International Education Studies; Vol. 8, No. 11; 2015
ISSN 1913-9020 E-ISSN 1913-9039
Published by Canadian Center of Science and Education

Ten-Structure as Strategy of Addition 1-20 by Involving Spatial Structuring Ability for First Grade Students

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Received: May 14, 2014 Accepted: January 4, 2015 Online Published: October 27, 2015

doi:10.5539/ies.v8n11p16 URL: http://dx.doi.org/10.5539/ies.v8n11p16

Abstract

The aim of this study is to design learning activities that can support students to develop strategies for the addition of number 1 to 20 in the first grade by involving students' spatial structuring ability. This study was conducted in Indonesia by involving 27 students. In this paper, one of three activities is discussed namely ten-box activity. This activity was aimed to introduce and develop ten-structure to be a students' strategy in addition of number 1 to 20. The method was design research by designing learning activities involving spatial structuring ability. PMRI underlined the context and activity. The result of the study indicates that ten-box activities can help students to develop ten-structure as a strategy in addition of number 1 to 20. As a recommendation, PMRI can be implemented as an approach of teaching and learning addition 1 to 20.

Keywords: addition, PMRI, spatial structuring

1. Introduction

Numbers is one of important topics because it will be the basis for the mastery of other mathematical concepts (Freudenthal, 1973; NCTM, 2000). In the early stages of learning about addition like in elementary school, there are many students who have not been able at doing the sums of numbers. This is because in the early stages of studying the numbers, more students use strategies that involve the calculation of one by one (Van Nes, 2007; Collins, 2009; Son, 2011) and rely on real objects to be counted (Collins, 2009). In Indonesia, the teachers still give the standard algorithm or procedure without any emphasis on teaching the concept (Armanto, 2002). Therefore, it is necessary to get a strategy that can help students not only to understand but also to be proficient in performing addition in particular addition 1 to 20.

To support the understanding of students about addition of number 1 to 20, researchers are encouraged to develop and to use spatial structuring ability to help students do the addition. The spatial structuring ability is the mental operation of constructing an organization or form for an object or set of objects (Battista & Clements, 1998).

A lot of research that has studied on how students' spatial and beginning numerical ability can influence the development of students' mathematical thinking (Clements, 2004; Mulligan, 2006). The spatial structuring ability takes an important role to help students for developing and simplifying their strategy to define, compare, and perform addition, subtraction, and multiplication (Battista, 1998; Papic, 2005; Van Nes, 2009). Meanwhile, a study conducted by Mulligan et al. (2006) found that students with sophisticated awareness of patterns and structures excelled in mathematical thinking and reasoning compared to peers and vice versa.

Based on that background, the researchers designed the context and learning activities that can help students to develop their strategies in solving addition of numbers 1 to 20. Pendidikan Matematika Realistik Indonesia (PMRI) underlying the design context and activities undertaken. In this paper, we present one of the three instructional activities conducted in a second cycle on the teaching experiment. The purpose of this paper is to explore how the ten-structure in ten-box activity can support students to develop their understanding about addition of number 1 to 20. Generally, the research question of this research is *how the ten-structure can support students to develop their understanding about addition of number 1 to 20?*

1.1 Spatial Structuring Ability

The spatial structuring ability is the mental operation of constructing an organization or form for an object or set of objects (Battista et al., 1998). The spatial structure that subsequently arises can help the student recognize (part of) the quantity and consequently abbreviate the counting procedure (Van Nes, 2009). Battista and Clements suggest that spatial structuring is the essential mental process underlying students' quantitative dealings with spatial situations (Battista et al., 1998). Spatial structure is considered to contribute to insight into important mathematical procedures and concepts such as patterning, algebra, and the recognition of geometric shapes and figures (Carraher, 2006; Clements & Sarama, 2007; Mulligan, Mitchelmore, & Prescott, 2006; Papic & Mulligan, 2007).

Research by Mulligan et al. (2006) found that students with sophisticated awareness of patterns and structures excelled in mathematics thinking and reasoning compared to their peers and vice versa. In this case, we can say that the students can understand more than to count the sum.

