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An Investigation into the Number Sense Performance of Secondary School Students in Turkey

Recai Akkaya

Correspondence: Recai Akkaya, Elementary Mathematics Department, Abant Izzet Baysal University, Bolu, Turkey.

Tel: +90 254 10 00–16 17

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Abstract

The purpose of this study is to investigate the number sense performance of secondary school students according to grade level, gender and the components of number sense. A descriptive survey design was used to collect data. A total of 576 secondary school students (291 girls and 285 boys) participated in the study. The results revealed that the number sense performance of secondary school students was very low for each grade level. Among the components of number sense, the lowest mean score was observed in multiple representations. Moreover, the number sense performance scores of the students differed significantly in terms of their grade level, though no significant difference was found according to gender. The investigation concluded that the number sense performance of secondary school students is not satisfactory and needs to be improved.

Keywords: number sense, secondary school students, Turkey

1. Introduction

In recent years, the concept of number sense has become a major research topic in the field of mathematics education at the international level. Number sense refers to the understanding of different relationships between numbers and operations and the flexible use of these relationships. Students who are unable to understand these relationships tend to perceive math concepts superficially, a shortcoming that can be attributed to the adoption of a rule-based approach in math classes, whereby students feel that they have to memorize a lot of rules to learn the relationships (Ekenstam, 1977). Looking at the results of international exams, such as TIMSS and PISA, it is seen that students capable of using the relationships between numbers and operations in a flexible way are more successful. These matters are important insofar as number sense is regarded as one of the key determinants in acquiring advanced mathematical skills (Geary, Bailey & Hoard, 2009; Mazzocco, Feigenson & Halberda, 2011). The importance of number sense is, therefore, emphasized by a large number of researchers and institutions (Cockcroft, 1982; National Council Teachers Mathematics [NCTM], 2000; National Research Council [NRC], 1989).

In Turkey, the concept of number sense has emerged as a research topic within the last ten years. Prior to that, though studies had been conducted that examine mental calculation and estimation skills in terms of number sense, the total number of these studies related to number sense was very limited. The most recent curriculum that has been developed serves to facilitate the students' understanding of mathematical concepts, relationships between the concepts, operations, and the fundamentals underlying the operations as well as the acquisition of math operation skills (Ministary of Natioal Education [MoNE], 2009). Although this approach places an increased emphasis on number sense, further development is still necessary to ensure children's acquisition of number sense (Umay, Akkuş-Çıkla & Duatepe, 2006). The small number of studies that have been carried out in Turkey on number sense revealed that secondary school students experience some difficulties (Şengül, Gülbağcı-Dede & Gerez-Cantimer, 2012; Şengül & Gülbağcı-Dede, 2012; Kayhan Altay, 2010; İymen, 2012; Harç 2010). As those studies examined certain grade levels and were subject-based, a more extensive investigation was conducted in the present study.

1.1 Theoretical Background

Because number sense is hard to define but easy to understand (Case, 1998), it is very unlikely to encounter two researchers offering the same definition (Gersten, Jordan & Flojo, 2005). The broadest definition of number sense includes a person's sense of numbers and operations (McIntosh, Reys & Reys, 1992; Yang, 2003). This sense is used flexibly when developing strategies essential to solving complicated problems and making mathematical judgements

(Burton, 1993; Reys & Yang, 1998). As number sense deals with numbers, operations and the relationships between these concepts as applied to everyday life situations, the relationship between math and everyday life is made easier to understand. Furthermore, it helps foster flexible, creative and effective mathematical thinking (Yang & Wu 2010).

