resources (Cohen, 1997a); increase student-student interdependence and shift checking for understanding to students (Ehrlich & Zack, 1997); and make students' life experiences, opinions, and points of view legitimate components of what is learned (Hand, 2003; Lotan, 2003).

Teachers delegate mathematical authority to students when they require students to convince their peers that their solutions make mathematical sense (Gresalfi, Martin, Hand, & Greeno, 2009) and when they require both individual and group products (Lotan, 1997). Delegating authority empowers students to argue, evaluate, and confirm the validity of their mathematical ideas. Students take on responsibility to explain mathematical concepts, answer questions, demonstrate multiple solution methods, and co-construct their overall conceptual understanding (Bianchini, 1999; Boaler & Staples, 2008; Gresalfi, 2009). When the teacher analytically scaffolds opportunities for students to participate in sustainable mathematical discourse, the delegation of authority is visible (Nathan & Knuth, 2003). Delegating authority can also manifest through (a) substantive student discourse, (b) task cards, (c) cooperative norms, and (d) use of procedural roles that set clear expectations for group and individual products (Cohen, 1994). The reporting of this study shows delegating mathematical authority as a significant pedagogical practice that offers equitable learning opportunities to students. Specifically, findings suggest that delegating mathematical authority to students can be (is) a significant aspect of striving toward equity in mathematics classrooms. Striving toward equity in mathematics education invokes constant, purposeful work from the teacher as she or he seeks to diminish differences in access, opportunities, and outcomes for students. This study builds on and contributes to current understandings of striving toward equity by investigating the pedagogical processes in which mathematical authority might be delegated from teacher to students (Esmonde, 2009b; Zahner, 2011). Two research questions guided the study:

1. What structures and practices did the teacher engage to delegate mathematical authority to students?
2. How was the teacher's process of delegating mathematical authority to students linked to striving toward equity?

Framing Learning

A sociocultural perspective on learning informs this study. This perspective allows the researcher to observe learners through their social and historical contexts as they develop and change in the classroom (Rogoff, 2003). In this study, students engaged in small group and whole class interactions. A sociocultural framing suggests students co-constructed their learning experiences, positioning themselves and their classmates at various levels of competence to complete mathematical tasks.

I draw on the sociocultural perspective in order to attend to learners' participation and interactions during group work; I simultaneously use status and positioning theories to explain the dynamic nature of learners' participation and interactions. Status theory explains which attributes have been valued in the classroom and offers an explanation for the ways students generalize their performance expectations for another student's contributions (Cohen & Lotan, 1997, 2014; Kalkhoff & Thye, 2006). Positioning theory attends to the dynamic process of positioning, or the ways the teacher and students actively position students to be mathematically competent (or not), and also offers the opportunity to understand discourse moves that established students' positions during social episodes in the activity of the classroom (Harré & van Langenhove, 1999). I couple these two theories to investigate how the teacher and students positioned one another to generalize expectations for mathematical competence.

Methods

Site and Participant Selection

As public school classrooms in the United States continue to increase in racial, ethnic, linguistic, and socioeconomic diversity, teachers are challenged to use pedagogies that are successful with heterogeneous student populations. Martin (2009) emphasized that examining race and racism, as well as other social and cultural markers of historical marginalized groups, are imperative considerations in research that strives toward equity. As such, I situated this study in an urban school with a racially, ethnically, linguistically, and socioeconomically diverse student population. I sought to understand how historically marginalized students (e.g., African Americans, Latinas/os, students impacted by poverty, immigrants, and girls in general) made sense of their mathematical learning within their classroom context. Because I was interested in learning primarily from a teacher whose students had not previously been successful in mathematics, I elected to work with a teacher and her students in a ninth-grade Algebra I class, knowing that many of the other ninth-graders were entering high school already prepared to take Geometry or Algebra II. (As I discuss later, it is significant that the students who were taking Algebra I in high school were less advanced in mathematics than some of their peers.)

I invited one teacher, Ms. Martin (all proper names are pseudonyms), to participate in this study because I believed that she intentionally made pedagogical choices to counter status issues in the classroom (Cohen, 1997b). She had been teaching Algebra I for five years when the study started; I came to know Ms. Martin about two years prior. Through these prior professional interactions, I learned that she cared about and intentionally developed sociomathematical norms (Kazemi & Stipek, 2001; Yackel & Cobb, 1996) which fostered interdependence during group work (Lotan, 2003); that she regularly engaged students in mathematics discourse (Cazden, 2001); and that she organized her classroom practices around student-centered learning. I also learned that she and I shared a common interest in supporting students who had previously been unsuccessful in mathematics.