The research conducted by Revina described that students' spatial structuring abilities provide the necessary input and organization for the numerical procedures that the students use to calculate an array of cubes. Using spatial structuring strategy allows students to determine the number of cubes in term of layers and then multiple or skip-calculate to obtain the total number of cubic units (Revina, 2011).

Learning by mathematical structures and pattern could stimulate students' learning and understanding of mathematical procedures and concepts. This coincides with Battista views about how students must learn to construct a meaningful structure and that students could improve their own use of structure if they recognize errors in their counting as the result of inadequate spatial structuring (Van Nes, 2009). Spatial structuring ability should be foster as a key factor in the development of number sense, particularly regarding insight into numerical relations (Van Nes, 2007).

1.2 Addition of Numbers 1 to 20

According to Freudenthal (1991) in his other book, he states that counting is the first student's verbal mathematics. Counting plays a very important role to develop in student's number sense students earlier. It is important especially ad a basic foundation of counting ability (Van den Heuvel-Panhuizen, 2008). Structuring in learning numbers which is consist of addition and subtraction are more precedence because if students have structuring number ability, the operational ability in calculation will they get by themselves.

Counting one by one is a basic strategy of students in solving addition problems (Nursyahidah, 2013). In the early counting phase, students will count the number of things one by one. In addition numbers 1 to 20, strategy which student uses can be different to the other students. There are some strategies that usually students use in addition number 1 to 20 (Thompson, 1999). Some of them are (1) counting on from first number, (2) counting on from larger, (3) double (near-double system, (4) using fives, and (5) bridging through ten.

Counting on from first number recognized as being the first strategy that children learn after counting all (Thompson, 1999). By using this strategy, students do not use counting-all-strategy, but they will count from the first number. Furthermore, in counting on from larger students will use strategy as same as counting on from the first number. However, students begin with the larger number. This strategy becomes a more useful and labour-saving procedure when the difference between the numbers being added is larger (Thomson, 1999).

Meanwhile, double strategy may help students to count by remind the addition for the same number. For instance, $2 + 7$ means students add 2 and 7 as same as repeated addition $2 + 2 + 2 + 1$, so $2 + 7 = 2 + 2 + 2 + 2 + 1 = 9$. Using-five-strategy and bridging-through-ten-strategy are strategies that need more critical thinking. Because in this step, students are required to be able to group numbers becomes group of five or ten and count the rest.

1.3 Pendidikan Matematika Realistik Indonesia (PMRI) Approach

Pendidikan Matematika Realistik Indonesia (PMRI) is an adaptation of the Realistic Mathematics Education (RME) approach (Putri, 2011). The RME has been developed in the Netherlands since the early 1970's. This approach emphasizes increasing pupils' understanding and motivation in mathematics (Zulkardi, 2002). Realistic Mathematics Education (RME) is determined by the view of Freudenthal about mathematics. Two his important views are "*Mathematics must be connected to reality and mathematics as human activity*". Firstly, mathematics should be close to the students and relevant to their reality. Secondly, Freudenthal (1991) emphasizes that mathematics is as human activity, so that students must be given opportunity to do activities in every topic in mathematics.

As the basis of this research, RME approach is defined elaborately through five tenets by Treffers (Bakker, 2004).

1) Phenomenological Exploration or the Use of Contexts

In RME, the starting point of mathematics instruction should be experientially real to the student, allowing them to become immediately engaged in the contextual situation (Zulkardi, 2002). Students are introduced to concepts and abstractions through concrete things and it is started from the experiences of students as well as derived from their environment.

Contextual problem as a topic of preliminary learning should be simple problems from students' daily activities. In the first activity, students were introduced with the five-structure and double-structure using the context of arranging balls in the box. In the second activity, the context of arrange the ball in the box of tens was used to discover and use the structure of tens as one strategy in addition.