Number sense is a complex process involving certain associations and skills related to numbers, so it is more helpful to examine the skills demonstrated by an individual during this process to fully understand the concept of number sense (Burns, 2007). Taking these skills into account, number sense was divided into sub-components (Li & Yang, 2010; Markovits & Sowder, 1994; McIntosh et al., 1992; Resnick, 1989; Reys et al.,1999; Sowder, 1992; Yang, 1995, 2002; Yang, Li & Li, 2008b; Yang & Tsai, 2010; Yang & Wu, 2010). Just as in the case of defining number sense, a common classification was not able to be established in defining its components. When the existing classifications were compared, the most frequently used components of number sense were found to be the knowledge of numbers, the use of multiple representations of numbers and the ability to grasp number magnitudes (Şengül & Gülbağcı-Dede, 2013). In these classifications, some components were divided into sub-components while others were not included in most classifications.

As number sense is a way of thinking, all aspects of it need to be addressed in math education (Reys, 1994). Children who have a strong appreciation of number sense are equipped with the ability to transfer the results of problem solutions into everyday life situations by using their skills of mental calculation and estimation. In other words, they are able to flexibly apply calculations and develop original strategies by performing mental operations when they encounter a problem in their everyday life. Such students devise plans when flexibly applying calculations, check their plans and decide whether the results are suitable or not (Mohamed & Johnny, 2010). This mental process helps students to see math not only as an academic subject taught at school, but also as a way of thinking that can be employed when solving problems faced in everyday life. It is for this reason that a number of studies have been carried out in recent years, on both the national and the international scale, focusing on different aspects of number sense. The results of these studies will be discussed below.

The most evident result to emerge from these earlier studies is that the number sense performance of secondary school students is poor (Harç, 2010; Işık &d Kar, 2011; Kayhan-Altay, 2010; Markovits & Sowder, 1994; McIntosh et al., 1992; Menon, 2004; Mohamed & Johnny, 2010; Reys et al. 1999; Reys & Yang, 1998; Şengül et al., 2012; Singh, 2009; Verschaffel, Greer & De Corte, 2007; Yang, 2005; Yang, Hsu & Huang, 2004; Yang & Li, 2008). One of the reasons underlying the poor performance is the emphasis that mathematics education programs and textbooks place on standard paper and pencil algorithmic calculations (Alajmi, 2004; Kayhan-Altay, 2010; Markovits & Sowder, 1994; Menon, 2004; Yang, 2005). Moreover, teachers' lack of pedagogical knowledge related to number sense plays a crucial role in the students' poor number sense performance (Şengül & Gülbağcı-Dede, 2014; Tsao & Lin 2011; Yang, Reys & Reys, 2009).

An analysis of the earlier studies indicated that secondary school students face a number of difficulties with number sense. The first of these difficulties involve the inability of students to make sense of situations pertaining to equivalent expressions (Kayhan-Altay, 2010; Singh, 2009; Haser & Ubuz, 2003; Orhun,2007; Şiap & Duru, 2004; Yetim & Alkan, 2010; O'Connor, 2001). A large number of students report that they rely heavily on standard rules, as they have difficulty performing mental calculations and are not capable of applying flexibility in calculations because they do not understand the equivalent expressions of numbers and figures. Another challenge is that students have demonstrated an inability to understand the impacts of operations on numbers (Harç, 2010; Mohamed & Johnny, 2010; Singh, 2009). Incorrect prior knowledge of the students could lead to serious problems in learning other mathematical subjects. Those who cannot remember the rules completely and accurately are more likely to learn the new concepts inaccurately as they formulate their own concepts. For instance, subjective generalizations like "multiplication makes the operation greater and division makes it smaller" could lead to flaws in understanding the effect of operations on numbers. Another significant finding from earlier studies involved the difficulty students had with interpreting the operations and their results. It has been reported that students largely prefer rule-based approaches when explaining the solution of a problem (Işık & Kar, 2011). This preference can be attributed to the particular importance mathematics education programs place on operation-based algorithms and procedures (Kayhan-Altay 2010; Markovits & Sowder, 1994; Şengül, 2013; Yang, 2005; Yang et al. 2008; Yang & Li, 2008). Another key finding was that number sense performance improved in accordance with grade levels (Işık & Kar, 2011; Kayhan-Altay, 2010; Pike & Forrester, 1996; Singh, 2009). Regarding the gender factor, some studies found no significant difference between males and females (Aunio et al., 2006; Kayhan-Altay, 2010; Menon, 2004), while others found a significant difference for male students (Singh, 2009).