Some mathematics educators and researchers have argued that effectively implemented group work is an important pedagogical tool that teachers (and students) can use to strive toward equity. For this reason, group work has become a consistent focus of research in the teaching and learning of mathematics (e.g., Boaler &

Staples, 2008; Cohen & Lotan, 1997, 2014; Esmonde, 2009b; Webb, 1991). "Effectively" implemented group work has been shown to increase student participation, engage students more deeply in their learning, develop their academic thinking (Boaler & Staples, 2008; Herrenkohl & Guerra, 1998), promote positive mathematics identities (Hand, 2006; Jilk, 2007; Nasir, Hand, & Taylor, 2008), and foster classroom relational equity (Boaler, 2006).

Ms. Martin's classroom practices feature group work facilitated by the principles of complex instruction (CI), a form of cooperative learning developed by Elizabeth Cohen and Rachel Lotan. CI is a form of ambitious teaching (Lampert, 1990; Lampert & Graziani, 2009), because the effective implementation of CI offers opportunities for every student to construct understandings of and reason about authentic problems. Much of Ms. Martin's learning and interpretation of CI was based on work that came out of Railside High School (Boaler & Staples, 2008), which had been presented to her during her teacher education program as a project that sought to understand equitable teaching and learning practices in mathematics classrooms. The presumption that every student is capable and competent in learning high-level mathematics is the key to a faithful implementation of CI. This presumption of competence drives CI teachers to the practice of regularly randomly assigning students to groups, which Ms. Martin did every 2 weeks during the study. CI teachers also aim to disrupt typical hierarchies that develop expectations for competence by using status theory and paying attention to the role of status as students work together (Cohen & Lotan, 1997, 2014). CI teachers facilitate learning in small groups by delegating mathematical authority to students and by assigning tasks that require multiple abilities and multiple entry points. Here, student time spent in class (i.e., classwork) was used to create opportunities for equal-status interactions in the classroom (Cohen & Lotan, 1997, 2014), thereby striving toward equity.

Data Collection

I used a qualitative case study approach to offer a thick and rich description (Corbin & Strauss, 2008) of the mathematical learning experience of one group of students. The data collection period took place over 50 classroom visits during one period in the fall semester of Ms. Martin's Algebra I class during the 2011–12 school year. This Algebra I class had 28 students. Twenty-four of the 28 students formally agreed to participate in the study. Seventeen students were African American or African immigrants,[1] five were Asian American, four were European American, and two were Latino/a. Twenty-two students were girls, six were boys. When

[1] Because student demographics were not made available to me, these calculations are on the basis of my own observation, so any error or misrepresentation of how students perceived their identities is my error.

compared with the school demographics, this Algebra I class had an overrepresentation of African Americans, Latinas, and girls in general, and an underrepresentation of Asian Americans, European Americans, and boys. I outline the racial and ethnic demographics here because it signifies the larger systemic inequities present in the opportunities these students had access to prior to arriving in this Algebra I class. In fact, the overrepresentation of African Americans, Latinas, and girls in general also mirrors a national trend in which, prior to entering high school, disproportionately more European Americans, Asian Americans, and boys successfully completed Algebra I, while disproportionately fewer African Americans, Latinos/as, and girls in general successfully completed Algebra I (Allexsaht-Snider & Hart, 2001; Oakes, 1990; Spielhagen, 2006; Tate, 1995). Above all, the data collection site for this study was motivated by the opportunity to feature successful mathematics learning opportunities for historically marginalized students.

I focused the data collection period around two units taught in Ms. Martin's Algebra I class during the fall semester. The first focal unit (Unit 1: Linears) was taught during 4 weeks at the beginning of the school year in the fall semester. The second focal unit (Unit 4: Lab Gear[2] and Solving Linear Equations) was taught for 2 weeks at the end of fall semester. The purpose of selecting one unit at the beginning and one unit at the end of the semester was to understand the nature of the students' experiences over time. During the focal units, I attended class every day. During the middle of the semester, I attended class about twice per week to collect field notes on student learning and to continue to build relationships with students.

Data collected for this study included: (a) field notes taken during 50 classroom observations; (b) qualitative records in which field notes were typed and fleshed out shortly after each classroom observation; (c) audio and video recordings of 26 classroom sessions focused on small-group participants during Units 1 and 4, placing one camera near the five small groups with study participants; (d) student and teacher artifacts, including but not limited to classwork, homework, projects, quizzes, and tests, gathered from observation days; (e) audio and video recordings of 22 semi-structured interviews with 12 of the 24 students who volunteered for interviews outside of class time; and (f) four semi-structured individual interviews and more than 100 informal conversations conducted with the teacher before, during, and after the study over a 4-year period between 2009 and 2013.