2) The Use of the Models or Bridging by Vertical Instruments

The terms of model relates to the situation model and mathematical model developed by students, themselves (*self developed models*) (Zulkardi, 2002). Its role is the bridge for students from real situation to the abstract situation or from informal mathematics to formal mathematics. It means that students make their own model in solving problem. First is the kind of situation close to the students' real world. Generalization and formalization of those models will turn to *model of* problem. Through mathematical reasoning, *model of* will be *model for* of the same problem. Eventually, it will be the model of formal mathematics. There are four levels of models in RME namely the situational level, referential level, general level, and formal level (Gravemeijer, 1994).

3) The Use of the Students own Productions and Constructions

Students should be asked to create things (Zulkardi, 2002). The ideas of students need to get noticed and appreciated in order to exchange idea in the learning process. Students produce and construct their idea, so that learning process will be constructive and productive. Students' ideas are communicated to other students and teacher so that mathematics learning appears from either individual activity or group activity. Both on the first activity and the second activity, the students were involved actively to state the way which they choose to solve addition problems.

4) The Interactive Character of the Teaching Process or Interactivity

In learning mathematics, good interaction has appeared among students concerning the result of student thinking. The teacher facilitates students in discussion interactively. So, the interaction among students, students and teacher, and students and learning tools is an important thing.

5) The Intertwinning of Various Learning Stands

The addition of number 1 to 20 could be taught by associating with students' spatial structuring ability. Teaching addition by involving spatial structuring ability can help students to find the strategy in order to make addition 1 to 20 more easily. Spatial structure can be use in learning addition such as finger patterns, five-structure, double-structure, and the ten-structure (Van Nes, 2007).

1.4 The Ten-Box Activity in Developing Ten-Structure as a Strategy in Addition 1 to 20

The ten-box activity was the third activity in this study. In this activity, students were asked to solve the addition problem represented in the third worksheet. Students have to find the total number of balls in two groups of balls without counting the balls one by one. To help students find the number of balls, teacher provided a ten-box which is consist of two lines and ten holes in every line. By arranging the balls in the box, they could find another way to find the number of balls.

Students expected to find the ten-structure when they arranging the balls in the box and use this structure to be a strategy in solving addition problem of number 1 to 20. Students' understanding about part-whole relationship by tens can bring up the strategy in solving addition problem by using structure (Faridah, 2013). A deeper experience about part-whole relationship also emphasizes on knowledge from all combination of ten and the next strategy is to make a ten-addition (Fosnot & Dolk, 2001).

2. Method

2.1 Participant

The research subjects in the teaching experiment are 27 first grade students of public school in Palembang, Indonesia, namely SD Negeri 179 Palembang. This school has been involved in PMRI Project since 2007. The students were about 6-7 years old and they had known some spatial structure in kindergarten and in the early of first grade in semester 1. There were 6 students involved in the first cycle and 21 students involved in the second cycle. The six students were selected based on their capability levels. The 21 students did the activities in pairs in the second cycle.

2.2 Research Design

This research method is a design research as an appropriate method to answer a research question. The focus of this research is developing *Local Instruction Theory* (LIT), cooperation between teacher and researcher to improve the quality of learning (Gravemeijer & Eerde, 2009).

Design research encompasses three phases: developing a preliminary design, conducting the teaching experiment, and carrying out the retrospective analysis (Gravemeijer, 2004). In the preliminary design, we study some literatures about addition 1 - 20, spatial structuring ability, the Pendidikan Matematika Realistik Indonesia (PMRI) approach, and analysis of addition 1 to 20 in Curriculum 2013. Learning trajectory and Hypothetical Learning Trajectory (HLT) were designed. HLT describes learning goals, learning activity, and media to support learning process. The sequence of learning activities is conjecture as guidance to anticipate students' strategy and thinking appeared and developed in the learning activities from informal level to formal level. The conjecture of student thinking is dynamic with the result that can be adjusted to the act of learning and revised during teaching experiment.