A further point raised in the studies was that students were able to use their skills of number sense when encouraged or given an opportunity to do so (Reys & Yang, 1998). This finding suggests that number sense should be particularly emphasized in education programs and lessons. Yang and Wu (2010) based the necessity of teaching and learning number sense on four rationales: i) number sense is a way to engage in flexible, creative, efficient and logical thinking,

ii) number sense is a concept that pertains to quantities, numbers, operations and their relationships with one another, iii) digital representation and mathematical reasoning in adults depend in part on number sense; Dehaene (1997) and Berch (2005) also pointed out that individuals should have an intuitive sense of numbers and that number sense is essential for advanced mathematical thinking and practices, iv) the focus on written calculation restricts not only the development of children's number sense but also their mathematical thinking and understanding. Therefore, the development of number sense in primary and secondary school students must be emphasized.

1.2 Purpose of Study

Number sense has a central role in relating mathematics to real life (Charles & Lester, 1984), for in real life approximate calculations are often made by performing mental operations. Considering this central role that number sense has in real life and the positive and significant relationship between number sense and mathematical achievement (Harç, 2010; Kayhan-Altay, 2010; Mohamed & Johnny, 2010), the need to conduct a thorough investigation of the number sense performance of students and to determine the areas where support is necessary has become increasingly important. With the help of all these data, the purpose of the present study is to examine the number sense performance of students in Turkey. With this purpose in mind, the following questions will be discussed.

1.3 Research Questions

1. How strong is the number sense performance of secondary school students?
2. How does the number sense performance of secondary school students differ according to the components of number sense?
3. Does the number sense performance of secondary school students differ according to their grade levels?
4. Does the number sense performance of secondary school students differ according to their gender?

2. Method

For this study, the survey method was used to conduct the research. With the use of this method, the aim is to describe a past or present situation in its own terms. It is unlikely for the issue being researched to change or be changed (Can, 2012). More particularly, it was decided that the descriptive survey design be used in the study, as the purpose was to determine the number sense performance of secondary school students who had received no prior education on number sense.

2.1 Sample Population

The study included 576 secondary school students attending four different schools in the province of Bolu during the 2014-2015 academic year. Of these students, 117 were 5th graders, 144 were 6th graders, 162 were 7th graders and 153 were 8th graders. The number of girls and boys were 291 and 285, respectively. The distribution of students according to their grades and gender is presented in Table 1.

Table 1. Distribution of students according to their grades and gender

	5th Grade	6th Grade	7th Grade	8th Grade	Total
Girls	54	75	78	84	291 (50.5%)
Boys	63	69	84	69	285 (49.5%)
Total	117 (20.3%)	144 (25%)	162 (28.1%)	153 (26.6%)	576 (100%)

2.2 Data Collection Tools

The "Number Sense Test" (NST) developed by Singh (2009) and adapted to Turkish by the researcher was used to assess the number sense performance of secondary school students. The NST is comprised of 50 multiple choice questions, with questions for each of the 5 sub-components of number sense: 14 questions on number concept, 7 questions on multiple representations, 8 questions on understanding and using the equivalent expressions of numbers and figures, 10 questions on understanding the effect of operations on numbers and 11 questions on flexibility in making calculations. Table 2 presents the sub-component categories of number sense and the number of questions in each category, along with their item numbers.