During the semester of the study, I became a participant observer in the class. Field notes attended to students' capital-*D* discourse (see Gee, 1996), positioning moves, storylines, displays of status characteristics, the teacher's pedagogical choices and positioning moves, evidence of status generalization, and equity. At times, I sat near groups to hear their conversations with one another; at times, I re-

[2] Lab Gear was created by Henri Picciotto, published by Wright Group/McGraw Hill – Creative Publications; for more information on using Lab Gear in a mathematics classroom, see http://www.mathedpage.org/manipulatives/lab-gear.html.

sponded to questions about their work, and, as often as possible and per Ms. Martin's expectation, I redirected them to work with one another when questions arose. As I started building relationships, some students called me over to ask questions more than others. When this occurred, I redirected them to use one another as mathematical resources. Positioning students to use one another as mathematical resources was critical; one of Ms. Martin's focal goals was to support students in finding themselves and one another as mathematically competent, often for the first time in their mathematics career. Positioning students to use one another as mathematical resources also became a way that Ms. Martin delegated mathematical authority to her students. In all cases, to understand the teacher's classroom structures and the process for delegating authority, I maintained a focus on the activity of learning mathematics in groups.

Data Analysis

Because mathematical learning and delegating authority were co-constructed between Ms. Martin and her students, my unit of analysis is the activity of delegating authority as students worked together in their groups. This group focus does not take away the importance of the community and personal planes of analysis (Rogoff, 2003). Rather, the examination of perceptions of learning from the mathematics community in this classroom and from individual learners provides a frame for studying student interactions in groups. Data collected on the nature of group interactions contributes to the empirical evidence of status, positioning, and equitable secondary mathematical learning practices.

Initial analyses of classroom observations and interview data took place by creating qualitative records. The qualitative records contained typed and tagged versions of field notes and coded video sessions. The field notes and coded video sessions reflected Ms. Martin's interpretation and use of CI. The video sessions were coded for teacher moves such as "assigning competence," positioning moves such as "positive self-positioning," and discursive interactional moments such as "on-task verbal group interaction" and "heads in." I wrote memos related to equitable pedagogical practices, students' perceptions of competence, and discursive positioning moves. I took multiple passes through the data, reading field notes, qualitative records, coding schemas of classroom sessions and interviews, coding for common themes and interesting moments. I used open coding (Emerson, Fretz, & Shaw, 1995; Merriam 2009) to allow codes such as "pointing and explaining" to develop. I found the teacher frequently reinforced the kind of group interaction she was hoping to see. I used linguistic microanalysis (Corbin & Straus, 2008) during a second phase of analysis; I studied the instances in which pointing and explaining developed, and eventually found its relationship to the delegation of mathematical authority from Ms. Martin to the small groups.

Once preliminary findings were determined, I questioned the data, so to speak, at multiple subsequent stages of analysis to better understand the nature of the delegation of mathematical authority in this classroom (Corbin & Straus, 2008). Questioning the data allowed me to probe more deeply into a particular interaction in class that I was aiming to understand and helped me when I was stuck at a certain stage of analysis. For example, I may have suspected, preliminarily, that when students made eye contact with one another, it meant they were interested in what the other person was saying. This assumption would have caused premature assignment of positioning codes. Questioning the data through multiple stages of analysis allowed me to determine that when eye contact was coupled with other discursive acts, it could take on different meanings between individuals, including interest, positioning, and even dominance.

I used the constant comparison method (Corbin & Strauss, 2008; Merriam, 2009) to analyze segments of data alongside each other to seek confirming or disconfirming evidence. I used constant comparison to analyze classroom observation and video data for evidence of assigning competence and positioning students as competent. I also used constant comparison to examine interviews for themes and trends. I searched for similarities and differences in how students talked about mathematical competence, and I triangulated their perceptions with specific moments from the data. I used constant comparison to focus on more than a single case and to examine my findings in greater detail.

Delegating Mathematical Authority – Three Cases

The teacher's practices for delegating mathematical authority included positioning all students to offer genuine mathematical contributions and cultivating a classroom community in which all students were given opportunities to display competence. In the following sections, I address how Ms. Martin framed her work with students and I offer examples of delegating authority through student presentations, Shuffle Quizzes, and Participation Quizzes.

Teacher Positioning of Students as Competent Sense-makers

During one interview, Ms. Martin explained to me that she knew that all of her students were competent to contribute meaningful mathematics ideas:

> Zoom out completely and know that I come into this whole-heartedly knowing that all of my kids have something genuine to contribute. All my kids are capable of learning math. I hope that exudes from me, as a teacher, everywhere I go in that classroom. And over time, I've gotten other kids convinced that yes, they're equal contributors to this class. ... I believe that big teacher belief that all kids can learn, and learn in equally valuable ways. And I'm going to do what I can to get them there.

Ms. Martin here illustrated that she knew all students were able to make authentic mathematical contributions, and that part of her job was to help students discover ways they were capable of contributing to the class. Ms. Martin's perspective, that all students "have something genuine to contribute" and are "capable of learning math," aligns with Cohen's (1997a) assertion that equitable classrooms start with teachers who position all students as "capable of learning both basic skills and high-level concepts" (p. 4) and with NCTM's (2000) expectation that all students be given opportunities to learn mathematics. By saying she convinced students they were "equal contributors" to the class, she indicated a vision for students to hold mathematical authority in a supportive classroom environment, in line with Gresalfi and Cobb's (2006) suggestion that mathematical authority is visible by who is "in charge" of the mathematical contributions.