In the second step namely teaching experiment, the learning design from preliminary design is tested to explore and to know students' strategies in learning addition of numbers 1 to 20. There are two stages, pilot experiment and teaching experiment. Pilot experiment is as the first cycle. HLT was tested to 6 students who did not come from teaching experiment class. It conducted to improve the quality of HLT which can be used in actual teaching process as the second cycle. Those six selected students were chosen by teacher who knew their abilities. They consisted of low, medium and high ability students.

In this study, there are three instructional activities which were implemented in the second cycle to develop students' strategies in addition of number 1 to 20. However, in this paper, we only discuss and focus on one of those three instructional activities, namely Ten-Box Activity. In this activity, students were asked to count balls in worksheet. There are two kinds of balls, red balls and blue balls. To bridging students into ten-structure, teacher and researcher provided the ten-box in worksheet.

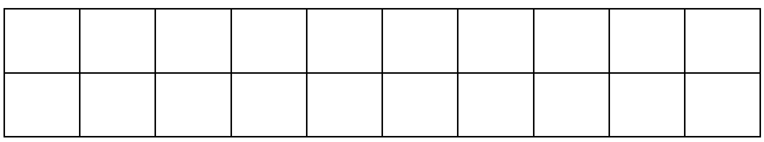

Figure 1. The ten-box

There were three problems on this activity. All of these problems were implemented in one meeting. The first problem was to determine the number of red and blue balls provided in the worksheet. In this problem, the activity began with contextual problem in order to make students interesting in following the learning activity. They have to find the number of balls by arranging the balls in the box. Teacher asked them to find easier arrangement that they can arrange so they will be easier to find the number of balls. From this problem, we hope students could understand and find that they can use another way, ten-structure, to find the number of balls by involving spatial structure.

After finding that ten-structure could they use to find the number of balls, students continued their activity by determining the number of unstructured balls in the boxes. The purpose of this activity is to explore students understanding of ten-structure and apply ten-structure to find the number of unstructured objects. In the worksheet, there were 2 groups of unstructured balls. Students have to find the number of balls in the boxes without counting one by one. They could apply some structure as a strategy to answer this problem.

In the last problem, students were asked to solve some problems of addition of number 1 to 20. In this problem, students not only determined the number of objects but also write down their strategies to solve that problem. Students were expected to be able to use the strategies which they have learned in the previous activities. Students did all this problems in 70 minutes.

Data was collected by interviewing the teacher and some students, observing the students' activities, taking some pictures and videos, and using field notes. In the first cycle, researcher was interviewed the teacher to find some information about the students' condition, materials, and teacher's statements about our HLT. Moreover, the students were interviewed to know the strategies that they always use to solve addition problem. In the second cycle, did the same interview to the teacher and six selected students who have different level of knowledge. Observation was conducted to the research subject in teaching experiment phase. Data from observation were collected by using observation sheets and field notes. In the beginning of the first and the second cycle, researcher gave pre-test for students before doing the interview. To support the data collection, some pictures and videos were taken during the activities either for focused-group or whole classroom. After teaching experiment, all data collected during learning process were analyzed.

3. Results

The findings of this study are (1) Ten-box can help students to find and explore ten-structure in determining the number of structured and unstructured objects; (2) Ten-structured is one of the spatial structure that can students use as a strategy to solve problem in addition of number 1 to 20; (3) spatial structure plays an essential role in bridging students to arrange structures and patterns that is useful for students in solving numerical problem especially addition of number 1 to 20.

Through a series of activities carried out in the first and the second activities, it appears that spatial structure can help students to develop their counting ability and addition strategies. Although students assisted with arranging ball in the box to build those structures, it appears that students can classify objects without being influenced by the box arrangement

In the first problem, students have been asked to find the number of red balls. Students explored by pasting and drawing red balls and blue balls into the box to determine the amount. In the beginning of the activity, students arranged and count the total number of 8 red balls and 9 red balls. Students used their own strategies that they usually do to solve addition problem by using their finger pattern. Some students did not aware that ten-box can help them to find the total number of red balls. Therefore, teacher repeated her explanation about what students have to be done. There was no problem when students arranged the balls on the boxes because they count the same color balls. Most of students put 10 balls on the first row of box and the rest in the second row. In this way, they could find that they can change the form of addition of 8 + 9 becomes 10 + 7. Because they put 10 red balls in the first row and 7 red balls in the second row. The problem appeared when there was one group that made that arrangement but looked confused to find the rest of balls that they can put on the second row. The following conversation shows that problem.