Table 2. Sub-components of number sense and item numbers

		Item Numbers	The number of questions	Total
Sub-components	Number Concept	1, 3, 4, 6, 9, 10, 15, 18, 19, 22, 25, 29, 36, 39	14	50
	Multiple representations	7, 8, 13, 14, 30, 31, 40	7	
	Effect of operation	16, 17, 20, 21, 24, 27, 28, 38, 48, 49	10	
	Equivalent expression	11, 23, 26, 32, 33, 34, 37, 45	8	
	Counting and Computation	2, 5, 12, 35, 41, 42, 43, 44, 46, 47, 50	11	

To begin, the 50 questions in the NST were first translated into Turkish by the researcher. Next, two mathematics education experts and a linguist translated the test into Turkish. The most suitable translation form was produced for the test items by conducting a co-analysis of these translated works. A pilot study was then conducted, just as it was in the original version, to decide on the duration of the test. A total of 111 secondary school students took part in the pilot study, with the range in grade level being as follows: 25 from grade 5, 28 from grade 6, 32 from grade 7 and 26 from grade 8. In the pilot study, the questions were projected onto a board one at a time and students were asked to solve the problems. After the completion of the pilot study, the duration of the test was set up as 40 minutes for 5th and 6th graders and 35 minutes for 7th and 8th graders. Students received 1 point for every correct answer and 0 points for every wrong answer. The final scores therefore had a range of 0 (minimum) to 50 (maximum).

2.3 Data Collection and Analysis

The means and standard deviations for the performance scores of the secondary school students were computed based on the number sense test and its sub-components. Performance scores, means and standard deviations were then calculated separately for each grade level. Results of the analyses indicated that the number sense performance scores of the students were normally distributed. In addition to descriptive statistics, the data collected was also analyzed using independent samples t-test, one-way analysis of variance for independent samples and Scheffe's test. In the analyses, .05 was set as the level of significance.

3. Results

This part includes the analyses performed to determine the number sense performance scores of secondary school students and whether these scores differed according to grade levels, gender and the components of number sense.

3.1 NST Scores According to Components of Number Sense and Grades

The total NST scores of the students and the mean scores they obtained from the components of number sense were computed separately based on grade levels. Table 3 shows the related results.

Table 3. Distribution of NST Scores According to Components of Number Sense and Grades

	5th Grade			6th Grade			7th Grade			8th Grade			Total		
Components of Number Sense	X	Sd	%	X	Sd	%	X	Sd	%	X	Sd	%	X	Sd	%
Number Concept	2.08	0.11	14.8	2,69	0.13	19.2	2.91	0.15	20.8	3.22	0.19	23	**2.78**	**1.94**	**19.8**
Multiple representation	0.44	0.06	6.2	1,12	0.09	16	1.23	0.1	17.6	1.67	0.11	23.8	**1.16**	**1.27**	**16.5**
Effect of operation	2.35	0.13	23.5	2.62	0.12	26.2	2.51	0.11	25.1	3.27	0.15	32.7	**2.71**	**1.63**	**27.1**
Equivalent expression	2.01	0.12	25.1	2.37	0.13	29.6	2.57	0.12	32.1	3.21	0.14	40.1	**2.58**	**1.61**	**32.2**
Counting and Computation	1.89	0.12	17.2	2.57	0.11	23.3	2.63	0.12	23.9	3.64	0.16	33.1	**2.73**	**1.72**	**24.8**
Total	**8.78**	**0.3**	**17.6**	**11.38**	**0.41**	**22.7**	**11.87**	**0.42**	**23.7**	**15.02**	**0.6**	**30**	**11.96**	**5.97**	**23.9**

The data presented in Table 3 shows that the total performance score that secondary school students achieved in the number sense test was 11.96. Considering that the maximum score was 50, the number sense performance of secondary school students was found to be very poor. For the components of number sense, the lowest mean score (1.16) was obtained in multiple representations and the highest (2.78) in number concept.

According to the grade levels, the mean scores were calculated as follows: 8.78 for 5th grade students, 11.38 for 6th grade students, 11.87 for 7th grade students and 15.02 for 8th grade students. One-way-analysis of variance for independent samples was used to compare the performance scores of students, the results of which are presented in Table 4.