Ms. Martin also regularly cited status theory (Cohen & Lotan, 1997, 2014) for positioning students as competent to offer authentic mathematical contributions. She described responding to status issues by publicly assigning competence to students whenever she could:

> In order to combat status, I try to assign competence to kids. And that only works if it's genuine and totally absolute. So there may be a day when I know a student has low status in his group, and I try really hard to get him back in the conversation, or get kids to turn him. But [maybe that day] it doesn't happen genuinely to actually bring him in. The cool part is that I haven't seen a kid where I've never been able to assign him competence. I've always been able to figure out how a kid is smart, or [find they had] some secret thing written down on their paper they didn't share with their group and [I'll be able to] make a big deal about it. Like maybe they had a perfect graph and I can use it as the example for the whole class.

In this excerpt, Ms. Martin discussed combatting status issues by positioning students as competent and assigning competence to low-status students for ways they authentically contributed to the mathematics of their group. She noted that assigning competence "only works if it's genuine and totally absolute." In this way, Ms. Martin stated that this strategy was contingent on assigning competence for genuine mathematical contributions. She indicated that it was always possible to find a way that a student was competent in mathematics.

Evidence that Ms. Martin's classroom structures were shaping students' productive dispositions (National Research Council [NRC], 2001) surfaced when students described their role in working with others. In one interview, a student shared:

> You do need to be able to like, explain thoroughly and like, several different ways. And just, so then, everybody can get it. Or at least like, if both of you have the right answer, you both have to explain how you got it, to each other.

This student was developing accountability to themselves and others, acknowledging that mathematics learning means explaining methods and understanding group members' methods. Other students discussed expectations to justify mathematics, saying for example, "C'mon you guys, we have to explain to each other. She's gonna ask us to explain how we know!" These excerpts illustrate how these students were seeing the utility of explaining and understanding mathematics in multiple ways.

Case 1: Delegating Mathematical Authority Through Student Presentations

Ms. Martin began to delegate mathematical authority during the first student presentation of the year. In her classroom, student presentations were an opportunity for students to listen to a peer's justification of his or her mathematical thinking. Most often two or more students would present in a row, and each student would have different ways of justifying the presented problem. Student presentations were a classroom structure that allowed Ms. Martin to delegate mathematical authority to students; and, in turn, asking students to justify the mathematics through student presentations reinforced the delegation. In another example, students were working on a problem where they were meant to assume a fictitious student had made mistakes on a graphing problem. The task directions were "circle the mistake(s) and tell what the student did wrong" (see Figure 1).

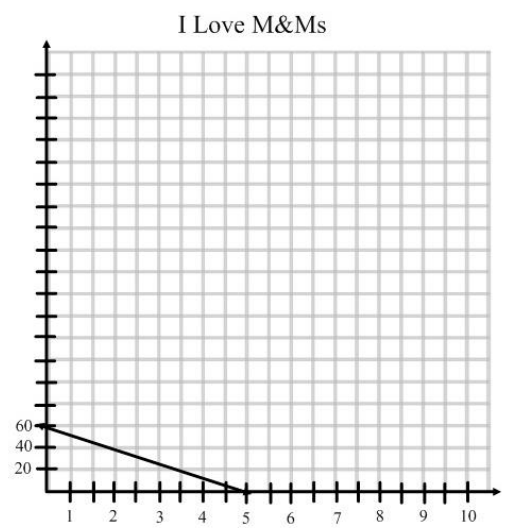

Figure 1. I Love M&Ms.

During this student presentation, Ms. Martin asked for a volunteer to go up to the front of the room to lead the discussion on the "I Love M&Ms" problem featured in Figure 1. When Ms. Martin asked a volunteer to be the first student to present that year, Naima raised her hand. Naima's willingness to volunteer is salient because in spite of failing Algebra I as a freshman in another class, on the eighth day of this Algebra I class, she was willing to take a role where she was justifying mathematics ideas in front her peers. Naima started:

Naima: Are they s'posed to tell me?
Ms. Martin: Sure, you can do whatever you want. Someone tell her. Or, you run the class.
Naima: You guys, raise your hands to me.