Teacher : How many balls in the second row?

Students: Nine

Teacher : Hm? How many balls do you put on the first row?

Students: Ten

Teacher : Ten, so how many the will you put on the second row?

Students: (*looked confused*)

Teacher : How many the red balls all over?

Students: Twenty (*Her answer was wrong)*

Teacher : Hm? How many 8 red balls and 9 red balls?

Students : Seventeen

Teacher : On the first row?

Students: Ten

Teacher : Ten. So on the second row?

Students: It is… (*think hard*)

Teacher : How many do you still need?

Students: Seven

Teacher : So, in the second row, you need?

Students: Seven.

From this conversation, we can find information that ten-box can help students to rearrange the addition form of 8 + 9. Although students looked confused to find the rest of the red balls in the second row, they still could find that 8 + 9 has result as same as 10 + 7.

In the next activity, students arranged balls which have different color. They counted how many 9 blue balls and 2 red balls in a total. Some students can find the arrangement which what teacher expected by putting 9 blue balls and 1 red balls in the first row. The rest of red balls, one, they put in the second row. So they can find 9 + 2 is equal to 10 + 1. They understood that they can find another form of addition 9 + 2. Addition 10 + 1 is an easier form to count. The following figure is the student's arrangement.

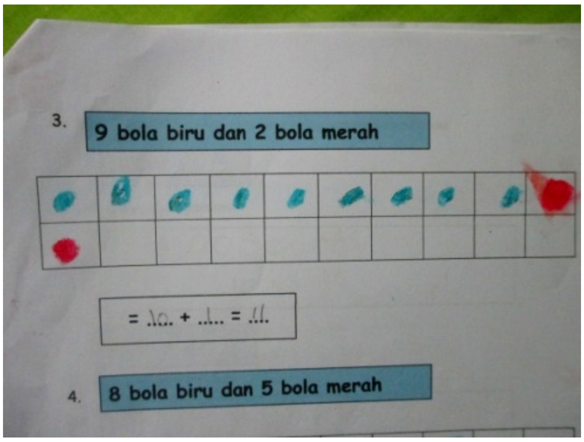

Figure 2. Student's arrangement in second problem

The difficulties that student faced in this activity was that students tend to be affected with the color of the balls. They still put the red balls and the blue balls into different row of box. For instance, in order to find the total number of 9 blue balls and 2 red balls, they put blue balls in the first row of box and 2 red balls in the second rows. Furthermore, they did not use the structure of ten-box to find a better strategy (ten-structure). Students prefer to use their own strategy by counting with finger pattern or counting one by one. Not only using finger pattern, but also students use the strategy that their teacher always uses namely *counting on from the first number* or *counting from the large number.* The following picture shows the students mistakes in arranging the balls in ten-box.