Table 4. Comparison of number sense scores according to grades

	Sum of Squares	Df	Mean Square	F	p
Between Groups	2666.481	3	88.827	28.599	.000
Within Groups	17777.434	573	31.079		
Total	20443.915	575			

The results of the analysis revealed a significant difference between the performance scores of students in regard to their grades ($F_{(3,573)}$= 28.599, p<.05). In other words, number sense performance scores of secondary school students differed significantly according to their grade levels. Scheffe's test was used to determine where the difference lied, the results of which are given in Table 5.

The data in Table 5 indicate that there was a significant difference between the 5^{th} and 6^{th} grades, 5^{th} and 7^{th} grades, 5^{th} and 8^{th} grades, 6^{th} and 8^{th} grades and 7^{th} and 8^{th} grades. This finding suggests that students improve their performances as their grade level increases.

Table 5. The results of Scheffe's test

Dependent Variable	I (Factor)	J (Factor)	Mean Difference (I-J)	Std. Error	p
Number Sense Test	5	6	-2.60*	.693	.003
		7	-3.09*	.676	.000
		8	-6.24*	.684	.000
	6	5	2.60*	.693	.000
		7	-.48	.638	.900
		8	-3.63*	.647	.000
	7	5	3.09*	.676	.000
		6	.48	.638	.900
		8	-3.15*	.628	.000

Taking the grade levels into account, the means and standard deviations of the number sense performance scores were first computed to assess whether the number sense performance scores of secondary school students differed in terms of their gender. Independent samples t-test was used to determine whether a significant difference existed between the performance scores of girls and boys. The results are shown in Table 6.

Table 6. Independent Samples t-test for Number Sense Points by Grades

Grades	Gender	N	X	S	Sd	t	p
5	Girls	54	8.74	2.67	115	.112	.911
	Boys	63	8.80	3.75			
6	Girls	75	10.88	4.53	142	1.284	.201
	Boys	69	11.92	5.25			
7	Girls	78	11.51	4.98	161	.812	.418
	Boys	84	12.20	5.75			
8	Girls	84	14.54	6.07	151	.866	.388
	Boys	69	15.59	8.82			
Total	Girls	291	11.71	5.28	574	.997	.319
	Boys	285	12.20	6.58			

No significant difference was observed between the mean scores in terms of the number sense performance of secondary school students, as shown in Table 6 ($t_{(574)}$=.997; p>.05). The performance mean scores of boys (X=6.28) were observed to be higher than those of girls (X=5.28). Moreover, the analyses conducted for each grade level found no significant difference in terms of gender.

3.2 Analysis of the Performance of Secondary School Students in the Components of Number Sense

This section summarizes the descriptive statistics on the performance scores of students according to the components of number sense included in the number sense test. First, the number of correct answers to each question was counted, and a then the percentage of success was calculated separately for each grade. The results revealed that the lowest percentage of success (16.5%) was found in the component of multiple representations and the highest (32.2%) in equivalent expression. The percentage of the items with correct answers in each component category was calculated and is shown in Table 7.

3.2.1 Number Concept

With the average rate of success being 19.8% in the number sense component, the secondary school students were found to demonstrate a lack of ability in fractions, rational numbers and decimals. In this component, as in the overall test, the rate of success for each question increases with the rise in grade level. In terms of the rate of success for these questions, it was found that the items with less than 10% success (3, 6, 9, 15, 18, 19 and 39) were related to the fractions and decimals. It is therefore evident that secondary school students are particularly weak in the areas of fractions and decimals.