A few students laughed when Naima said line 4, possibly because Naima had taken, for the first time, what is often seen as a teacher role. Ms. Martin's encouragement in lines 2–3 indicated a delegation of responsibility from teacher to student. In this way she delegated authority to Naima to decide how to proceed. In line 4, Naima asked students to raise their hands, and when the first student did, Naima called on him and he talked about missing subtitles. Naima circled the empty spaces along the *x* and *y* axes, and wrote "no labels (subtitles)" next to the graph. The class conversation continued, and when the class again became quiet, Ms. Martin added:

Ms. Martin: What else? (*Pause.*) There's more. There's other big ideas too. (*Naima quietly wrote in the left-hand corner: "needs to start @ 10."*)
Ms. Martin: Why does it need to start @ 10? Tells us about that.
Naima: Because you can start this one and go from 0 to 20. (*Pointing to the y axis.*) This one, you go from 0 to 1.
Student: It's going up by 20. (*Naima looked back at the graph, shrugged, and erased "needs to start @ 10".*)
Ms. Martin: I was actually, I was thinking it was a good idea, "needs to start @ 10." Or some other number?
Student: It's going up by 20!
Student: …doesn't start at 0…
Ms. Martin: Yeah, okay, let me back that up. It's going by 20s, right? So the spacing is okay.
Naima: But there's stuff in between it (*Pointing*.). It would be easier to graph if you did it by 10s, because there's still stuff in the 10s spot.
Ms. Martin: Yeah. Look how BIG that grid is. And look how much space the line takes up. (*Naima rewrote "needs to start @ 10."*) So if we went by 10s, or even by 5s, we could space that 60 out, we could use the whole *y*-axis. So that's actually really important. I'll ask you to make a full-page graph. You want to use the whole graph to show the line, to show the whole curve. So, spacing by another number would actually help us to see the mistake up there. What else? …

In line 7, Ms. Martin fostered the immediate expectation for Naima to provide mathematical justification for "needs to start @ 10." By using non-evaluative language, "Why does it need to start at 10? Tell us about that," Ms. Martin positioned Naima as genuinely able to justify her thinking while simultaneously conveying the expectation that students contribute mathematical ideas. Starting the origin at 10 with no other axes changes would be mathematically problematic because the spacing would be 10, 20, 40, 60. Ms. Martin, however, took Naima's assertion "needs to start @ 10" in line 6 as an opportunity for her to explain her thinking, which Naima did in lines 8–9 and 18–19. Ms. Martin's non-evaluative question in line 7 and revoicing in lines 20–24 positioned Naima as a competent sense-maker. This excerpt illustrates how Naima received the mathematical authority to highlight spacing as a common graphing mistake.

The "needs to start @ 10" example addresses one way Ms. Martin used student presentations to delegate mathematical authority to students. She maintained a specific role of keeping the cognitive demand of the talk rigorous while orienting students to one another's mathematical ideas. Ms. Martin also reframed Naima's contribution as valid, by offering the opportunity to justify mathematical thinking. In this way, "needs to start @ 10" brought authentic mathematical competence to the conversation. During the 26 class days captured on video for this study, 15 different students were delegated mathematical authority to lead whole-class conversations through student presentations. Student presentations, then, were one of the classroom structures that delegated mathematical authority to students by positioning students to learn from one another's mathematical ideas.

Case 2: Delegating Mathematical Authority Through Shuffle Quizzes

Ms. Martin also used a second classroom structure, Shuffle Quizzes, to delegate mathematical authority and build equitable learning opportunities for students. A Shuffle Quiz is a strategy that Ms. Martin learned through her training in complex instruction. She described a Shuffle Quiz as a small-group structure in which one member of a small group would be chosen randomly and asked to explain a mathematics concept or problem their group was working on. Prior to Ms. Martin approaching a group for the quiz, the groups were expected to work together, reach mathematical consensus, and to practice justifying their understanding of the mathematics in question. When she approached, she would shuffle their papers, and the student whose paper landed on top would be responsible for explaining the given work. The reason for randomly calling on students was two-fold. First, she told me that she wanted them to work together to explain the mathematics to one another. Second, she was positioning them all as capable to represent the mathematics ideas of their group. When a group's response was deemed mathematically valid, the group was allowed to move on to the next part of the assignment, thus "passing" the Shuffle Quiz.

The interaction between Ms. Martin and one group during the first unit demonstrated how she used Shuffle Quizzes to delegated mathematical authority to students. Ms. Martin started the semester by developing students' expectations for justifying mathematics during Shuffle Quizzes. Part of developing the expectations for justification meant leaving groups when their justifications were not yet mathematically sound, setting the expectation that they needed to continue working to understand the mathematics in a particular way, and then returning to groups for another opportunity to explain the mathematics and pass the Shuffle Quiz. After previously setting this group to a more in-depth discussion about explaining the meaning of variables, Ms. Martin re-approached the group for their second attempt to pass this Shuffle Quiz:

Ms. Martin: Okay, did we continue that variable conversation?
Students: Sort of.
Ms. Martin: Sort of? Okay. I feel like there's a lot of places where you, like, you sort of have the right idea, but you're not a hundred percent on, so I'm going to bug you guys to be really detailed about it. (*Jaelyn's paper lands on top, indicating she begins the Shuffle Quiz.)* So, Jaelyn, part *a*, what do *T* and *D* mean in this situation?
Jaelyn: *T* means total amount of money. *D* means number of days.
Ms. Martin: Okay, so when I was here last, we had a discrepancy about what, what do you mean about total amount of money? Can you put more words around that?
Jaelyn: No… It means total amount of money! Like, in this case, it means, would equal, total amount of money. Correct?
Ms. Martin: Yeah, well, I'm not sure that you guys actually had this conversation. Did you guys have this conversation? No, no, keep going. Try again. Total amount of money, say more about that. You're right, but it needs more detail.
Tamira: Okay. She wants us to give more detail. Like, stretch it out!
Jaelyn: Okay. I'm just putting words to it. Total amount of money he has, each week.
(*Pause.*)
Jaelyn: Okay. I give up.
Ms. Martin: Okay, so I think this is a misconception, I'm not sure. But, *T* and *D* are variables, right? That means that the numbers they represent vary? They change? So what does *T* represent?
Tamira: The varying, the change.
Ms. Martin: Every, each day. It's 80 on day 0, so total amount of money left on each day.
Jaelyn: What did we start with? 80 it would be at the beginning.
Ms. Martin: 10 is happening each day. Where is 10 in the table?
(*Continued Shuffle Quiz with other students.*)
Ms. Martin: Okay, so there were a couple hiccups here, where you had about the same thing, but not exactly the same thing. So stuff like your tables are important to check, and stuff like talk about *a* and *b*, where it asks you in *a* and *b* to explain. I'm going to ask you not just to read, but to say more about it.

This excerpt illustrates one of the times Ms. Martin asked students to work together to achieve mathematical consensus. By telling them to further discuss the significance of variables when she left, and then by walking away, monitoring their conversation from afar, and returning when she thought they might be ready, she held the group accountable for justifying mathematical understanding to one another before justifying it to her, as their teacher. When Ms. Martin returned, she first asked, in line 27, whether they were accountable to explaining mathematics to one another, "Ok, did we continue the variable conversation?" She then made it clear that students had the responsibility to justify the mathematics, saying, in lines 30–31, "I'm going to bug you to be really detailed about it." When asked to describe what *T* and *D* represented, Jaelyn repeatedly replied, "total amount of money" and "number of days" (Lines 34, 38–39, and 45–46). Ms. Martin continued to press Jaelyn to explain more, saying, in lines 35–37, "What do you mean about total amount of money? Can you put more words around that?" and then in lines 42–43, "Say more about that. You're right, but it needs more detail." After Jaelyn said, "I give up" in line 48, Ms. Martin explained to the group in lines 49–51 that she was pressing Jaelyn to describe *T* and *D* as variables. Because this was the first Shuffle Quiz of the year, Ms. Martin explained, in lines 58–62, her expectations that she was going to ask students to work together to achieve mathematical consensus, "There were a couple hiccups here, where you had about the same thing, but not exactly the same thing." This excerpt illustrates how Ms. Martin emphasized justifying mathematics and working toward mathematical consensus as ways to delegate mathematical authority to students in groups.

The Shuffle Quiz excerpt displays the ways Ms. Martin used Shuffle Quizzes to delegate authority to students. Shuffle Quizzes became opportunities for Ms. Martin to regularly hold students accountable for individual and group sense-making. Holding students accountable for being on task, providing mathematical contributions, and justifying mathematics are all examples of the ways that delegating mathematical authority was realized in her classroom. Shuffle Quizzes, then, became a second classroom structure that Ms. Martin used to position students to learn from one another's mathematical ideas and therefore delegate mathematical authority.

Case 3: Delegating Mathematical Authority Through Participation Quizzes

Ms. Martin's use of Participation Quizzes represented a third classroom structure that she used to delegate mathematical authority to students. The Participation Quiz was a public record, displayed on a Smartboard at the front of the classroom, recording students' group interactions while they worked together on mathematics. See Figure 2 for an example of one Participation Quiz from early in the school year.