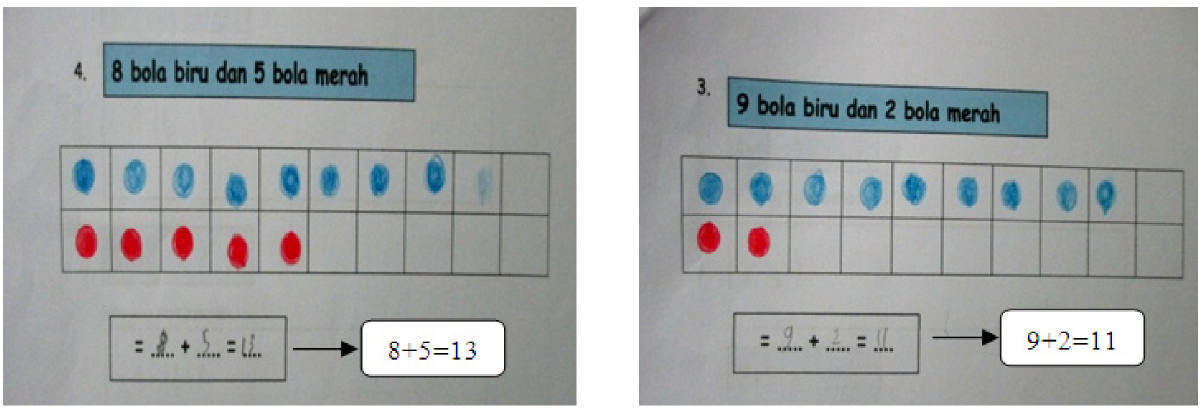

Figure 3. Students' mistake in arranging the balls in the ten-box

In determining the number of unstructured balls in the box, some of students used ten-structure by encircling and grouping ten balls into one group and add it with the rest. Some strategies students used to find those ten balls. There were students applying double-structure which they learned in the previous activity. However, the other students sometimes counted one by one to find the ten balls. The Figure below showed student's strategies to count the unstructured balls.

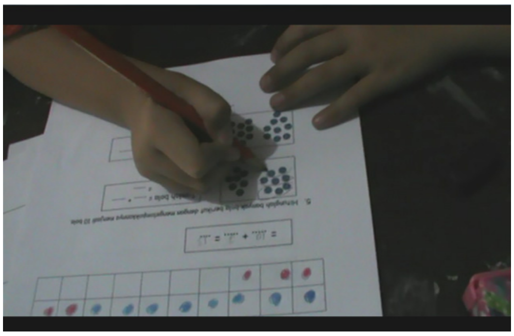

Figure 4. Student counted unstructured balls

From this activity, we can find that students were able to apply some structures to find another structure. Like what student did in the Figure 4 where student used double-structure and five-structure to make ten-structure to facilitate them in counting the number of unstructured balls. In the previous activities, students have explored their knowledge about five-structure and double structure in different activities. These structures are able to help them to find ten-structure rather than using counting one by one. The first way that students did was to make a double-structure of objects, for example grouping the objects into three and encircling them to separate them with other objects. Then, if ten objects were enough, they combined some groups of three objects, so they can find another group which consists of ten objects. Thus, they just added ten objects and the rest. From this information, we can find that students have knowledge to connect some structures in developing better strategies in counting the number of objects.

The other arrangement in the second problem was determining the sum of 9 blue balls and 6 red balls. Students classified blue balls and red balls in the form of structure of ten. However, they did not put ten balls in the first row and the rest in the second row, like in Figure 2. It also appeared strategy which connected with the structure of five and double structure.

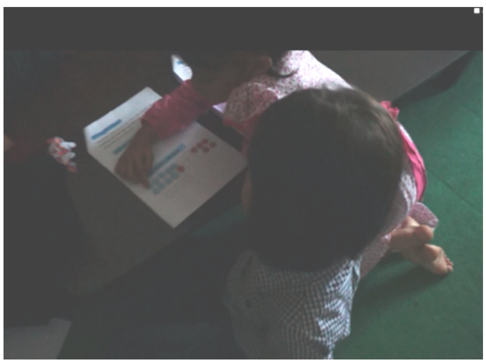

Figure 5. The other arrangement of the structure of ten arranging by students

In the last problem, students tried to solve some problems in addition number 1 to 20. In the worksheet, there were some problems that students have to solve and they also need to write down their strategies. Some strategies students used in this part. Most of them applied the ten-structure by represented the number by drawing some balls and rearranging them into group of ten. While doing those activities, the researcher tried to explore the student's reasons why they used the structure of ten as a strategy to solve the formal form of addition in student worksheet. The following is the conversation between the researcher and students.

Researcher: Why can 7 plus 9 become 10 plus 6?

Venia : Because they are also very easy.