Table 7. Items analysis for Number Concept Component across Grades

Number Sense Component	Items	% Correct				
		5th Grade	6th Grade	7th Grade	8th Grade	Average
Number Concept	1	17.9	19.4	12.3	13.1	15.5
	3	4.27	6.94	8.64	11.1	7.99
	4	43.6	67.4	71.6	75.2	65.8
	6	0.85	6.25	5.56	9.8	5.9
	9	1.71	4.86	11.1	5.23	6.08
	10	11.1	20.8	22.2	24.2	20.1
	15	0	2.78	3.09	6.54	3.3
	18	0	4.17	8.02	16.3	7.64
	19	0.85	2.08	6.17	7.84	4.51
	22	32.5	29.2	36.4	36.6	33.9
	25	18.8	14.6	24.7	26.1	21.4
	29	47	59	48.8	52.3	51.9
	36	25.6	26.4	29	26.8	27.1
	39	4.27	5.56	4.32	11.8	6.6
	Average	**14.8%**	**19.2%**	**20.8%**	**23%**	**19.8%**

3.2.2 Multiple Representations

The data in Table 8 shows that secondary school students had the most difficulty with multiple representations. The percentage of the correctly answered questions in this component (16.9%) was smaller than the average rate of success (23.6%). Students had the most trouble with items 40, 14, 31 and 13. An example of the type of questions this component featured can be shown by question 40, where students were required to order the numbers 0.595, 2/4, 61%, 0.3 and 30.5% from least to greatest. At the basic level, a success rate of 5%, even among the 8th graders, clearly implies that students have trouble understanding the relationships between percents, fractions and decimals. Particularly, students in the 5th grade had a very low level of success regarding this component.

Table 8. Items analysis for Multiple Representations across Grades

Number Sense Component	Items	% Correct				
		5th Grade	6th Grade	7th Grade	8th Grade	Average
Multiple Representations	7	12.82	42.36	40.74	45.75	36.81
	8	17.09	32.64	25.93	37.91	28.99
	13	2.56	11.81	16.67	17.65	12.85
	14	4.27	4.17	9.26	15.03	8.51
	30	5.13	10.42	14.81	26.14	14.76
	31	0.85	6.94	8.64	19.61	9.55
	40	1.71	3.47	7.41	5.23	4.69
	Average	**6.2%**	**16%**	**17.6%**	**23.8%**	**16.5%**

3.2.3 Effect of Operation

Compared to the first two components, secondary school students achieved a higher rate of success in this component. The rate of success in this component ranged from 23.5% to 32.7%, and the average rate of success was 27.1%. Even though the percentage of the questions answered correctly in this component was higher when compared to that of other questions, it was still not at the desired level. Items 16, 20, 21, 27 and 28, which involved multiplication or division of an integer by a decimal smaller than zero, were the questions with the lowest rate of success.

Table 9. Items analysis for Effect of Operation Component across Grades

Number Sense Component	Items	% Correct				
		5th Grade	6th Grade	7th Grade	8th Grade	Average
Effect of Operation	16	25.64	35.42	22.22	31.37	28.65
	17	35.04	47.92	42.59	52.94	45.14
	20	16.24	23.61	16.67	27.45	21.18
	21	21.37	15.28	12.35	17.65	16.32
	24	23.08	23.61	23.46	33.99	26.22
	27	22.22	22.92	25.93	20.26	22.92
	28	22.22	25.69	24.07	21.57	23.44
	38	29.06	36.81	40.12	63.4	43.23
	48	17.09	11.81	15.43	32.68	19.44
	49	23.08	19.44	28.4	25.49	24.31
	Average	**23.5%**	**26.2%**	**25.1%**	**32.7%**	**27.1%**

3.2.4 Equivalent Expression

As evident from Table 10, equivalent expression was the component where the secondary school students achieved the highest scores. Nonetheless, the students still experienced difficulty with items 11, 23, 33 and 34, as can be seen in the percentage of correct answers given by the students to these questions. For question 11 particularly, 87% of the secondary school students failed to provide an equivalent expression for 0.5 x 840. It was surprising to see that 8th grade students also obtained low scores on this question. Compared to the other grade levels, the success rate of 5th graders was higher for question 32.