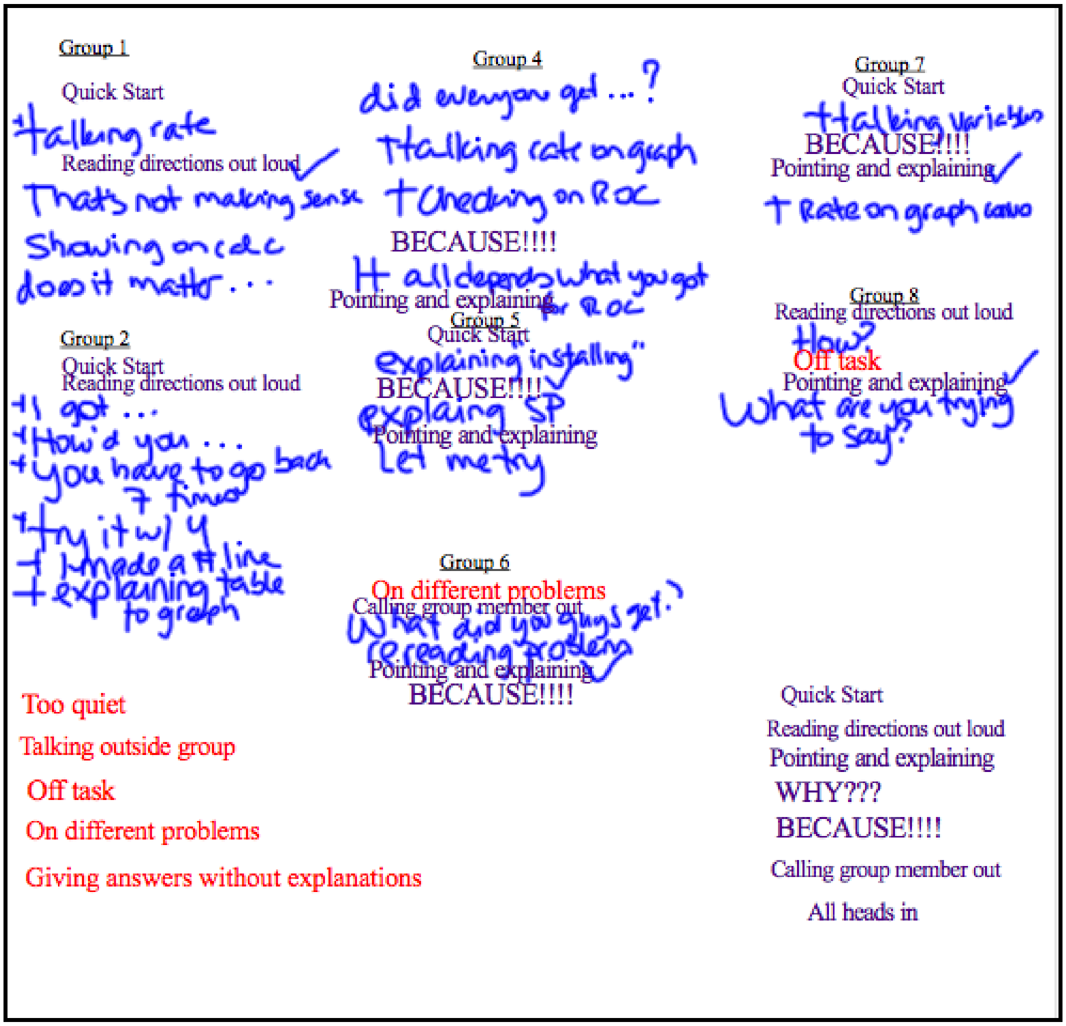

Figure 2. Using Participation Quizzes to delegate mathematical authority.

Before a Participation Quiz started, Ms. Martin prepared the purple and red phrases shown in Figure 2 at the bottom of an otherwise blank Smartboard screen. When students interacted in ways Ms. Martin found appropriate, she copied the prepared purple phrases into the space for their group. When students interacted in undesirable ways, she copied the prepared red phrases into the space for their group. When something happened that was desirable but not part of her prepared list, she wrote it in by hand, in blue. Desirable ways of interacting from this Participation Quiz included, for example, "Quick Start" and "Reading directions out loud." Undesirable ways of interaction during this Participation Quiz included, for example, "Too quiet," and "Talking outside group." Students' group grade was meant to represent how well they worked together as a group. A group's significant number of blue and purple ways of interacting, along with no red, was worth full credit on a Participation Quiz.

The Participation Quiz was a classroom structure that Ms. Martin used to reinforce students working together in particular ways. She described her intent with Participation Quizzes in one interview, “I visibly grade students and show them what they’re doing well. Making it public is really important, as is giving immediate feedback on the behaviors that contribute to good group work and ultimately good learning.” This quote indicates that Ms. Martin used Participation Quizzes to create and publicly reinforce “good group work.” For example, under “Group 1” in this Participation Quiz, she wrote (a) “Quick Start,” (b) “talking [about] rate,” (c) “Reading directions out loud,” (d) “That’s not making sense,” (e) “showing on calc[ulator],” and (f) “does it matter…[?]” The purple and blue phrases emphasize students’ ways of interacting, including (a) getting started quickly on mathematics, (c) reading directions out loud, and (e) showing what is displayed on a calculator to group members. The phrases also highlight students’ ways of speaking, including (b) talking about the mathematics topic, such as rate, (d) asking whether something matters, and (f) talking about whether something makes sense. The Participation Quiz was a classroom structure that Ms. Martin used to introduce and reinforce group work interactions that positioned students to use one another as mathematics resources. Making the Participation Quiz a public representation was also one of the ways that students had the authority to monitor the expectations for mathematical justification.