Researcher: Very easy? 7 plus 9 is change into 10 plus 6?

Venia : Yes.

Researcher: Hm. Why become 10? Where does it come from?

Venia : Here.. (*while pointing her worksheet*). Because 10, plus 1, so it is ten right? Because it is ***expelled***. ***Expelled 1*** (*while pointing 7 on the problem*).

Researcher: ***Expelled*** where?

Venia : Eh no, no. Do not.

Putri : We save it

Venia : This 1 is moved here, so it will be 10. Then this one becomes 6. So, 16

It appears above that the student, Venia, was able to express her reason why and from where the structure of ten she used. The student was able to bring it by using their own language. She used the term "expelled" and "moved" to explain that 10 in 10 plus 6 derived from 7 on the problem. By using terms "expelled" and "moved", she showed how she was able to arrange this structure of ten without using ball box. She could arrange their own structure in her mind that can help her find the answer. She does not need to rearrange the balls into ten-structure, but she can just remember the pattern how to make and change the addition pattern into ten-structure.

4. Discussion

The pretest and the interview provided information that most of students used one-by-one counting strategy to solve addition problems. Most of students used the strategy which teacher gave to them in the class. They counted by using finger pattern. The first number will be remembered then the second number will be added by the help of finger pattern. After doing Ten-box Activity, students are able to use and to choose the best structure to be their strategy in solving addition problems.

Based on the activities undertaken by students, it can be concluded that spatial structuring ability can help students to organize objects so that they can find easily the better strategy to count the objects. Students no longer use one-by-one counting, but organize objects by arranging structures that is easier to count. The use of arranging ball context in ten-box activity can help students to discover the ten-structure as strategies in addition number 1 to 20. In the beginning of this activity, students got some difficulties and did not aware that ten-box can help them to arrange ten-structure. Some of them still arrange the balls in the boxes by separate them by the color. So, in order to find the answer of the total number of objects, they still use their own strategies. But after solving some problems, student can understand and discover that ten-structure can help them to find another strategy in addition of number 1 to 20.

The use of ten-structure also students applied in determining the number of unstructured objects. In solving this problem, students could explore their knowledge and use some structures to find the number of unstructured objects. Students not only used ten-structured but also they combined some structures, for instance finger pattern, five-structure, and double structure to arrange ten structure. In this way, students were aware that to find the number of more objects, counting one by one was not effective. So they have to find another strategy to abbreviate their numerical procedures. By applying the structured that they have learned in the previous meeting, such as five-structure, and double structure, they can arrange ten-structure and find the answer easily.

The interesting thing that researcher and teacher found during this activity is that the students are very attractive and happy to explore the structure that they can use in abbreviating their numerical procedures especially in addition of number 1 to 20. However, students were not confident to share their idea bravely. Sometimes, teacher needs to give some question and provokes students to speak. In order to help students in sharing their ideas, teacher invited students to discuss their answer on the worksheet and gave opportunities to students in front of the class. During this discussion, teacher always gave motivation so students looked very enthusiastic to follow this activity. Many students raised their hands to get the opportunities to give and explain their answer in front of the class. But sometimes, teacher asked some students who look unconfident to answer the questions. In this activity, teacher played her role as a motivator and facilitator very well. She can handle the class and give the opportunities to students without differentiate the students. Moreover, most of opportunities are for unconfident students in order to build up their motivation to study mathematics.

As a recommendation, the RME approach can be a basis and an approach in teaching addition 1 to 20. However, learning addition in lower grade is still considered as the most difficult topic for some students. By designing an interactive learning, students will understand the concept and develop strategies in addition number 1 to 20 better. Those ways are applying the RME approach and involving spatial structuring ability of students. By linking the addition of number 1 to 20 with spatial structures, students are expected to develop their abilities in counting number 1 to 20 and not to be influenced to use one-by-one counting.

Acknowledgements

The authors would like to express our gratitude to Ms. Siti Aisyah and her students for participating in this study. The activities were conducted within the IMPoME program and were funded by DIKTI, Indonesia.

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