Table 10. Items analysis for Equivalent Expression Component across Grades

Number Sense Component		**% Correct**				
	Items	**5th Grade**	**6th Grade**	**7th Grade**	**8th Grade**	**Average**
Equivalent Expression	11	3.42	8.33	14.81	21.57	12.67
	23	7.69	11.11	20.37	15.69	14.24
	26	35.04	45.83	47.53	58.82	47.57
	32	68.38	59.03	66.05	64.71	64.41
	33	19.66	11.81	16.67	24.84	18.23
	34	15.38	30.56	24.07	26.8	24.65
	37	26.5	34.72	35.8	52.29	38.02
	45	24.79	36.11	32.1	56.21	38.02
	Average	**25.1%**	**29.6%**	**32.1%**	**40.1%**	**32.2%**

3.2.5 Counting and Computation

The average success rate of secondary school students was 24.8% for the counting and computation component. The lowest success rate was observed in items 43 and 44. In all grades, the success rate was noticeably lower on question 44. The 6th, 7th and 8th grade students attained the highest success rate on question 2.

Table 11. Items analysis for Counting and Computation Component across Grades

Number Sense Component		**% Correct**				
	Items	**5th Grade**	**6th Grade**	**7th Grade**	**8th Grade**	**Average**
Counting and Computation Component	2	57.26	81.25	81.48	86.93	77.95
	5	0	9.03	7.41	17.65	9.03
	12	35.04	50.69	50	52.94	47.92
	35	1.71	9.03	2.47	22.88	9.38
	41	21.37	25.69	22.22	32.68	25.69
	42	22.22	27.78	26.54	47.71	31.6
	43	4.27	1.39	8.02	8.5	5.73
	44	0	0.69	4.32	6.54	3.13
	46	26.5	33.33	33.95	35.29	32.64
	47	17.09	11.81	17.28	23.53	17.53
	50	3.42	6.25	9.26	29.41	12.67
	Average	**17.2%**	**23.3%**	**23.9%**	**33.1%**	**24.8%**

4. Discussion and Conclusion

In this study, number sense performance of secondary school students was examined in terms of the variables of number sense components, grade levels and gender. To this end, the Number Sense Test was administered to a total of 576 secondary school students, of whom 117 were 5th graders, 144 were 6th graders, 162 were 7th graders and 153 were 8th graders.

The results of the analyses of data collected from secondary school students showed that the mean scores students obtained from the number sense test ranged between 8.78 and 15.02. From this finding, it is demonstrably clear that number sense performance of secondary school students is very poor, particularly considering that the maximum attainable score was 50 for the test. This result shows similarities with the findings of earlier studies (Harç, 2010; Işık & Kar, 2011; İymen, 2012; Kayhan-Altay, 2010; McIntosh et al., 1992; Reys et al., 1999; Singh, 2009; Verschaffel et al., 2007). The studies carried out on the international scale confirm that poor number sense performance of students is common, not only in Turkey, but also in many other countries. Therefore, additional studies are needed at the international level to better understand and improve the number sense performance of students.

The research data revealed that number sense performance of secondary school students differed significantly with regard to grades level. In other words, number sense performance appears to increase in line with the grade level. This finding is supported by the results of the studies conducted by Singh (2009), Aunio et al., (2006) and Işık and Kar (2011), while Kayhan-Altay (2010) and Mohammed and Johnny (2010) obtained dissimilar results. The inconsistency in the results can be explained in different ways. It could be that the experiences teachers have with number sense and components increase in accordance with the grade level and thus positively affects the number sense performance of students. Or, another explanation could be that since calculation methods based on rules, memorization and paper and pencil (as opposed to mental processes) are used more frequently as the grade level increases, the level of performance decreases as some students embrace such methods (Markovits & Sowder 1994; McIntosh et al. 1992). The differences in the research studies underline the need to conduct more extensive and detailed studies to explore whether number sense performance does in fact differ based on the grade level of students.