Ms. Martin used the Participation Quiz to delegate authority to students by reinforcing specific kinds of high-quality group interactions, including “Get off to a quick start,” “Point and explain,” and using “WHY?!” and “BECAUSE!” sentences. The Participation Quizzes displayed the ways in which students were enacting the expected ways of interacting while working together, and they reinforced the type of group interactions Ms. Martin expected to see, thus serving to reinforce the ways Ms. Martin expected individuals to engage as students and as mathematicians.

Summary of Ms. Martin’s Processes for Delegating Mathematical Authority

The cases presented in this section illustrate how Ms. Martin used three classroom structures—student presentations, Shuffle Quizzes, and Participation Quizzes—to position students as competent, to attend to status issues, to require students to justify mathematics, and to position students to work with one another. Student presentations were used as opportunities for students to stand in front of the room and take intellectual and mathematical risks. Ms. Martin often used opportunities like “needs to start @ 10” to position students’ ideas as authentically mathematically valid. Shuffle Quizzes became opportunities for students to rehearse justifying mathematics with one another before being required to do so by Ms. Martin. Participation Quizzes were opportunities for students to receive public acknowledgement and course credit for interacting in the ways that Ms. Martin expected. Cases 1, 2, and 3 offer evidence of these classroom structures as processes Ms. Martin used to delegate mathematical authority to her students.

Delegating Mathematical Authority – Striving Toward Equity

This study was purposefully situated in an Algebra I class where underrepresented and historically marginalized students were delegated mathematical authority to engage in high-level algebraic thinking and learning. In the context of a nation struggling to reposition students who have been historically perceived as having less competence than their peers, Ms. Martin's classroom structures offered her students opportunities to engage in high-level mathematics sense-making. These classroom practices aligned with the NCTM (2000) equity principle that all students must have opportunities to learn rich mathematics.

Furthermore, while this study focuses on three specific classroom structures, each was chosen to highlight a different facet of the ways in which delegating authority can position and reposition students as competent sense-makers. In Case 1, Ms. Martin used non-evaluative language in a student presentation by inviting Naima to explain her initially unclear but valid mathematical thinking about "needs to start @ 10." In Cases 2 and 3, Ms. Martin held students accountable to justify mathematics to one another through Shuffle Quizzes and Participation Quizzes.

Ms. Martin used these three particular classroom structures to position students to do the intellectual heavy lifting when it came to sharing mathematical ideas, facilitating equitable opportunities to students who had not previously been successful in mathematics. She delegated authority by featuring classroom structures that positioned students as competent sense-makers, positioned students to use one another as mathematical resources, and required students to use valid mathematical justification. One student, Helen, was particularly eloquent in summarizing Ms. Martin's enactment of delegating authority to students:

> At first I was like, *Ms. Martin is secretly teaching us!* At first I had to try to get used to the fact that it was about working with groups. But it's helped me a lot to improve my math skills and stuff. All of my other teachers will show you the formulas and show you the little tricks about how they will do the problem. But Ms. Martin will make *us* try and figure it out.

Ms. Martin's "secret teaching" offered students the mathematical authority to share their own methods for understanding mathematics problems. She created a learning environment in which students were comfortable sharing their mathematics thinking during whole-class and small-group interactions. Through repeatedly drawing attention to many students' different competencies, she increased the number of smart contributions available in the room. Once the number of smart contributions increased, the amount of smartness students had access to, while learning high-level Algebra I content, increased.

Closing Thoughts

Although clearly illustrating how a mathematics classroom teacher might create processes that delegate mathematical authority to students, the study reported here is limited by this explicit focus on Ms. Martin's classroom practices. While I collected data on students' practices and perspectives, those data are not included here. Nevertheless, the nature of the questions I asked during students' interviews often uncovered students' self-reflections of what was needed to improve their own mathematics experiences. For example, one two-part question I asked was, "Think of a moment when you learned something really well. How did you know that you learned it well?" The process of interviewing students in this way may have been positively correlated with their changed perceptions of their experiences in Ms. Martin's mathematics classroom. Another limitation of the study is that data were collected during lessons in one class period and from one teacher who had training in CI. The study, therefore, does not attend to the scope or complexity that would be offered by simultaneously studying the delegation of authority in multiple pedagogies, classrooms, teachers, schools, and/or districts.

In the end, the analysis reporting here contributes to the ongoing conversation about the importance of striving for equitable learning opportunities, while simultaneously revealing classroom structures that delegate mathematical authority. While previous research has analyzed mathematics classroom structures and practices alone (e.g., Esmonde, 2009a; O'Connor & Michaels, 1993; Staples, 2008), this article describes the pedagogical processes associated with delegating mathematical authority to students used by one teacher in one classroom. It further describes how delegating mathematical authority to students, through classroom structures like student presentations, Shuffle Quizzes, and Participation Quizzes, can strengthen student learning.

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