Another telling finding of the study showed that there was no significant difference in the number sense performance of secondary school students in terms of gender. However, the performance scores of boys were found to be higher than those of girls in all grades. This result is in agreement with the study findings provided by Singh (2009), Kayhan-Altay (2010) and Menon (2004). Viewed within the framework of mathematics education, the fact that both girls and boys had low performance scores, a finding that has also been demonstrated in studies conducted on an international scale, shows that this matter is a critical issue, one that implies that students still consider mathematics to be a school subject wherein certain written rules and procedures are followed. However, mathematics needs to be approached as a problem-solving process, based on the modeling of real life situations, rather than as a lesson in which students are taught certain abstract concepts and skills (De Corte, 2004). By instituting such an approach, rule and procedure-based education can be abandoned and replaced by more meaningful learning processes.

The analysis of the results related to number sense components demonstrated that the component of multiple representations was the most challenging for secondary school students. The responses to the items in this component showed that students had difficulty converting between fractions, decimals and percents. They also had difficulty producing the representations of fractions, decimals and percents on the number line. In examining the mathematics education program in Turkey, it was observed that the relationships between fractions, decimals and percents are taught starting from grade 5, but as different subjects, with no emphasis on the relationships between them. It was also reported in other studies that secondary school students had difficulty converting and comparing fractions, decimals and percents (Haser & Ubuz, 2003; Orhun, 2007; Şiap & Duru, 2004; Yetim & Alkan, 2010). O'Connor (2001) reported that students perceive fractions, decimals and percents as two different numbers. Students assume that there is always a decimal number between two decimals or a fraction between two fractions. Therefore, they overlook the fact that fractions, decimals and percents are simply different representations of one another. This problem negatively affects the number sense performance of students and makes it harder for them to be flexible in making calculations or estimations. For that reason, the importance of the relationships between fractions, decimals and percents needs to be stressed in education programs and textbooks.

Another area wherein secondary school students experienced difficulty was the magnitude of numbers. It was found that the correct answer rate of the NST items related to the magnitude of numbers was very low. For instance, the correct answer rate for the question "How many different decimals are there between 1.52 and 1.53?" was 11.11% for 8^{th} graders and as low as 4.27% for 5^{th} graders. Similarly, the question "How many different fractions are there between 2/5 and 3/5?" had a correct answer rate of 9.8% for 8^{th} graders and 0.85% for 5^{th} graders. In studies showing similar findings, it was reported that this lack of understanding in the area of magnitude can be attributed to the assumption held by a majority of the students that fractions and decimals are ordered in sequence, just like natural numbers (Moss, 2005; Ni & Zhou, 2005). Untrue generalizations or previously acquired false or incomplete knowledge could negatively impact later learning. It is therefore necessary to identify these untrue generalizations and the common and conflicting points between the former and recent knowledge and discuss them using examples that are appropriate or inappropriate for the situation so that the students can recognize the differences. The relationships of fractions, decimals and percents to each other should be addressed and discussed at each grade level.

Various reasons can account for the poor number sense performance of secondary school students. One reason could be the mathematics and classroom teachers' lack of pedagogical knowledge related to number sense. Teachers play a critical role in increasing students' performance on number sense through the teaching methods and activities they design (Tsao & Lin, 2011). However, earlier studies have shown that mathematics and classroom teachers have a low level of number sense performance and do not know how to help students improve their number sense performance (Şengül & Gülbağcı Dede, 2014; Yang et al., 2009). To address this, the pedagogical knowledge of teachers can be enhanced with the provision of in-service seminars on topics such as number sense and its importance, number sense strategies and how to help students develop number sense. Moreover, teacher training programs covering similar topics should be considered for teacher candidates.

Mathematics education programs and textbooks are other factors affecting the development of number sense. Mathematics education programs have a major place in the development of number sense. While the concept of number sense and its importance is emphasized in the education programs of many countries, this concept is virtually absent in the mathematics education program used in Turkey, and there are no objectives and activities to develop number sense (Umay et al., 2006). The same applies to textbooks. Therefore, it is believed that the integration of number sense activities into mathematics education programs and textbooks will play a crucial role in increasing the performance levels of students on number sense.

In future research, it would be useful to design different education methods to improve number sense performance of secondary school students and then investigate how these methods affect the number sense performance.

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